56th Czech-Slovak Conference on Graph Theory

Rajecké Teplice, 23–27 August 2021

Booklet of abstracts

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(Editors)

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Contents

About

The 56th Czech-Slovak Conference on Graph Theory 2021, held in Rajecké Teplice on 23–27 August 2021, continues along tradition started by the conference in Liblice in 1961; for the history of these events see e.g., `www.iti.zcu.cz/csgt18/en`. This year's edition has been organized by the Slovak University of Technology in Bratislava in collaboration with the Slovak Mathematical Society. Further particulars about the conference can be found at `http://www.math.sk/wiki/CSGT21`.

This booklet contains a schedule of the event, abstracts of presentations, and a list of participants.

Invited speakers

Michael Drmota	University of Technology, Vienna
Andrzej Grzesik	Jagiellonian University, Kraków
Pavol Hell	Simon Fraser University, Burnaby
Klavdija Kutnar	University of Primorska, Koper
Michał Pilipczuk	University of Warsaw, Warsaw
Paweł Rzążewski	Warsaw University of Technology, Warsaw
Maria Saumell	Czech Academy of Sciences & Czech Technical University

Organizers

Štefan Gyürki, Ivona Hrivová, Pavol Jánoš, Martin Knor, Soňa Pavlíková, Jozef Širáň (Slovak University of Technology, Bratislava).

Timetable

Monday, 23 of August

13:00–15:00	**Registration**	
15:00–15:10	**Opening**	
15:10–16:00	**Pavol Hell** Simon Fraser U.	In praise of loops
16:00–16:20	**Přemysl Holub** UWB Plzeň	On S-packing colouring of distance graphs
16:20–16:40	**Coffee break**	
16:40–17:00	**Tatiana Jajcayová** Comenius Univ.	Rearrangement problem of bicolor arrays by prefix reversals
17:00–17:20	**Petr Kovář** VŠB–TU Ostrava	Supermagic graphs with arbitrary degree differences
17:20–17:40	**Jakub Závada** VŠB–TU Ostrava	Scheduling of incomplete tournaments
17:40–18:00	**Václav Blažej** CTU Prague	m-eternal domination number of cactus graphs
20:00–21:00	**Welcome reception**	

Tuesday, 24 of August

09:00–12:00	Morning	
09:00–09:50	**Klavdija Kutnar** Univ. Primorska	Vertex-transitive graphs via automorphisms with or without fixed vertices
09:50–10:10	**Martin Bachratý** STU Bratislava	Orientably-regular maps with no non-trivial exponents
10:10–10:40	**Coffee break**	
10:40–11:00	**Robert Jajcay** Comenius Univ.	A connection between a question of Bermond and Bollobás and Ramanujan graphs
11:00–11:20	**Katarína Čekanová** Šafárik Univ.	Light edges in embedded graphs with minimum degree 2
11:20–11:40	**Mária Skyvová** UWB Plzeň	Classification of finite group actions on orientable surfaces
11:40–12:00	**Peter Czimmermann** Univ. Žilina	Voltage graphs in the services of location science

15:00–18:00	Afternoon	
15:00–15:50	**Andrzej Grzesik** JU Kraków	Generalized Turán problem for cycles
15:50–16:10	**Jozef Rajník** Comenius Univ.	Uniquely 3-edge-colourable graphs with cyclic connectivity 4
16:10–16:40	**Coffee break**	
16:40–17:00	**Róbert Lukoťka** Comenius Univ.	Circular flow number of Goldberg snarks
17:00–17:20	**Roman Nedela** UWB Plzeň	Cubic graphs of defect 3
17:20–17:40	**Edita Máčajová** Comenius Univ.	Perfect matching index of cubic graphs with defect 3
17:40–18:00	**Ján Karabáš** MI SAS, B. Bystrica	Girth, oddness, and colouring defect of snarks

Wednesday, 25 of August

09:00–12:00	**Morning**	
09:00–09:50	**Maria Saumell** CAS + CTU Prague	Hamiltonicity of Delaunay graphs: An overview
09:50–10:10	**Petr Vrána** UWB Plzeň	Hamilton-connected {claw,net}-free graphs
10:10–10:40	**Coffee break**	
10:40–11:00	**Martin Knor** STU Bratislava	Distance based indices in generalized nanotubical graphs
11:00–11:20	**Martin Škoviera** Comenius Univ.	Determining the perfect matching index of a snark is NP-complete
11:20–11:40	**Ján Kratochvíl** Charles Univ.	Covers of disconnected graphs: What are they? And why should we care?
11:40–12:00	**Onur Çağırıcı** Masaryk Univ.	Unit disk visibility graphs

