A note to the existence of n-dimensional s-maps on quantum logics

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In our contribution we will study multidimensional states on more general structures as Boolean algebra. We will focus to orthomodular lattice with at least one state. It is called quantum logic.

One possible approach to modelling of non-compatible events is studying of one-dimensional states, which represent probability measures. However, not all states have Jauch-Piron property (e.g.[2, 9, 10]). On the other hand, this property is fulfilled for any additive measure on any measurable space: $P(A) = P(B) = 1$ implies $P(A \cap B) = 1$.

Multidimensional states represent measures of intersection (s-map), union and symmetric difference in case of compatibility [4, 6, 8]. The modified Jauch-Piron property is fulfilled for each 2-dimensional s-maps on quantum logic [1]. Moreover, we can use multidimensional states to study non-compatible observables and stochastic causality of probability spaces (e.g.[3, 5]). For Boolean algebras we get the same results as in the classical probability theory.

Multidimensional states always exist for any countable dimensions on the classical probability space. However, it is not true in a quantum logic. Existence of 3-dimensional s-map with known 2- dimensional marginal s-map is solved in [7]. We will show the conditions of existence of n-dimensional s-map if we know the marginal $n-1$-dimensional s-map $(n > 3)$.

Indeed, we solve the following problem: Let $x_1, ..., x_n$ be discrete observables on quantum logic. Let p be $(n-1)$-dimensional s-map. The joint distribution always exists for observables $x_{k_1}, ..., x_{k_{n-1}}$, where $k_1, ..., k_{n-1} \in \{1, ..., n\}$. It is given by

$$p_{x_{k_1},...,x_{k_{n-1}}}(E_{k_1}, ..., E_{k_{n-1}}) = p(x_{k_1}(E_{k_1}), ., x_{k_{n-1}}(E_{k_{n-1}})),$$

where $E_i \in \mathcal{B}(\mathbb{R})$. This function is called *marginal joint distribution* of dimension $(n-1)$ for observables $x_1, ..., x_n$. We show conditions for the existence of n-dimensional s-map $p_{x_1,...,x_n}$ such that $p_{x_1,...,x_{k_{n-1}}}$ is its marginal s-map.

Acknowledgment
This contribution is supported by the VEGA grant agency under the contracts No. 1/0103/10 and and 1/0297/11.

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