Mathematics in Landscape Design and Architecture

- **doc. Mgr. Mária Ždímalová, Phd.**
- Associate professor
- Slovak University of Technology in Bratislava, Slovakia
- Faculty o Civil Engineering
- Department of Mathematics and Descriptive Geometry
- zdimalova@math.sk, maria.zdimalova@stuba.sk

- We discuss following topics:
- **Fractals and its application in garden and landscape architecture.**
- **Golden section.**
- **Symmetries in garden and landscape architecture.**
- **Polygons and geometric solids and its applications to gardens architecture.**
- **Surfaces and curves of analytic and “free” form in garden and landscape architecture.**

- Garden and landscape architecture is a part of architecture and urban design.

- **Mathematical aims:**

- To study the mentioned topics and its applications in gardens: polygons, tessellations, geometric solids, symmetries, mappings, fractals, surfaces, curves and surfaces of free form, lights and shadows, geometrical shapes, golden section…in garden and landscape architecture.

Fractlas in garden and landscape architecture

- The Kaos Garden (Ponte de Lima, Portugal) is an interpretation of the theme belongs to the Fractal Geometry. “The Kaos of the Universe” is a fractal that turns into plants, paths, floors, and globes that offer, through their apparent confusion, the idea of a Cosmic Chaos. Aromatic species, herbs and covering plants are combined with light silver and coloured globes that want to obtain an ideal representation of the stars and the planets of the universe. This leads to a question:

- What is a fractal?

- Is a natural phenomenon or a mathematical set that exhibits a repeating pattern that is displayed at every scale. If the replication is exactly the same at every scale, it is called a self-similar pattern. Fractals can also be nearly the same at different levels. It is also known as expanding symmetry or evolving symmetry. If the replication is exactly the same at every scale, it is called a self-similar pattern. Easily said fractals includes the idea of a detailed pattern that repeats itself. This latter pattern is illustrated in the small magnifications of the Mandelbrot set.

- Fractals are different from other geometric figures because of the way in which they scale. Doubling the edge lengths of a polygon multiplies its area by four, which is two (the ratio of the new to the old side length) raised to the power of two (the dimension of the space the polygon resides in). Likewise, if the radius of a sphere is doubled, its volume scales by eight, which is two (the ratio of the new to the old radius) to the power of three (the dimension that the sphere resides in).

- But if a fractal's one-dimensional lengths are all doubled, the spatial content of the fractal scales by a power that is not necessarily an integer. This power is called the fractal dimension of the fractal, and it usually exceeds the fractal's topological dimension.

- As mathematical equations, fractals are usually nowhere differentiable. An infinite fractal curve can be conceived of as winding through space differently from an ordinary line, still being a 1-dimensional line yet having a fractal dimension indicating it also resembles a surface.

- The mathematical roots of the idea of fractals have been traced throughout the years as a formal path of published works, starting in the 17th century with notions of recursion, then moving through increasingly rigorous mathematical treatment of the concept to the study of continuous but not differentiable functions in the 19th century by the seminal work of Bernard Bolzano, Bernhard Riemann, and Karl Weierstrass, and on to the coining of the word fractal in the 20th century with a subsequent burgeoning of interest in fractals and computer-based modelling in the 20th century.

- The term "fractal" was first used by mathematician Benoît Mandelbrot in 1975. Mandelbrot based it on the Latin fractus meaning "broken" or "fractured" and used it to extend the concept of theoretical fractional dimensions to geometric patterns in nature. Fractal patterns have been modeled extensively, albeit within a range of scales rather than infinitely, owing to the practical limits of physical time and space.

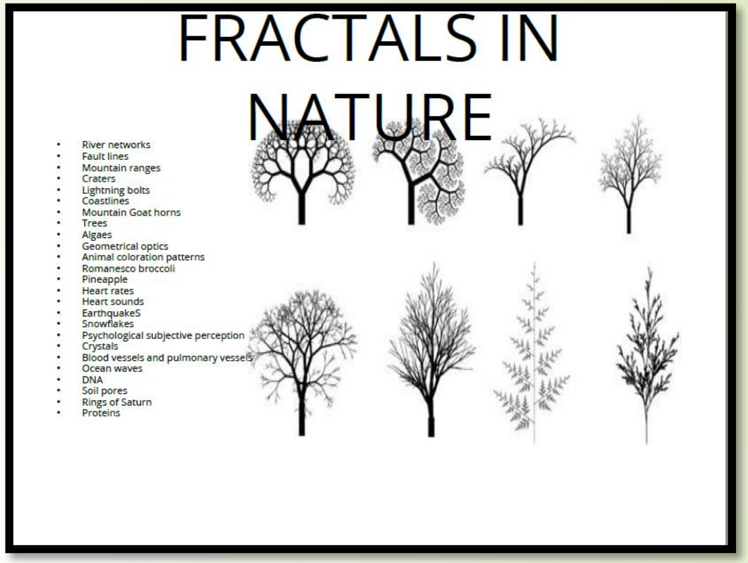
- Models may simulate theoretical fractals or natural phenomena with fractal features. The outputs of the modelling process may be highly artistic renderings, outputs for investigation, or benchmarks for fractal analysis. Some specific applications of fractals to technology are listed elsewhere. Images and other outputs of modelling are normally referred to as being "fractals" even if they do not have strictly fractal characteristics, such as when it is possible to zoom into a region of the fractal image that does not exhibit any fractal properties. Also, these may include calculation or display artifacts which are not characteristics of true fractals.

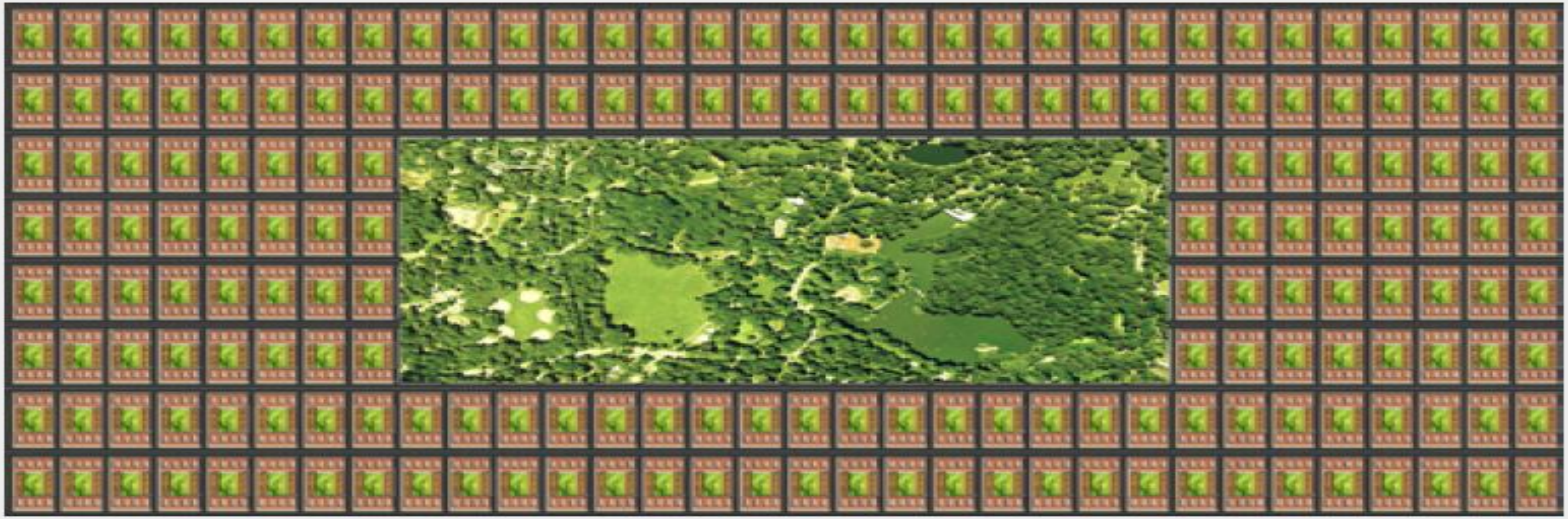
- Modeled fractals may be sounds, digital images, electrochemical patterns, circadian rhythms, etc. Fractal patterns have been reconstructed in physical 3-dimensional space and virtually, often called "in silico" modeling. Models of fractals are generally created using fractal-generating software that implements techniques such as those outlined above. As one illustration, trees, ferns, cells of the nervous system, blood and lung vasculature, and other branching patterns in nature can be modeled on a computer by using recursive algorithms and L-systemstechniques.