13:00–20:00	**Hiking afternoon**

Thursday, 26 of August

09:00–12:00	Morning	
09:00–09:50	**Paweł Rzążewski** WUT + UW	H-free graphs: from structure to algorithms
09:50–10:10	**Tomáš Kaiser** UWB Plzeň	Kempe equivalence in odd-hole-free graphs
10:10–10:40	**Coffee break**	
10:40–11:00	**Stanislav Jendroľ** Šafárik Univ.	Loose edge-connection of graphs
11:00–11:20	**Tomáš Madaras** Šafárik Univ.	Facial homogeneous colourings of graphs
11:20–11:40	**Alfréd Onderko** Šafárik Univ.	Edge homogeneous colorings
11:40–12:00	**Zuzana Šárošiová** Šafárik Univ.	Interval vertex coloring of trees

15:00–18:00	Afternoon	
15:00–15:50	**Michał Pilipczuk** Univ. Warsaw	Structural graph theory through the lens of First Order logic
15:50–16:10	**Peter Zeman** Charles Univ.	Testing isomorphism of chordal graphs of bounded leafage is in FPT
16:10–16:40	**Coffee break**	
16:40–17:00	**Jakub Balabán** Masaryk Univ.	Twin-width is linear in the poset width
17:00–17:20	**Filip Pokrývka** Masaryk Univ.	Twin-width of circle graphs
17:20–17:40	**Petr Hliněný** Masaryk Univ.	Isomorphism testing for T-graphs in FPT
17:40–18:00	**Deniz Ağaoğlu** Masaryk Univ.	Proper T-graph isomorphism in FPT-time
19:00–21:00	**Conference dinner**	

Friday, 27 of August

09:00–12:00	Morning	
09:00–09:50	**Michael Drmota** TU Wien	Subgraph statistics in series-parallel graphs and related graph structures
09:50–10:10	**Štefan Gyürki** STU Bratislava	Vertex transitive directed strongly regular graphs of order up to 32
10:10–10:40	**Coffee break**	
10:40–11:00	**Pavol Jánoš** STU Bratislava	On a relation between G-graphs and lifting construction
11:00–11:20	**Soňa Pavlíková** STU Bratislava	Upper bounds on the HOMO-LUMO spectral gap of graphs
11:20–11:40	**Jozef Širáň** STU Bratislava	Classification of orientably-regular maps of genus $p+1$ for prime p without computational assistance

List of Abstracts – Invited talks

Subgraph statistics in series-parallel graphs and related graph structures

Michael Drmota[1], **Lander Ramos**[2], **Juanjo Rué**[3]

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Series-parallel graphs can be characterized in several ways. For example, they can be generated by a successive series-parallel extension of a forest, or they are precisely those graphs with no K_4 as a minor. The (asymptotic) enumeration problem of series-parallel graphs was solved in [1] in the vertex-labeled case and in [3] in the unlabeled case; the corresponding results and methods will be reviewed in the first part of the talk.

However, the main purpose of this talk is to establish asymptotic properties of the subgraph counting problem in random series-parallel graphs, where we assume that every series-parallel graph G with n vertices appears equally likely. Let H be a fixed connected series-parallel graph. The main result says that the number of occurrences of H (as a subgraph) in a random series-parallel graph of size n follows asymptotically a normal limiting distribution with linear expectation and variance [2]. Actually the same result holds in so-called *sub-critical graph classes*.

The main ingredient in the proof is the analytic framework developed by Drmota, Gittenberger and Morgenbesser to deal with infinite systems of functional equations [4]. As a case study, we get explicit expressions for the number of triangles and cycles of length four.

References

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Generalized Turán problem for cycles

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The generalized Turán problem asks for the largest number of copies of a graph H in an n-vertex graph not containing a graph F as a subgraph. In the talk we will discuss this problem in the case when both H and F are cycles by presenting known bounds and recent developments. In particular, we will sketch the proof providing the first exact solution for the generalized Turán problem for cycles in the sparse setting.

