ON THE STRUCTURE OF ASSOCIATIVE n-DIMENSIONAL COPULAS

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The associativity of n-dimensional copulas in the sense of Post is studied. The structure of associative n-dimensional copulas is clarified. Recall that for $n \in \mathbb{N}$, $n \geq 2$, a function $C\colon [0,1]^n \to [0,1]$ is called an n-dimensional copula (n-copula, for short) if it satisfies the properties:

(C1) $C(x_1, \ldots, x_n) = x_i$ whenever $\forall j \neq i$, $x_j = 1$,

(C2) $C(x_1, \ldots, x_n) = 0$ whenever $0 \in \{x_1, \ldots, x_n\}$,

(C3) the n-increasing property, i.e., $\forall\, \mathbf{x}, \mathbf{y} \in [0,1]^n$, $x_i \leq y_i$, $i = 1, \ldots, n$, it holds

$$\sum_{J \subset \{1,\ldots,n\}} (-1)^{|J|} C\left(u_1^J, \ldots, u_n^J\right) \geq 0, \text{ where } u_i^J = \begin{cases} x_i, & \text{if } i \in J, \\ y_i, & \text{if } i \notin J. \end{cases}$$

By the Post definition of associative n-ary functions [4], an n-copula C is associative whenever for all $x_1, \ldots, x_n, \ldots, x_{2n-1} \in [0,1]$ it holds

$$C\left(C(x_1, \ldots, x_n), x_{n+1}, \ldots, x_{2n-1}\right) = C\left(x_1, C(x_2, \ldots, x_{n+1}), x_{n+2}, \ldots, x_{2n-1}\right)$$

$$= \ldots = C\left(x_1, \ldots, x_{n-1}, C(x_n, \ldots, x_{2n-1})\right).$$

We show that associative n-copulas are n-ary extensions of associative 2-dimensional copulas with special constraints. The main result solves an open problem formulated by R. Mesiar in [1].

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References

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