Projections and ideals in a synaptic algebra

David J. Foulis and Sylvia Pulmannová *

This lecture is based on the works [1, 2, 3]. A synaptic algebra is an abstract version of the partially ordered Jordan algebra of all bounded Hermitian operators on a Hilbert space. We review the basic features of a synaptic algebra and then focus on the interaction between a synaptic algebra and its orthomodular lattice of projections. Each element in a synaptic algebra determines and is determined by a one-parameter family of projections—its spectral resolution. We observe that a synaptic algebra is commutative if and only if its projection lattice is boolean, and we prove that any commutative synaptic algebra is isomorphic to a subalgebra of the Banach algebra of all continuous functions on the Stone space of its boolean algebra of projections. We show that the projections in a synaptic algebra form an M-symmetric orthomodular lattice, and give several sufficient conditions for modularity of the projection lattice. We study quadratic ideals in synaptic algebras and show conditions under which a quadratic ideal is generated by its projections.

References

[1] Foulis, D.J., Synaptic algebras, *Math. Slovaca* **60** (2010), 631-654.

[2] Foulis, D.J., Pulmannová, S., Projections in a synaptic algebra, *Order* **27** (2010), 235-257.

[3] Pulmannová, S., Ideals in synaptic algebras, *Math. Slovaca*, accepted.

*University of Massachusetts, Amherst, USA and Mathematical Institute, Slovak academy of Sciences; e-mail: foulis@math.umass.edu and pulmann@mat.savba.sk