In praise of loops

Pavol Hell[1]

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I will discuss several (unrelated) examples of graph and digraph concepts that have interesting extensions if loops are allowed (but not required). This will illustrate how allowing loops can unify and extend existing concepts and results. The examples will include versions of the game of cops and robbers, versions of domination, versions of chordality, and versions of interval graphs. I will include some recent and some not so recent joint results with T. Feder, C. Hernández Cruz, J. Huang, J. Lin, R. McConnell, J. Nešetřil, and A. Rafiey.

Vertex-transitive graphs via automorphisms with or without fixed vertices

***M.D.E. Conder*[1], *A. Hujdurović*[2], *K. Kutnar*[3], *D. Marušič*[4]**

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When dealing with symmetry of combinatorial objects – or in any other setting for that matter – one will come across two different kinds of nonidentity automorphisms of these objects: those fixing as large as possible subset of points, on the one hand, and those fixing no points at all on the other.

For example, given an automorphism α of a graph X, we let $\mathrm{Fix}(\alpha)$ be the set of all those vertices of X which are fixed by α. When $\mathrm{Fix}(\alpha)$ is empty, then α is called a *derangement*, and furthermore, it is said to be *semiregular* if all of its cycles in the cycle decomposition are of the same length. When $\mathrm{Fix}(\alpha)$ is non-empty, then an α-*rigid cell* is a connected component of the subgraph induced by $\mathrm{Fix}(\alpha)$. In this lecture, I will aim for two goals:

1. give a complete description of rigid cells in cubic arc-transitive graphs, and
2. discuss some recent developments in regards to the problem of deciding which cubic arc-transitive graphs admit the so called *simplicial automorphisms*, that is, semiregular automorphisms whose quotient “multigraphs” are in fact simple graphs – a special case of the well known semiregularity problem regarding existence of semiregular automorphisms in vertex-transitive graphs.

Structural graph theory through the lens of First Order logic

Michał Pilipczuk[1]

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We say that a graph H can be *transduced* from a graph G if H can be interpreted in a coloring of G using First Order logic FO. This is a basic concept of embedding one graph into another that is well-suited for investigating problems connected to FO on graphs, as that H can be transduced from G implies that H is not more complicated than G in terms of how complicated structures can be defined in FO.

During the talk we will discuss what happens if one attempts to construct a structural theory for graphs with transductions set to be the main underlying notion of embedding, and how this theory connects to contemporary work in structural graph theory. In particular, we survey recent results on graph classes that are *monadically stable* and *monadically dependent*, which appear to be the main dividing lines in the emerging theory. No prior exposure to logic in computer science is required.

H-free graphs: from structure to algorithms

Paweł Rzążewski[1,2]

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One of the active areas of algorithmic graph theory is to investigate how the restrictions imposed on the set of input instances influence the complexity of computational problems. Quite often we can witness an interesting interplay between graph-theoretic and algorithmic results: a good understanding on the structure of instances may help in the design of efficient algorithms.

During the talk we will mostly focus on H-*free* graphs, i.e., graphs that do not contain a fixed graph H as an induced subgraph. We will survey some old and some new results concerning the complexity of well-known problems in H-free graphs, for various graphs H.

Hamiltonicity of Delaunay graphs: An overview

Maria Saumell[1,2]

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Proximity graphs, and in particular Delaunay graphs, are a central object in computational geometry due to their great number of real-life applications and their use as a tool to solve a variety of interesting mathematical problems. Unfortunately, the standard Delaunay graph of a set of points in the plane is not always Hamiltonian, which has lead to the question of which variants or generalizations of this graph are.

In this talk, I will present the history of this problem, including the recent result proving that there exist universal (constant) values of k such that the k-order Delaunay graph with respect to any arbitrary convex shape contains a Hamiltonian cycle for any set of points [1].

References

[1] P. Bose, P. Cano, M. Saumell, R.I. Silveira, *Hamiltonicity for convex shape Delaunay and Gabriel graphs*, Comput. Geom. **89**, 101629 (2020).

List of Abstracts – Contributed talks

Proper T-graph isomorphism in FPT-time

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For a fixed graph H, an H-graph is the intersection graph of connected subgraphs of a suitable subdivision of H [3]. An H-graph is *proper* if the representing subgraphs of H can be chosen incomparable by the inclusion. The isomorphism problem for (proper) H-graphs is GI-complete even when H is a star with d rays, i.e. S_d-graphs. As we have shown that S_d-graph isomorphism can be solved in FPT-time parameterized by d in [1], we prove that proper T-graph isomorphism problem can also be solved in FPT-time parameterized by the size of a fixed tree T. In contrast to our group-theoretical approach for S_d-graph isomorphism, we give a simple and fully combinatorial algorithm for proper T-graph isomorphism, the details of which are given in [2].

