OPERATOR EFFECT ALGEBRAS IN HILBERT SPACES

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In the clasical (Kolmogorovian) probability theory the set of events form a Boolean sigma-algebra. In a Boolean algebra every two elements are mutually compatible. Moreover,element of a Boolean algebra are sharp, meaning that elements have yes-no character. Another situation arises in quantum mechanics. For example, from Heisenberg uncertainty principle it follows that the position and the momentum of an elementary particle cannot be measured simultaneously with arbitrarily prescribed accuaracy (we say that they are noncompatible).Moreover, due to unsharpness of elements the orthocomplementation is weakened to supplementation. Common generalizations of Boolean algebras including noncompatibility as well as unsharpness of elements are effect algebras. Effect algebras has been introduced by Foulis and Bennett for modelling unsharp measurenments in quantum mechanical systems. The prototype of the abstract definition of effect algebras was the set E(H) of Hilbert space effects, meaning all self-adjoint operators between null operator O and identity operator I on H .The set E(H) equipped with a partial binary operation which coincides with the usual sum of operators if it is in the interval [O,I] is an effect algebra. If a quantum mechanical system is represented in the usual way by a complex Hilbert space then operators in the set E(H) represents measurements that may be unsharp.The subset P(H) of the set E(H) consisting of orthogonal projections represent measurements that are sharp. Mutually equivalent generalizations of effect algebras without top element were introduced by Foulis and Bennett, Hedlíková and Pulmannová, Kalmbach and Riečanová and Kopka and Chovanec. An example of operator generalized effect algebra is the set V(H) of all positive operators densely defined on an infinite-dimensional complex Hilbert space with partial binary operation which coincides with the usual sum of operators if, for a pair of operators,it exists. Every interval in this operator generalized effect algebra is an operator effect algebra.These operator generalized effect algebras and operator effect algebras may include also unbounded operators. Clearly on the set V(H) of operators we can define varied partial sum of operators. We study properties of these different operator generalized effect algebras and effect algebras. Namely the stady ,set of states and the existence of a faithful state. Namely we show that on the set V(H) of all positive densely defined operators in an infinite-dimensional Hilbert space there exists partial sum of operators which coincides with the ususal sum of operators and under which on every interval of obtained generalized effect algebra there exists an ordering set of states as well as a faithful state. Thus every operator effect algebra which is an interval in that generalized effect algebra is an operator interval effect algebra. That means that this effect algebra is isomorphic to some interval in some partially ordered Abelian group.

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