- The recursive nature of some patterns is obvious in certain examples—a branch from a tree or a frond from a fern is a miniature replica of the whole: not identical, but similar in nature. Similarly, random fractals have been used to describe/create many highly irregular real-world objects.

- Benoit B. Mandelbrot (20 November 1924 – 14 October 2010) was a Polish-born, French and American mathematician with broad interests in the practical sciences, especially regarding what he labeled as "the art of roughness" of physical phenomena and "the uncontrolled element in life."He referred to himself as a "fractalist". He is recognized for his contribution to the field of fractal geometry, which included coining the word "fractal"', as well as developing a theory of "roughness and self-similarity" in nature.

- He set is closely related to the idea of Julia sets, which produce similarly complex shapes.
- Its definition and name are due to Adrien Douady, in tribute to the mathematician Benoit Mandelbrot.
- Mandelbrot set images are made by sampling complex numbers and determining for each whether the result tends towards infinity when a particular mathematical operation is iterated on it. Treating the real and imaginary parts of each number as image coordinates, pixels are colored according to how rapidly the sequence diverges, if at all.





Daniel Lordick, Tu Dresden, Architectural Fractals

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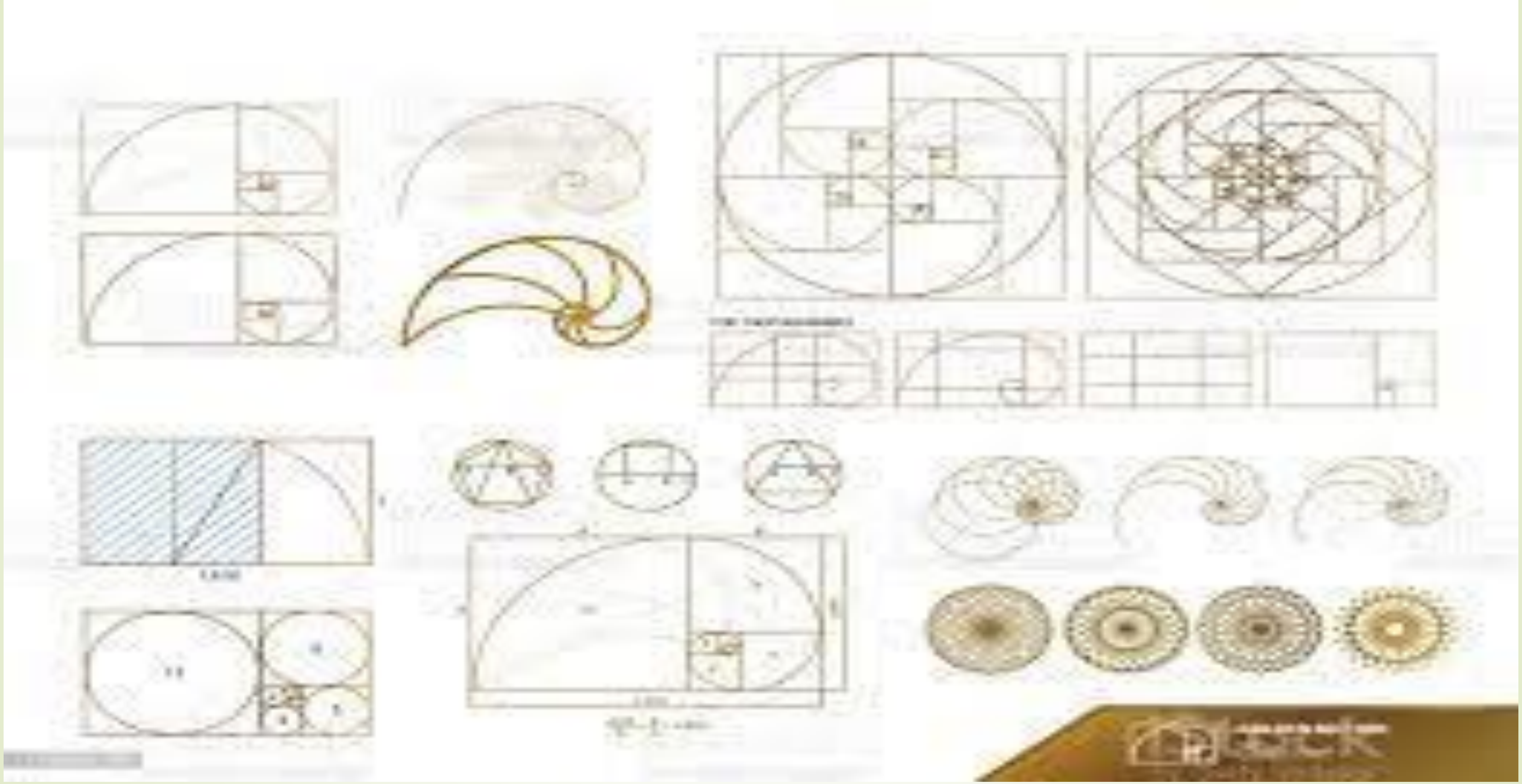
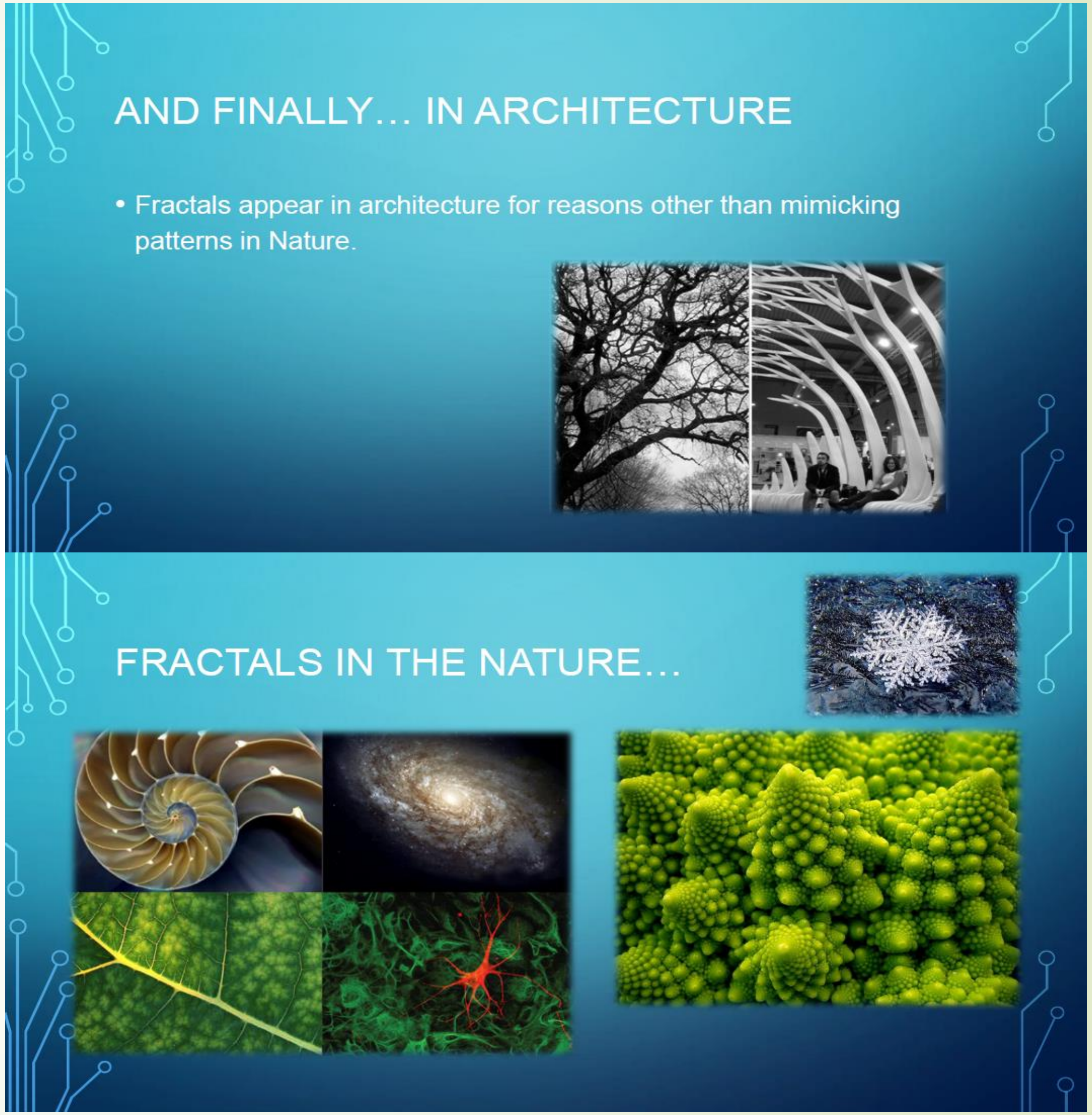


And finally in architecture:

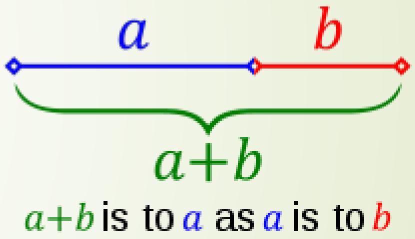
Fractals appear in architecture for reasons other than mimicking patterns in Nature.



