1. Zistite, či sú vektory lineárne závislé / nezávislé

A) $\vec{u} = (1,7)$ $\vec{v} = (2,-1)$ $\vec{w} = (1,1)$

$$k\vec{u} + l\vec{v} + m\vec{w} = \vec{0}$$

$$k(1,7) + l(2,-1) + m(1,1) = (0,0)$$
$$(k,7k) + (2l,-l) + (m,m) = (0,0)$$

$$k + 2l + m = 0 \Rightarrow k = -2l - m$$
$$7k + (-l) + m = 0$$

$$7(-2l - m) + (-l) + m = 0$$
$$-14l - 7m + (-l) + m = 0$$
$$-15l - 6m = 0$$

$\downarrow$

vieme nájsť nenulové l, m spĺňajúce toto?

$$l = 1$$
$$-15.1 - 6m = 0$$
$$-6m = 15$$
$$m = -\frac{15}{6}$$

Dopočítajme k:

$$k = -2l - m = -2 - \left(-\frac{15}{6}\right) =$$

$$= -2 + \frac{15}{6} = -\frac{12}{6} + \frac{15}{6} =$$

$$= \frac{3}{6} = \frac{1}{2}$$

Skúška správnosti

$$\frac{1}{2}\vec{u} + 1 . \vec{v} + \left(-\frac{15}{6}\right)\vec{w} = \vec{0} \quad ?$$

$$\frac{1}{2}(1, 7) + 1 . (2, -1) + \left(-\frac{15}{6}\right)(1, 1) =$$

$$\left(\frac{1}{2} + 2 - \frac{15}{6}, \frac{7}{2} - 1 - \frac{15}{6}\right) =$$

$$= \left(\frac{3}{6} + \frac{12}{16} - \frac{15}{6}, \frac{21}{6} - \frac{6}{6} - \frac{15}{6}\right) =$$

$$= (0, 0) \checkmark$$

(13) $\vec{u} = (1, 3, 2)$ $\vec{v} = (-1, 0, 1)$
$\vec{w} = (0, 1, 1)$

$$k\vec{u} + l\vec{v} + m\vec{w} = (0, 0, 0)$$

$$k(1, 3, 2) + l(-1, 0, 1) + m(0, 1, 2) = (0, 0, 0)$$

$$\left.\begin{aligned} k + (-l) + 0m &= 0 \\ 3k + 0l + m &= 0 \\ 2k + l + 2m &= 0 \end{aligned}\right\} \text{sloupce!} \; \text{''} \; \text{vektory}$$

$$k - l = 0 \implies l = k$$
$$3k + m = 0$$
$$2k + l + 2m = 0$$

$$3k + m = 0 \implies m = -3k$$
$$3k + 2m = 0$$

$$3k + 2.(-3k) = 0$$
$$3k + (-6)k = 0$$
$$(-3k) = 0$$
$$k = 0 \; ; \; m = 0, \; l = 0.$$

(2) Nech A, B, C, D sú matice

$$A = \begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Určte typy matíc

$A : 2 \times 2$ $\quad B = 2 \times 3$ $\quad C : 2 \times 1$

$D : 2 \times 2$

Vypočítajte (ak existujú)

$A + B$ (neex)

$A + D =$

$$\begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -2 & 9 \end{pmatrix}$$

$2B$

$$2\begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 4 \\ -2 & 0 & 2 \end{pmatrix}$$

$-2A + 3D = \ldots$ sami

$$A \cdot B \quad \begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 + (-3) & 1 + 0 & 2 + 3 \\ -2 + (-8) & -2 + 0 & -4 + 8 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 1 & 5 \\ -10 & -2 & 4 \end{pmatrix}$$

$B.A$

$$\begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix} = \text{neex.}$$

dĺžka riadku ≠ dĺžka stĺpca

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počet stĺpcov počet riadkov

$$\underset{m \times k}{U} . \underset{k \times n}{V} = \underset{m \times n}{W}$$

$A.A = A^2$

$$\begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix} = \begin{pmatrix} 1+(-6) & 3+24 \\ -2+(-16) & -6+64 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 27 \\ -18 & 58 \end{pmatrix}$$

$$A.D = \begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix}$$

$$D \cdot A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -2 & 8 \end{pmatrix}$$

③ Zvoľme „náhodné" dve matice typu 2×2 A, B

Vypočítajme AB, BA

$(AB)^T$ $B^T A^T$

④ Nech $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

nájdite maticu X takú, že $AX = C$

A) akého typu bude X?

1) Iste bude mať dva riadky (lebo A má 2 stĺpce)

$m \times k \quad k \times n \quad m \times n$

$2 \quad 2 \quad 2 \times 1 \quad 2 \times 1$

X mā 1 stâlpec!

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x_1 + 2x_2 = 1$$
$$2x_1 + x_2 = 1$$

$$-2x_1 - 4x_2 = -2$$
$$-3x_2 = -1$$

$$x_2 = \frac{1}{3}$$

$$2x_1 + \frac{1}{3} = 1$$

$$2x_1 = \frac{2}{3}$$

$$x_1 = \frac{1}{3}$$

$$X = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$