Acknowledgements This work has been supported by the Czech Science Foundation, project no. 20-04567S.

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Orientably-regular maps with no non-trivial exponents

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Given an orientable map $\mathcal{M}$, and an integer e relatively prime to the valency of $\mathcal{M}$, the e th rotational power $\mathcal{M}^e$ of $\mathcal{M}$ is the map formed by replacing the cyclic rotation of edges around each vertex with its e th power. If maps $\mathcal{M}$ and $\mathcal{M}^e$ are isomorphic, and the corresponding isomorphism preserves the orientation of the carrier surface, then we say that e is an exponent of $\mathcal{M}$.

In this talk, I will explain how canonical regular covers of maps can be used to prove that for every given hyperbolic pair (k, m) there exists an orientably-regular map of type $\{m, k\}$ with no non-trivial exponents. In particular, for each hyperbolic pair (k, m) we find a suitable orientable map of type $\{m, k\}$ such that the canonical regular cover of this map is an orientably-regular map (of the same type) with no non-trivial exponents.

Acknowledgements The second author is grateful for the support from the VEGA Research Grant 1/0238/19.

Twin-width is linear in the poset width

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Twin-width is a new parameter informally measuring how diverse are the neighbourhoods of the graph vertices, and it extends also to other binary relational structures, e.g. to digraphs and posets. It was introduced just very recently, in 2020 by Bonnet, Kim, Thomassé and Watrigant [1]. One of the core results of these authors is that FO model checking on graph classes of bounded twin-width is in FPT. With that result, they also claimed that posets of bounded width have bounded twin-width, thus capturing prior result on FO model checking of posets of bounded width in FPT [2]. However, their translation from poset width to twin-width was indirect and giving only a very loose double-exponential bound.

We prove that posets of width d have twin-width at most $9d$ with a direct and elegant argument, and show that this bound is asymptotically tight. Specially, for posets of width 2 we prove that in the worst case their twin-width is also equal 2. These two theoretical results are complemented with straightforward algorithms to construct the respective contraction sequence for a given poset.

References

[1] É. Bonnet, E.J. Kim, S. Thomassé, R. Watrigant, *Twin-width I: tractable FO model checking*, 2020 IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS), pp. 601-612 (2020).
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m-eternal domination number of cactus graphs

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Given a graph G, guards are placed on vertices of G. Then vertices are subject to an infinite sequence of attacks so that each attack must be defended by a guard moving from a neighboring vertex. The m-eternal domination number is the minimum number of guards such that the graph can be defended indefinitely. We present a recent result showing that an exact m-eternal domination number can be determined in polynomial for any cactus graph (a connected graph where each edge lies in at most one cycle).

We consider three variants of the m-eternal domination number: first variant allows multiple guards to occupy a single vertex, second variant allows at most one guard on each vertex, and in the third variant additional "eviction" attacks must be defended.

Unit disk visibility graphs

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Visibility graphs were studied considering various geometric sets such as a simple polygon [5], a polygon with holes [6], a set of points [1], a set of line segments [2], along with different visibility models such as line-of-sight visibility [3] and α-visibility [4].

We study unit disk visibility graphs, where the visibility relation between a pair of geometric entities is defined by not only obstacles, but also the distance between them. This particular graph class models real world scenarios more accurately compared to the conventional visibility graphs. We first define and classify the unit disk visibility graphs, and then show that the 3-coloring problem is NP-complete when unit disk visibility model is used for a set of line segments and for a polygon with holes.

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Voltage graphs in the services of location science

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Location of emergency stations, firehouses, depots, and charging stations belongs to the wide class of location problems. Mathematical representation of these problems includes weighted p-median, weighted p-center, uncapacitated facility location problem, and covering location problem. It is known that these problems are from the class of hard combinatorial problems, and improving heuristic algorithms (for example genetic algorithms) are often used to solve them. These algorithms usually work with the input set of feasible solutions, which is repeatedly transformed into a set of new (and better) feasible solutions. It is known that the above-mentioned algorithms are more efficient when the starting set of feasible solutions has high diversity. Hence we decided to test a t-uniformly deployed set (t-uds) as a starting set in these algorithms. A set is called t-uds with parameters n and p, if its elements are vectors from $\{0, 1\}^n$ that contains exactly p ones and the minimal Hamming distance between each pair of vectors is at least $2p - 2t$.