Golden ration

- In mathematics, two quantities are in the **golden ratio** if their ratio is the same as the ratio of their sum to the larger of the two quantities. The figure on the right illustrates the geometric relationship. Expressed algebraically, for quantities a and b with $a > b > 0$

$$\frac{a+b}{a} = \frac{a}{b} = \varphi.$$



- where the Greek letter phi φ represents the golden ratio. It is an irrational number with a value of φ.

- Two quantities *a* and *b* are said to be in the *golden ratio* φ if

$$\frac{a+b}{a} = \frac{a}{b} = \varphi.$$

- One method for finding the value of φ is to start with the left fraction. Through simplifying the fraction and substituting in b/a = 1/φ,

$$\frac{a+b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\varphi}.$$

- Therefore,

$$1 + \frac{1}{\varphi} = \varphi.$$

- Multiplying by φ gives

$$\varphi + 1 = \varphi^2$$

- which can be rearranged to

$$\varphi^2 - \varphi - 1 = 0.$$

- Using the quadratic formula, two solutions are obtained:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.61803\,39887\ldots$$

- and

$$\varphi = \frac{1-\sqrt{5}}{2} = -0.6180\,339887\ldots$$

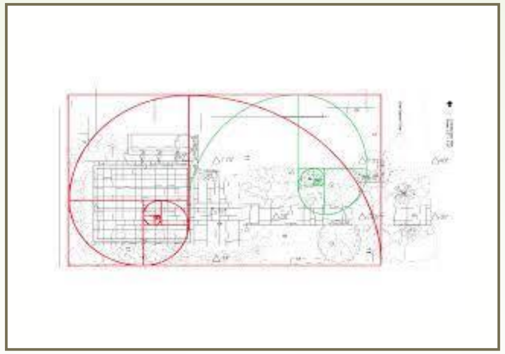
- Because φ is the ratio between positive quantities φ is necessarily positive:

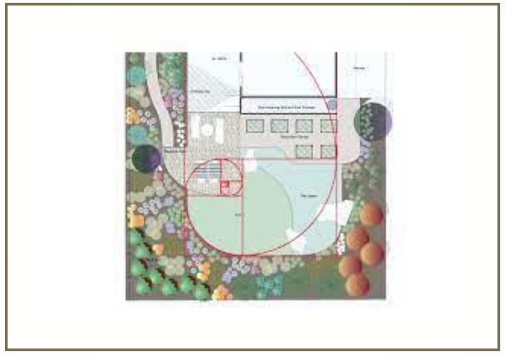
$$\varphi = \frac{1+\sqrt{5}}{2} = 1.61803\,39887\ldots$$

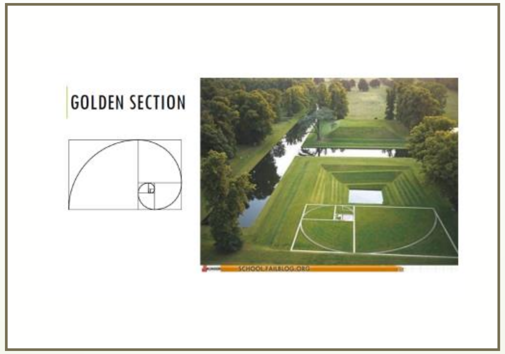
- The golden ratio is also called the golden mean or golden section (Latin: sectio aurea). Other names include extreme and mean ratio medial section, divine proportion, divine section (Latin: sectio divina), golden proportion, golden cut, and golden number.

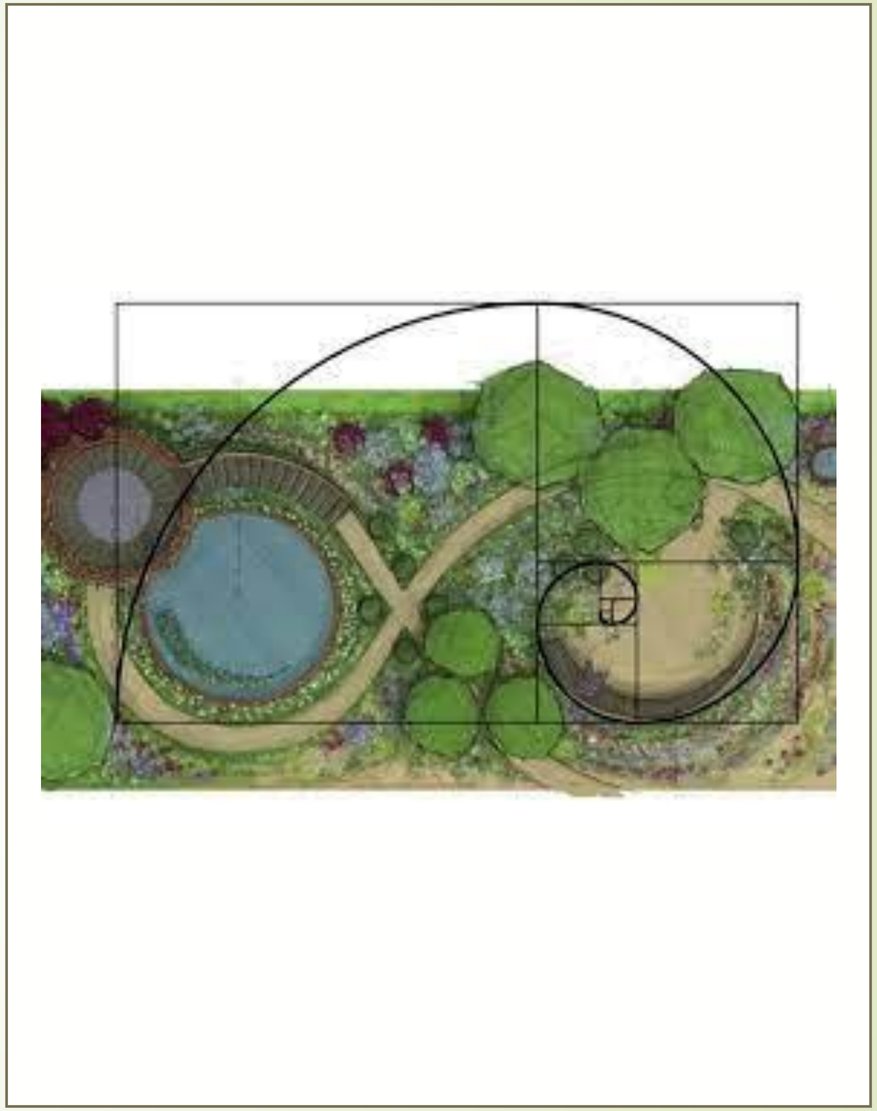
- Some twentieth-century artists and architects, including Le Corbusier and Dalí, have proportioned their works to approximate the golden ratio—especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the golden ratio—believing this proportion to be aesthetically pleasing. The golden ratio appears in some patterns in nature, including the spiral arrangement of leaves and other plant parts.

- Mathematicians since Euclid have studied the properties of the golden ratio, including its appearance in the dimensions of a regular pentagon and in a golden rectangle, which may be cut into a square and a smaller rectangle with the same aspect ratio. The golden ratio has also been used to analyze the proportions of natural objects as well as man-made systems such as financial markets, in some cases based on dubious fits to data.









