Dimension of fractal D-posets

Ferdinand Chovanec[1], Mária Jurečková[2]

[1]*Armed Forces Academy, Liptovský Mikuláš, Slovak Republic*
ferdinand.chovanec@aos.sk
[2]*Banking Institute - College of Banking, Banská Bystrica, Slovak Republic*
maria.jureckova@bivs.sk

In recent years there has been recognized that the mathematics of non-smooth objects like irregular sets provide good representation of many natural phenomena. A general framework for the study of such irregular sets provides fractal theory. When we think about fractals, we can use two points of view. We can perceive them just as static objects and on the other hand we can study fractals in dynamic processes which created them.

There are various definitions of the fractals, but in all cases two characteristics are dominant. It is a self-similarity and a fractal dimension. Very simply we can say, that a self-similar object is an object that is composed of multiple smaller copies that look exactly, or more or less the same as the original object. This property can be observed exactly in fractals generated by computers, especially in fractals that are visualized by the geometrical shapes [3]. In real physical structures the self-similarity property of fractal shapes can be observed at several successive levels of magnification [4].

Talking about fractals we usually think of their fractal dimension, which characterizes some aspects of the complexity of the fractal objects. From the variety of the definitions, the basic type is the well known Hausdorff - Besicovitch dimension, which is based on the definition of the Hausdorff measure.

In this paper we will apply a fractal approach in the MV-algebra pastings process to construct a special algebraic structure [2]. The motivation for this were the typical linear fractals and furthermore Greechie diagrams which are a very useful tool for a graphic representation of orthomodular structures.

According to our above mentioned ambition we draft the construction of two linear fractals - the Cantor set and the Koch curve and the calculation of their fractal dimension.

The basic geometric construction of the Canntor set begins with a line segment, the interval [0,1], which is divided into three equal subintervals. By removing the middle interval we obtain two subsegments. The geometric construction is iterated by removing the middle thirds of these, and so on. This frame leads to the so called ternary Cantor set. We can generalize this construction for the Cantor n-ary set (see [1]). Generally, the Cantor set is the set of points which will not be taken out no matter how often we carry out this removal process.

In the case of the Koch curve we start again with an initiator, a closed unit interval, which is divided into the thirds. The middle third of the interval we replace with two line segments to make a triangle. This curve is a generator in this frame. We will repeat this procedure on all of the existing line segments. Analogously, as it was in the case of the Cantor set we can generalize this construction.

To calculate fractal dimension of these linear fractals we use above mentioned Hausdorff - Besicovic dimension which is given by the formula

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log \frac{1}{\varepsilon}},$$

where $N(\varepsilon)$ is the number of self-similar structures of linear size ε and $\frac{1}{\varepsilon}$ is the number of pieces we need to cover the initiator.

It is well-known, that the Cantor set has the topological dimension 0 and Koch curve has the topological dimension 1. But if we calculate their Hausdorff - Besicovitch fractal dimensions we obtain non-integer numbers which are higher than their topological dimensions. Moreover, in both cases it is easy to prove that their dimensions do not depend on the step of the iteration process.

In our contribution we will implement these fractal constructions to the MV-algebra pastings. Pasted MV-algebras generate special types of diference posets. We will define the dimension of fractal D-posets and show that the fractal structures discussed in this paper have a non-integer fractal dimension. Furthermore we show an interesting agreement between the fractal dimension of some classical linear fractals (Cantor set, Koch curve) and the dimension of fractal D-posets (Cantor fractal D-poset, Koch fractal D-poset).

Acknowledgment
This work was supported by the Slovak Research and Development Agency under the contract VEGA 2/0032/09 and VEGA 2/0046/11.

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