In our contribution, we present the construction of large t-uds with parameters n and p from appropriate voltage (di)graphs. The elements of these sets are rows of an adjacency matrix of abelian lifts of these (di)graphs. We also present a fast algorithm for the construction of t-uds by this method.

Light edges in embedded graphs with minimum degree 2

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Let $\mathcal{G}$ be a class of graphs. The weight $w(e)$ of an edge e is the sum of the degrees of its endvertices. We say that K_2 is *light* in $\mathcal{G}$ if there exists a constant k such that every graph $G \in \mathcal{G}$ contains an edge e with $w(e) \leq k$.

Kotzig [4] proved that every 3-connected plane graph contains an edge with weight at most 13. Ivančo [1] described upper bounds for weights of edges in the class of graphs embeddable on orientable surfaces with higher genus. Jendroľ and Tuhársky [2] examined the existence of light edges in the class of graphs embeddable on the non-orientable surfaces with higher genus. Later, Jendroľ, Tuhársky and Voss [3] described exact types of edges in embedded graphs with minimum degree 3, minimum face size 3 and sufficiently many vertices.

In this talk we describe types of edges in connected embedded graphs with minimum degree at least 2, minimum face size 3, and with prescribed number of vertices. We also discuss the optimality of all parameters in our results.

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Vertex transitive directed strongly regular graphs of order up to 32

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A *directed strongly regular graph* (DSRG) with parameters (n, k, t, λ, μ) is a regular directed graph on n vertices with valency k such that every vertex is incident with t undirected edges; the number of directed paths of length 2 directed from a vertex x to another vertex y is λ, if there is an arc from x to y and μ otherwise.

In this talk we will focus on the DSRGs whose group of automorphisms acts transitively on the vertex set. Enumeration of all such DSRGs up to order 20 was given in [1] by Fiedler, Klin and Muzychuk. As a continuation, here we provide such an enumeration up to order 32.

Acknowledgements This research was supported by the APVV Research Grants 17-0428 and 19-0308, and the VEGA Research Grants 1/0238/19 and 1/0206/20.

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Isomorphism testing for T-graphs in FPT

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A T-graph (as a special case of a chordal graph) is the intersection graph of connected subtrees of a suitable subdivision of a fixed tree T. We deal with the isomorphism problem for T-graphs which is *GI-complete* in general – when T is a part of the input and even a star S_d. We prove that the T-graph isomorphism problem is in FPT when T is the fixed parameter of the problem. This can equivalently be stated that isomorphism is in FPT for chordal graphs of (so-called) bounded leafage. While the recognition problem for T-graphs is not known to be in FPT wrt. T, we do *not* need a T-representation to be given (a promise is enough). To obtain the result, we combine a suitable isomorphism-invariant decomposition of T-graphs with the classical tower-of-groups procedure of Babai [2], and reuse some of the ideas of our FPT-time isomorphism algorithm for S_d-graphs [1].

Acknowledgements This work has been supported by the Czech Science Foundation, project no. 20-04567S.

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On S-packing colouring of distance graphs

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Let G be a simple undirected graph and $S = (s_1, s_2, s_3, \dots)$ a non-decreasing sequence of positive integers. A mapping $c : V(G) \to \{1, 2, \dots, k\}$ is an S-packing k-colouring of G if, for any pair of distinct vertices $x, y \in V(G)$ with $c(x) = c(y) = i$, the distance between x and y in G is greater than s_i. The smallest k such that G has an S-packing k-colouring is the S-packing chromatic number of G, denoted $\chi_S(G)$. For a set D of positive integers, a distance graph $G(D)$ is the graph with $\mathbb{Z}$ as its vertex set, with two vertices i, j being adjacent if and only if $|i - j| \in D$.

In this talk we focus on the distance graphs with distance sets $D = \{1, t\}$, $D = \{2, t\}$ and $D = \{1, 2, t\}$ and we determine the S-packing chromatic numbers of these distance graphs, where S is any sequence with $s_i \in \{1, 2\}$ for all i.

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A connection between a question of Bermond and Bollobás and Ramanujan graphs

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In our talk, we present an interesting connection between two seemingly unrelated problems:

- the *Degree/Diameter Problem*, a well-known problem from Extremal Graph Theory; and
- the question of the existence of infinitely many non-bipartite *Ramanujan graphs* of any degree k; with Ramanujan graphs defined via the properties of their spectra.