- The golden ratio has been claimed to have held a special fascination for at least 2,400 years, although without reliable evidence. According to Mario Livio:

- Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece, through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler, to present-day scientific figures such as Oxford physicist Roger Penrose, have spent endless hours over this simple ratio and its properties.

- But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.

- Ancient Greek mathematicians first studied what we now call the golden ratio because of its frequent appearance in geometry. The division of a line into "extreme and mean ratio" (the golden section) is important in the geometry of regular pentagrams and pentagons. Euclid's Elements provides the first known written definition of what is now called the golden ratio:

- A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.

- Euclid explains a construction for cutting (sectioning) a line "in extreme and mean ratio" (i.e., the golden ratio).[Throughout the Elements, several propositions (theorems in modern terminology) and their proofs employ the golden ratio.

- The golden ratio is explored in Luca Pacioli's book De divina proportione (1509).

- The first known approximation of the (inverse) golden ratio by a decimal fraction, stated as "about 0.6180340", was written in 1597 by Michael Maestlin of the University of Tübingen in a letter to his former student Johannes Kepler.

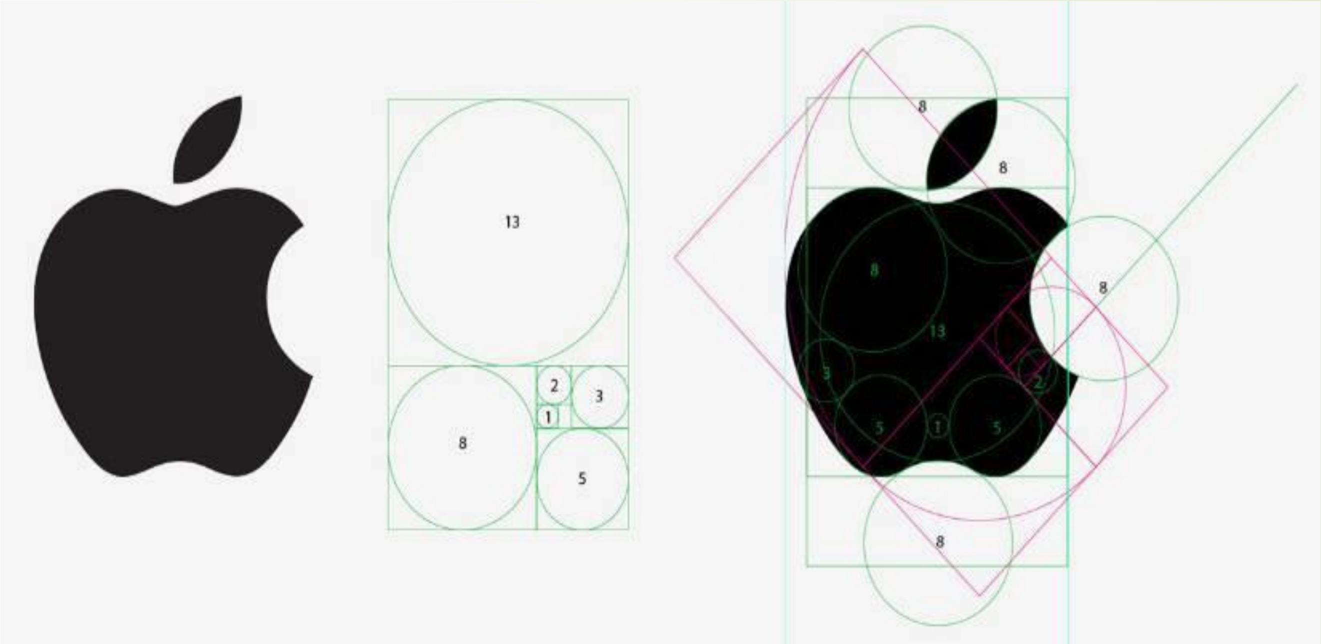
- Since the 20th century, the golden ratio has been represented by the Greek letter φ (phi, after Phidias, a sculptor who is said to have employed it) or less commonly by τ (tau, the first letter of the ancient Greek root τομή—meaning cut).

Book Design

- There was a time when deviations from the truly beautiful page proportions 2:3, 1:√3, and the Golden Section were rare. Many books produced between 1550 and 1770 show these proportions exactly, to within half a millimeter.

Brand Design

- Some sources claim that the golden ratio is commonly used in everyday design, for example in the shapes of postcards, playing cards, posters, wide-screen televisions, photographs, light switch plates and cars.



Perceptual studies

- Studies by psychologists, starting with Fechner, have been devised to test the idea that the golden ratio plays a role in human perception of beauty. While Fechner found a preference for rectangle ratios centered on the golden ratio, later attempts to carefully test such a hypothesis have been, at best, inconclusive.
- **Irrationality**

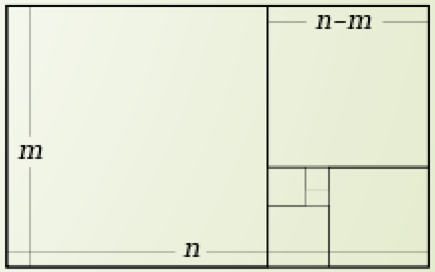
  The golden ratio is an irrational number. Below are two short proofs of irrationality:
- **Contradiction from an expression in lowest terms**

  Recall that:

  the whole is the longer part plus the shorter part;the whole is to the longer part as the longer part is to the shorter part.If we call the whole n and the longer part m, then the second statement above becomes

  n is to m as m is to $n - m$,or, algebraically

$$\frac{n}{m} = \frac{m}{n-m}. \qquad (*)$$



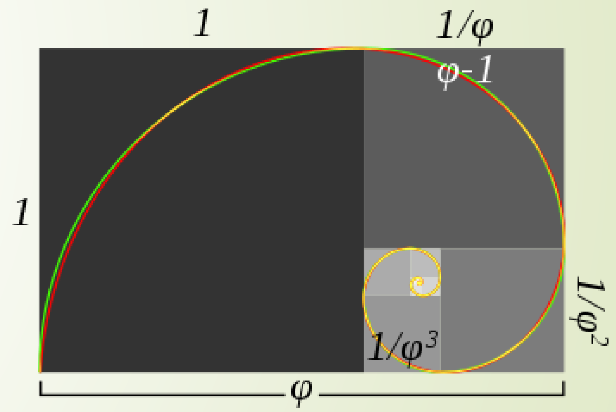
- To say that φ is rational means that φ is a fraction n/m where n and m are integers. We may take n/m to be in lowest terms and n and m to be positive. But if n/m is in lowest terms, then the identity labeled (*) above says $m/(n - m)$ is in still lower terms. That is a contradiction that follows from the assumption that φ is rational.

Fibonacci numbers and Geometry

- The convergents of these continued fractions (1/1, 2/1, 3/2, 5/3, 8/5, 13/8, ..., or 1/1, 1/2, 2/3, 3/5, 5/8, 8/13, ...) are ratios of successiveFibonacci numbers.

- The number φ turns up frequently in geometry, particularly in figures with pentagonal symmetry. The length of a regular pentagon's diagonal is φ times its side. The vertices of a regular icosahedron are those of three mutually orthogonal golden rectangles.

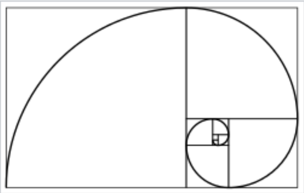
- There is no known general algorithm to arrange a given number of nodes evenly on a sphere, for any of several definitions of even distribution (see, for example, *Thomson problem*). However, a useful approximation results from dividing the sphere into parallel bands of equal surface area and placing one node in each band at longitudes spaced by a golden section of the circle, i.e. 360°/φ ≅ 222.5°. This method was used to arrange the 1500 mirrors of the student-participatory satelliteStarshine-3.



- The mathematics of the golden ratio and of the **Fibonacci sequence** are intimately interconnected.

- The Fibonacci sequence is:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987,....



A Fibonacci spiral which approximates the golden spiral, using Fibonacci sequence square sizes up to 34. The spiral is drawn starting from the inner 1×1 square and continues outwards to successively larger squares.

