Quantum Levy area

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Levy's stochastic area [Levy]is defined in classical probability as the area $L(t)$ subtended at the origin at time t by the trajectory of a two-dimensional Brownian motion, or equivalently by the joint trajectory in the Cartesian plane of two stochastically independent one-dimensional Brownian motions X and Y. The Levy stochastic area formula is for the characteristic function in the probabilistic sense. It has many interesting connections, for example to Meixner distributions, integrable systems, Bernoulli and Euler polynomials, Riemann zeta values, etc. For a recent review and references see [IkTa]

In quantum stochastic calculus [HuPa] the "momentum" and "position" Brownian motions P and Q do not commute, so it is meaningless to speak of their stochastic independence. But they do have a property that for classical processes is tantamount to independence, namely factorisation of the notional joint characteristic function into the product of individual characteristic functions. In this talk I will investigate what becomes of Levy area and the Levy area formula when X and Y are replaced by P and Q. For the area formula this involves explicit construction of a causal double quantum stochastic product integral. In the Fock case the characteristic function is identically 1; to get something interesting we must use non-Fock calculus [HuLi].

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