The Degree/Diameter Problem is the problem of finding the order $n(k,d)$ of a largest graph of maximum degree k and diameter d. If we let $M(k,d)$ denote the corresponding Moore bound, then $n(k,d) \leq M(k,d)$, for all $k \geq 3, d \geq 2$. This inequality has been shown to be strict for all but very few parameter pairs k and d, however, the exact relation between the values $n(k,d)$ and $M(k,d)$ is unknown. The uncertainty of the situation is captured by an open question of Bermond and Bollobás who asked whether it is true that for any positive integer $c > 0$ there exists a pair k and d, such that $n(k,d) \leq M(k,d) - c$.

Ramanujan graphs are k-regular graphs whose second largest eigenvalue (in modulus) does not exceed $2\sqrt{k-1}$. It is an open question whether infinitely many non-bipartite k-regular Ramanujan graphs exist for any degree k.

The surprising connection exhibited in our talk is based on a result in which we show that if the answer to the question of Bermond and Bollobás were negative, and if there existed a $c > 0$ such that $n(k,d) \geq M(k,d) - c$, for all $k \geq 3, d \geq 2$, then, for any fixed k and all sufficiently large even d's, the largest graphs of degree k and diameter d would all have to be Ramanujan graphs. This would imply a positive answer to the question of the existence of infinitely many non-bipartite k-regular Ramanujan graphs for any degree k.

Rearrangement problem of bicolor arrays by prefix reversals

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In our presentation we will consider a rearrangement problem of two-dimensional bicolor arrays by prefix reversals as a generalization of the burnt pancake problem [1,2]. We study an equivalence relation on the set of bicolor arrays induced by prefix reversals, and the rearrangement problem is then to characterize the equivalence classes of this relation. While previously studied rearrangement problems for unicolor arrays [3] made use of the classical group theoretic tools, in the present problem rearrangements are described by partial injections, and thus the equivalence classes are characterized in terms of a groupoid action. We also mention graphs appearing in the study of rearrangment problems as well as outline algorithms and give estimates for a minimum number of rearrangements needed.

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On a relation between G-graphs and lifting construction

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Constructions of regular graphs with given properties are of special interest in the degree-girth problem, introduced by Tutte [3], where one has to find the smallest possible order $n(d, g)$ of a d-regular graph of girth g. Several of these constructions are based on groups; a prominent example are lifting constructions, which can be regarded as a generalisation of the well known Cayley graphs. Bretto and Faisant presented in [1] another construction of graphs related to groups and having highly regular properties, called G-graphs.

The purpose of this talk is to compare these two constructions and to show under which circumstances the G-graphs can be obtained as lifts of dipoles. We also provide the lifting constructions of graphs of girth 6 and 8, originally constructed in [2] as p-regular G-graphs for an arbitrary prime p, which we were able to extend for prime powers.

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Loose edge-connection of graphs

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In the last years, connection concepts such as rainbow connection and proper connection appeared in graph theory and obtained a lot of attention. In this talk, we investigate the loose edge-connection of graphs. A connected edge-coloured graph G is loose edge-connected if between any two of its vertices there is a path of length one, or a bi-colored path of length two, or a path of length at least three with at least three colours used on its edges. The minimum number of colours, used in a loose edge-colouring of G, is called the loose edge-connection number and denoted $\mathrm{lec}(\mathrm{G})$. We determine the precise value of this parameter for any simple graph G of diameter at least 3. We show that deciding, whether $\mathrm{lec}(G) = 2$ for graphs G of diameter 2, is NP-complete problem. Furthermore, we characterize all complete bipartite graphs $K_{r,s}$ with $\mathrm{lec}(K_{r,s}) = 2$.

Kempe equivalence in odd-hole-free graphs

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A *Kempe change* in a graph G with vertices coloured from a set C consists in choosing two colours $a, b \in C$ and interchanging them on some maximal set $X \subseteq V(G)$ such that all vertices in X are coloured a or b and the induced subgraph of G on X is connected. Two colourings of a graph G are *Kempe-equivalent* if one can be changed into the other by a series of Kempe changes.

We study Kempe-equivalence of colourings in the class of *odd-hole-free* graphs (that is, graphs containing no induced odd cycle of length at least 5). Let G be an odd-hole-free graph with maximum degree Δ and clique number ω, and let $t = \left\lceil \frac{\Delta+\omega+1}{2} \right\rceil$. We show that every colouring of G with at least t colours is Kempe-equivalent to a t-colouring. In particular, this provides a short proof of a special case of the well-known 'ω, Δ, χ' conjecture of Reed.