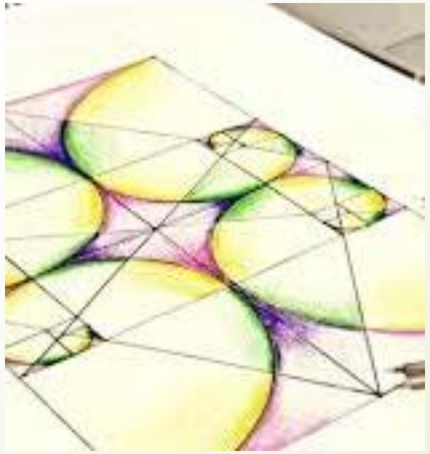
$$\lim_{n\to\infty} \frac{F(n+1)}{F(n)} = \varphi.$$

The closed-form expression for the Fibonacci sequence involves the golden ratio:

$$F(n) = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}.$$

The golden ratio is the limit of the ratios of successive terms of the Fibonacci sequence (or any Fibonacci-like sequence), as originally shown by Kepler.

Golden Spiral in Landscape Design and Architecture



Golden Spiral in Landscape Design and Architecture

Golden Section
Golden Spiral
in Gardens

Golden Spiral in Landscape Design and Architecture

Geometric symmetries

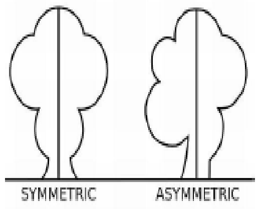
-are often presented in the gardens architecture like:

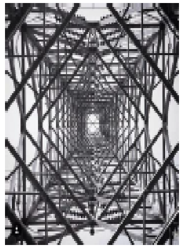
-point symmetrie: e.g. a water fontaine in the middle of the garden, big central pattern made from plants, a big central geometric shape represented by plants or trees.

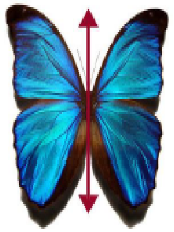
-reflection symmetrie: a path in the middle of the garden, with different colours, materials which produce contrast in the garden, path from small rocks, from trees cutted and formed trees in special shapes, a bridge over small river in the garden, path of flowers of the same colours and high. We also can consider garden mazes.

Symmetry

- Definition - an object is invariant to any of various geometric transformations : Reflection(Mirror), Rotation, Translation, Glide reflection
- symmetries preserve distances, angles, sizes, and shapes
- The mathematical study of symmetry is systematized and formalized in a group theory.
- Symmetric patterns occur in nature, and are invented by artists, craftspeople, musicians, choreographers, and mathematicians.

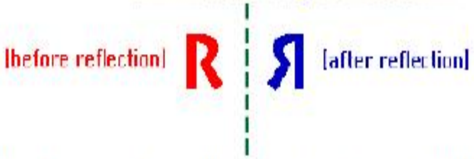






Reflection Symmetry

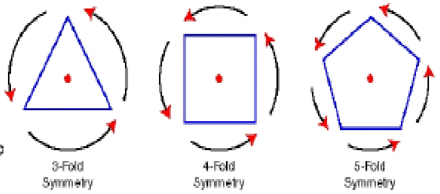
- = Line Symmetry = Mirror Symmetry
- An object is reflected across a line, which divides the shape into two identical parts.
- The line can be vertical, horizontal or diagonal.

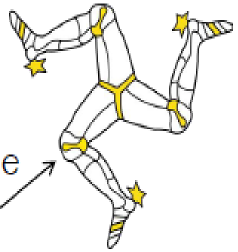


reflection of an "R" is a backwards "R"

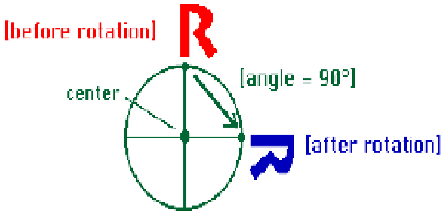
Rotation Symmetry

- The turning of an object or coordinate system by an angle about a fixed point (center).
- An object's degree of rotational symmetry is the number of distinct orientations in which it looks the same.





Because its appearance is identical in three distinct orientations, its rotational symmetry is three-fold.

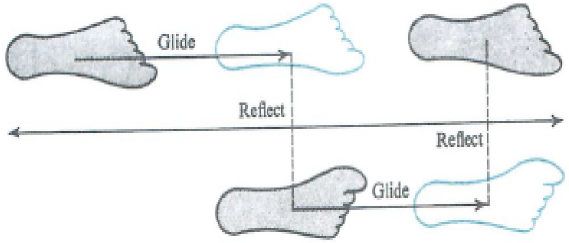


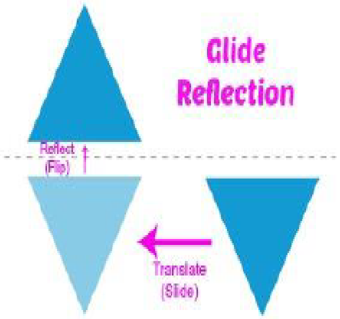
Symmetry in garden architecture

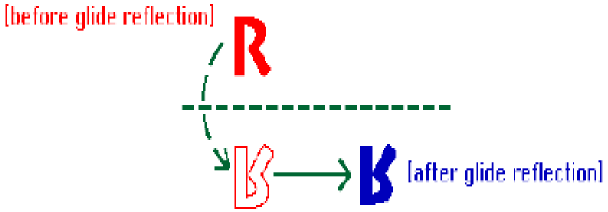
- Symmetry is one of the most common and used features in architecture in general - brings harmony and balance.
- It is the reflection of shared forms, shapes, or angles across a central line or point called the **axis.**
- Style of garden based on symmetry and the principle of imposing order on nature is e.g. **French formal garden** or **English garden.**

Glide reflection

- combines a reflection with a translation along the direction of the mirror line
- the order of these transformations does not matter
- only type of symmetry that involve more than one step



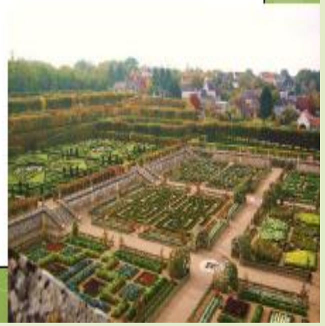




Examples of symmetrical gardens

Gardens of the Chateau de Villandry, France

- Formal French garden
- constructed by Jean Le Breton in 1536
- a large number of knot gardens, square gardens bordered by box hedging - filled in with different plants or vegetables, create very symmetrical gardens
- the gardens cover 9 hectares and include a water garden(ponds and fountain), an ornamental garden and vegetable garden



Examples of symmetrical gardens

Hidcote Manor Garden, England

- one of the best-known and most influential Arts and Crafts gardens in Britain
- linked "rooms„ (e.g. White Garden or Fuchsia Garden) of hedges, rare trees, shrubs and herbaceous borders
- Created by Lawrence Johnston
- owned by the National Trust

White Garden

Examples of symmetrical gardens

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Examples of symmetrical gardens

The Gardens of Versailles, France

- Classic French garden
- one of the most visited public sites in France
- designed by André Le Nôtre, the reign of Louis XIV
- 372 statues, 55 water features, 600 fountains and 35km of canals
- the garden covers 800 hectares which includes 300 hectares of forest and two 'French' gardens, the 'Petit Parc' and the 'Trianon'

Examples of symmetrical gardens

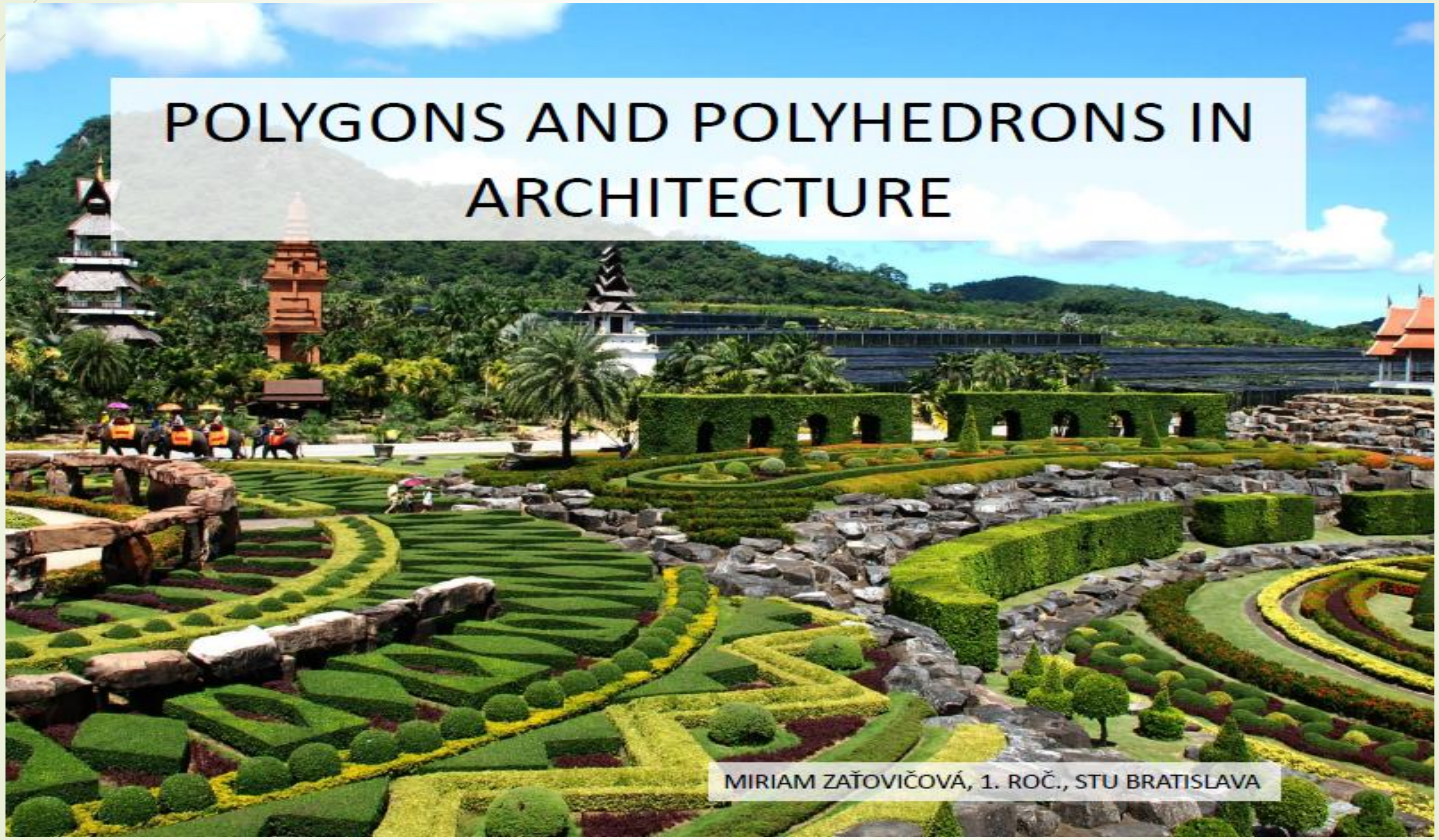
Monet's Garden at Giverny, France

- probably the most famous garden in France - 500 000 people visit every year
- two parts of garden : a) <u>flower garden(Clos Normand)</u> is about one hectare, full of perspectives, symmetries and colours -thousands of flowers - tulips, irises, oriental poppies and peonies

b) <u>water garden</u> – inspiration of Monet's wonderful water lily paintings, full of asymmetries and curves, inspired by the Japanese gardens

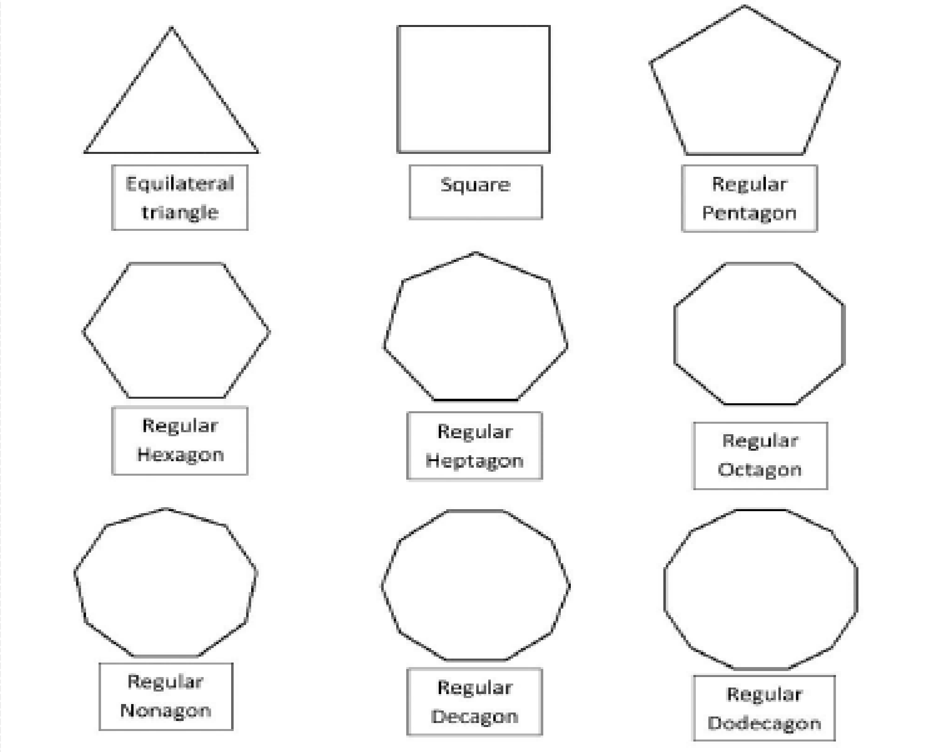
The Japanese bridge

Polygons, polyhedrons and geometric solids in landscape and garden architecture



POLYGON

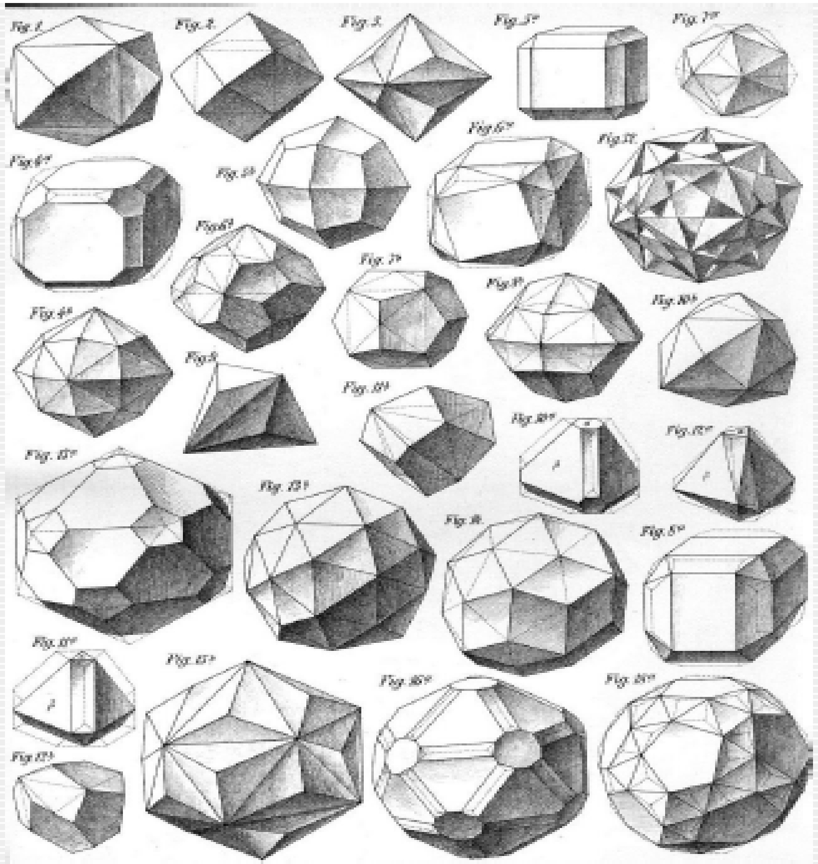
- is a plane figure that is bounded by a finite chain of straight line segments closing in a loop to form a closed chain or circuit
- segments are called its edges or sides, and the points where two edges meet are the polygon's vertices or corners
- an n-gon is a polygon with n sides