For perfect graphs (which form a subclass of the above), we can get a stronger result: Any two colourings with at least $t + 1$ colours are Kempe-equivalent. Some related problems and observations will be discussed in the talk.

Joint work with Marthe Bonamy, Sébastien Bonduelle and Clément Legrand-Duchesne.

Girth, oddness, and colouring defect of snarks

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The colouring defect of a cubic graph, introduced by Steffen in 2015, is the minimum number of edges that are left uncovered by any set of three perfect matchings. Since a cubic graph has defect 0 if and only if it is 3-edge-colourable, this invariant can measure how much a cubic graph differs from a 3-edge-colourable graph.

Our aim is to examine the relationship of colouring defect to oddness, an extensively studied measure of uncolourability of cubic graphs, defined as the smallest number of odd circuits in a 2-factor. We show that there exist cyclically 5-edge-connected snarks (cubic graphs with no 3-edge-colouring) of oddness two and arbitrarily large colouring defect. This result is achieved by means of a construction of cyclically 5-edge-connected snarks with oddness two and arbitrarily large girth. The fact that our graphs are cyclically 5-edge-connected significantly strengthens a similar result of Jin and Steffen (2017), which only guarantees graphs with cyclic connectivity at most three. At the same time, our result improves Kochol's original construction of snarks with large girth (1996) in that it provides infinitely many nontrivial snarks of any prescribed girth $g \geq 5$, not just girth at least g.

Distance based indices in generalized nanotubical graphs

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There are many graph invariants important in chemistry. They are used for instance for preliminary estimation of properties of complex molecules. Big group of these indices involve distances, and usually they are in a form

$$I^\lambda(G) = \sum_{u \neq v} f(u,v) dist^\lambda(u,v),$$

where λ is a real number and $f(u,v)$ is a symmetric nondecreasing function depending only on degrees of u and v.

A nanotube is a carbon structure of cylindrical form. It can be open, closed, or in some sense degenerated at the ends. For all the above mentioned chemical indices I^λ and for all generalized nanotubes we found estimations for $I^\lambda(G)$. These estimations differ for $\lambda < -1$, $\lambda = -1$ and $\lambda > -1$ and they all depend only on the circumference of the nanotube and not on its specific type.

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Supermagic graphs with arbitrary degree differences

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A supermagic labeling of a graph G, $G = (V, E)$, is a labeling of all edges in E by consecutive distinct integers such that each vertex $v \in V$ has the same weight, while the weight of vertex v is the sum of edge labels of all edges incident to v.

The concept of supermagic labelings has been explored for a half century. The majority of results naturally deals with regular or almost regular graphs. However, in a recently published paper [1] the authors have shown that a supermagic graph can have arbitrary many different degrees.

In this talk we extend the result. We show that given an arbitrary nondecreasing sequence of nonnegative integers $a_1, a_2, \ldots, a_{n-1}$ there exists a supermagic graph with degrees from the list $d_1, d_2, \ldots, d_n$ such that $a_i = d_{i+1} - d_i$ for all $i = 1, 2, \ldots, n-1$.

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Covers of disconnected graphs: What are they? And why should we care?

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The notion of graph covers is a discretization of covering spaces introduced and deeply studied in topology. In discrete mathematics and theoretical computer science, they have attained a lot of attention from both the structural and complexity perspectives. Nonetheless, disconnected graphs were usually omitted from the considerations with the explanation that it is sufficient to understand coverings of the connected components of the target graph by components of the source one. However, different (but equivalent) versions of the definition of covers of connected graphs generalize to nonequivalent definitions of disconnected graphs. The aim of this talk is to summarize this issue and to compare three different approaches to covers of disconnected graphs:

1. locally bijective homomorphisms,
2. globally surjective locally bijective homomorphisms (which we call *surjective covers*), and
3. locally bijective homomorphisms which cover every vertex the same number of times (which we call *equitable covers*).

The standpoint of our comparison is the complexity of deciding if an input graph covers a fixed target graph. We show that both surjective and equitable covers satisfy what certainly is a natural and welcome property: covering a disconnected graph is polynomial time decidable if such it is for every connected component of the graph, and it is NP-complete if it is NP-complete for at least one of its components. Despite of this, we argue that the third variant, equitable covers, is the right one, when considering covers of colored (multi)graphs, and we completely characterize the complexity of covering 2-vertex colored multigraphs with semi-edges.