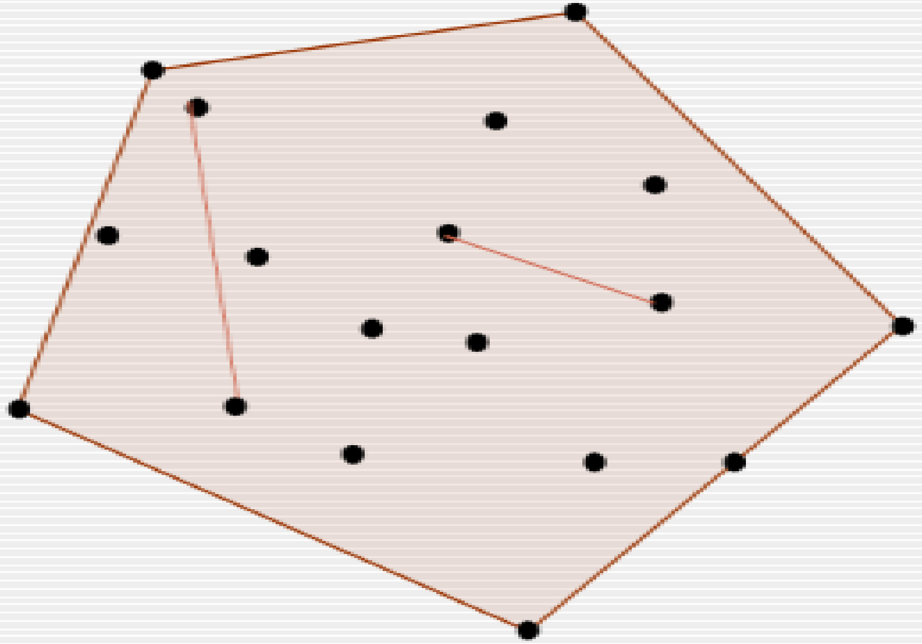
POLYHEDRON

- is a solid in three dimensions with flat polygonal faces

  meeting in pairs along their edges which are straight-line segments, and with the edges meeting in vertex points



CONVEX POLYGON

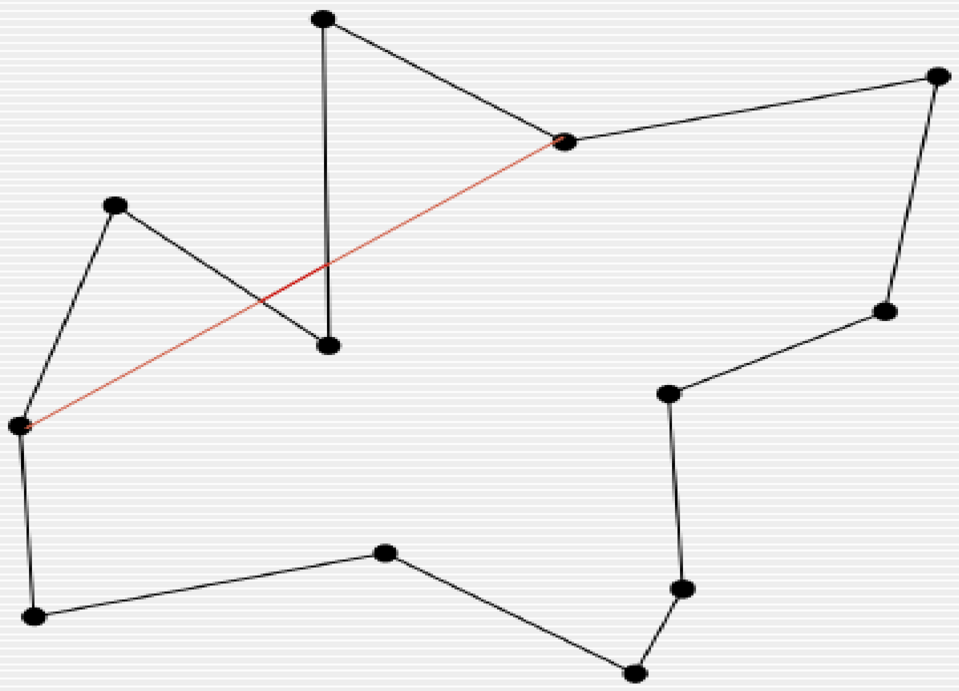


- any line drawn through the polygon (and not tangent to an edge or corner) meets its boundary exactly twice
- as a consequence, all its interior angles are less than 180°
- line drawn between two points inside polygon lies inside the polygon

NON-CONVEX POLYGON

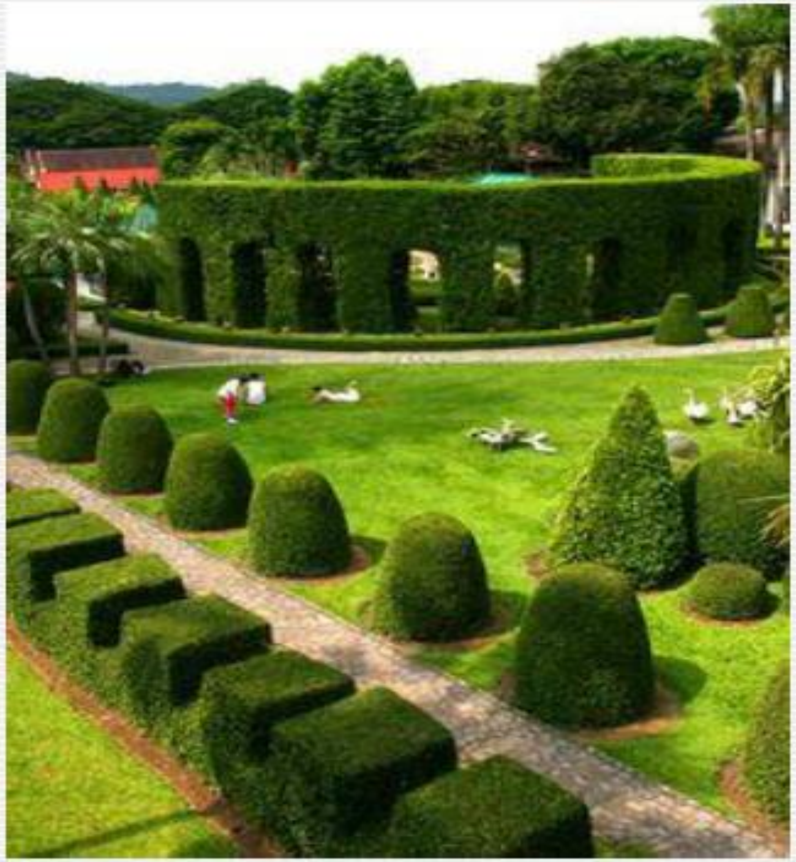
- a line may be found which meets its boundary more than twice

- equivalently, there exists a line segment between two boundary points that passes outside the polygon



NONG NOOCH TROPICAL GARDEN

- 500-acre (2.0 km^2) botanical garden and tourist attraction at kilometer 163 on Sukhumvit Road in Chonburi Province, Thailand
- Nongnooch Tansacha purchased the 600-acre (2.4 km^2) plot of land in 1954 with the intentions of developing the land as a fruit plantation
- However, instead decided to plant tropical flowers and plants as a wildlife conservation project



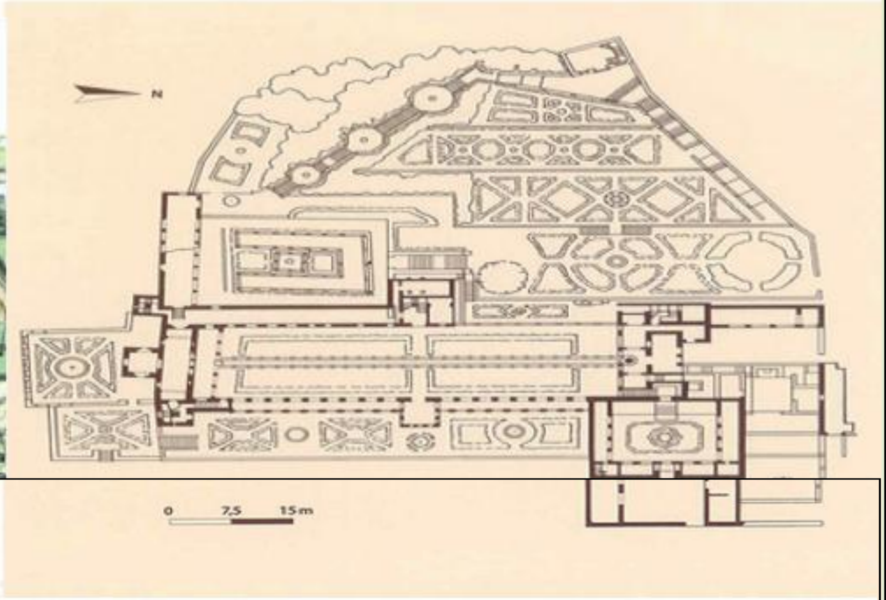
Garden of Cosmic speculation

The Garden of Cosmic Speculation is a 30 acre sculpture garden created by landscape architect and theorist Charles Jencks at his home, Portrack House, in Dumfriesshire, Scotland. Like much of Jencks' work, the garden is inspired by modern cosmology.

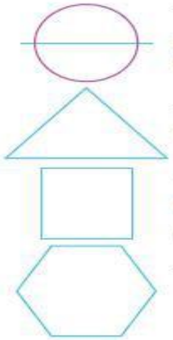
Gardens of Cosmic Speculation

Dubai Miracle Garden

Polygons in islamic gardens:



Alhambra did not have a master plan for the total site design, so its overall layout is not orthogonal nor organized. As a result of the site's many construction phases: from the original 9th-century citadel, through the 14th-century Muslim palaces, to the 16th-century palace of Charles V; some buildings are at odd positioning to each other. The terrace or plateau where the Alhambra sits measures about 740 metres (2,430 ft) in length by 205 metres (670 ft) at its greatest width. It extends from west-northwest to east-southeast and covers an area of about 142,000 square metres (1,530,000 sq ft). The Alhambra's most westerly feature is the alcazaba (citadel), a strongly fortified position. The rest of the plateau comprises a number of Moorish palaces, enclosed by a fortified wall, with thirteen towers, some defensive and some providing vistas for the inhabitants. The river Darro passes through a ravine on the north and divides the plateau from the Albaicín district of Granada. Similarly, the Assabica valley, containing the Alhambra Park on the west and south, and, beyond this valley, the almost parallel ridge of Monte Mauror, separate it from the Antequeruela district. Another ravine separates it from the Generalife.

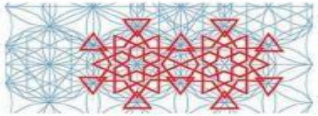


The Circle and its centre is symbol at which all Islamic patterns begin. It emphasizes one god.

The Triangle symbolises human and the principles of harmony.

The Square is the symbol of physical experience and the physical world or materiality.

The Hexagon symbolises heaven.



The Star symbolises the spread of Islam.

Surfaces and curves (of free form)

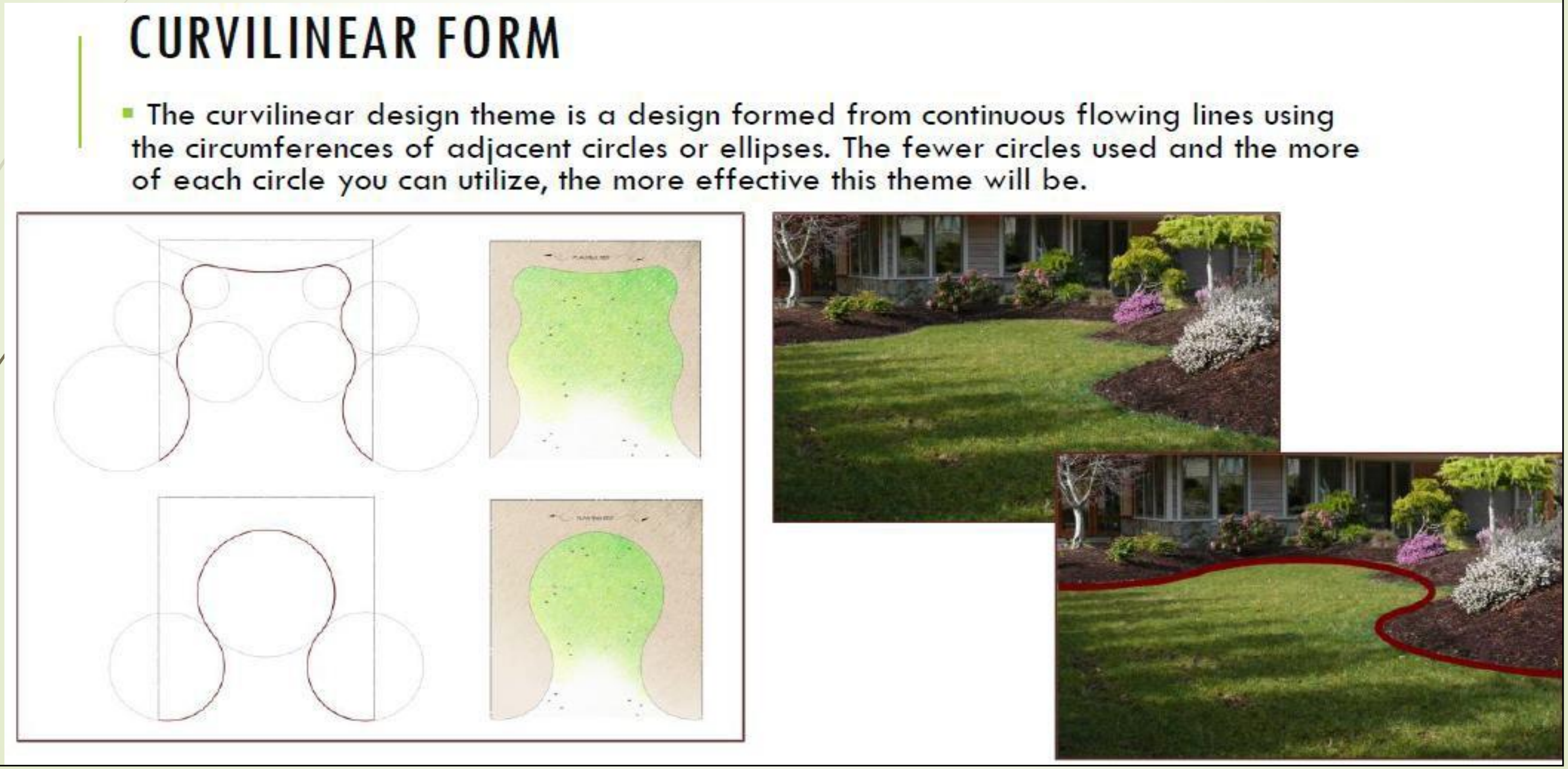
Surfaces and curves of implicit expression and also free form in gardens and landscape architecture

We can use curvilinears forms as well as different mathematical curves in the planning of garden and landscape architecture,

Example of mathematical curves in garden and landscape architecture



Curves of free form



- Example of mathematical curves in garden and landscape architecture

Geometrical curves and surfaces in landscape architecture:

Example of curves in garden architecture

Green wall in Emporia

GREEN WALL - EMPORIA

- Emporia is a shopping mall and one of the biggest ones in Scandinavia
- It is situated in the city of Malmö in Sweden
- Emporia opened on 25 October 2012 and the total construction expense was about 2 billion Swedish kronor
- the architect of the Emporia project is Gert Wingårdh of Wingårdh arkitektkontor

NANJING ZENDAI HIMALAYAS CENTER

- for an exhibition at The Venice architecture biennale 2014, MAD architects presents their latest project, the 'nanjing zendai himalayas center'
- the complex is estimated to be completed in 2017
- an achieved spiritual harmony between nature and humanity
- the site is composed of six zones, two of which are linked by a vertical public plaza. Curving pathways weave through the commercial complexes, which provide access from the busy ground level to the elevated park, where citizens can wander between buildings and gardens

Curves of free form and its application in garden and landscape design

Curves of free form and its application in garden and landscape design

THE AUSTRALIAN GARDEN

- World Landscape of the Year 2013
- by Taylor Cullity Lethlean and Paul Thompson
- The Royal Botanic Gardens is a division of the Royal Botanic Gardens, Melbourne. It is located in the suburb of Cranbourne, about 45 km south-east of the Melbourne city centre.
- The total area of this division of the botanic gardens is 363 hectares, including heathlands, wetlands and woodlands. The gardens also provide habitat for native birds, mammals and reptiles, including some rare and endangered species.
- A recent feature of the Royal Botanic Gardens is the specially constructed *Australian Garden*, opened to the public on May 2006
- The *Australian Garden* features a number of exhibition gardens, sculptures and displays aimed to bring the beauty and diversity of the Australian landscape and plants to the public

Thank you for your attention.