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Circular flow number of Goldberg snarks

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A *circular nowhere-zero r-flow* on a bridgeless graph G is an orientation of the edges and an assignment of real values from $[1, r-1]$ to the edges in such a way that the sum of incoming values equals the sum of outgoing values for every vertex. The *circular flow number* of G is the infimum over all values r such that G admits a nowhere-zero r-flow. We prove that the circular flow number of Goldberg snark G_{2k+1} is $4 + 1/(k+1)$, proving a conjecture of Goedgebeur, Mattiolo, and Mazzuoccolo [1].

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Facial homogeneous colourings of graphs

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A proper vertex k-colouring of a plane graph G is called *facial ℓ-homogeneous* if every face of G sees precisely ℓ colours. The case $k = \ell$ corresponds to proper polychromatic colouring (the general version of which was introduced by Alon et al. in 2008, see [1]). We present (rare) examples of plane graphs which are not facial homogeneously colourable at all, or require significantly more colours than chromatic number; in addition, we study various sufficient conditions (related to girth, face sizes or weak dual structure) for facial homogeneous colourability of plane graphs, its relation to other facial colourings, and the extension of this concept for embedded graphs.

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Perfect matching index of cubic graphs with defect 3

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The defect of a cubic graph is one of the parameters that measure how far a given cubic graph is from being 3-edge-colourable. It is defined as the smallest number of edges not covered with three perfect matchings. Steffen [1] proved that the defect of a cubic graph is at least 3 whenever the graph is not 3-edge-colourable.

Here we prove that the edges of every cyclically 4-edge connected cubic graph with defect 3 can be covered with four perfect matchings, implying that some of the long open problems are satsfied for this family of graphs. This result is best possible as there is an infinite family of cubic graphs with cyclic connectivity 3 whose edges cannot be covered with four perfect matchings.

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Cubic graphs of defect 3

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Every 3-edge-colourable cubic graph has a set of three perfect matchings that cover all of its edges. Conversely, three perfect matchings cover the edge set of a cubic graph only when they are pairwise disjoint, and consequently, the graph is 3-edge-colourable. It follows that if a cubic graph is not 3-edge-colourable, then any collection of three perfect matchings leaves some of its edges not covered. The minimum number of edges of a cubic graph G left uncovered by any set of three perfect matchings will be called the *colour defect* of G.

The concept of colour defect of a cubic graph was introduced in 2015 by Steffen who used the notation $\mu_3(G)$ but did not coin any term for it. Among other things he proved that every bridgeless cubic graph which is not 3-edge-colourable – a snark – has colour defect at least 3. In the present talk we first derive some basic properties of the defect. After that we investigate the behaviour of cubic graphs of defect 3 with respect to reductions along edge-cuts of size at most four. A surprising behaviour of the defect with respect to 3-cuts is studied in detail.

Edge homogeneous colorings

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We define several types of edge colorings defined by the requirement of equal number of colors appearing, in particular ways, on maximal stars and maximal double-stars without the central edge (these are denoted by S_v and H_e, for a central vertex v and central edge e, respectively). We characterize graphs, which are colorable in such a way that, for each edge e, q colors are used on each connected component of H_e. We present sufficient condition for a graph to admit 2-coloring in a way that, for each edge e, the number of colors used on H_e equals 2. We also show that the set of all integers k, such that G admits a k-coloring in which the number of colors used on edges of each maximal star S_v equals q, forms an integer interval.

Upper bounds on the HOMO-LUMO spectral gap of graphs

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In quantum chemistry, the smallest positive and the largest negative eigenvalue of a graph representing a molecule determine (up to a constant multiple) the energy of the highest occupied molecular orbital (HOMO) and the lowest unoccupied molecular orbital (LUMO), respectively; their difference is the HOMO-LUMO separation (or spectral) gap.

In our contribution we will focus on the HOMO-LUMO spectral gap of weighted graphs. In particular, we introduce a construction of a new weighted graph from a pair of old ones by bridging them over a bipartite graph, and analyze the construction from the point of view of maximizing the spectral gap of the resulting graph with respect to the bridging graph.

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Twin-width of circle graphs

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In 2020 Bonnet et al. introduced parameter of graphs called twin-width [1]. The series of results followed, showing that graph classes of bounded twin-width have tractable first-order model checking and then followed by examples of classes of bounded twin-width [2]. One of the most interesting graph class result is that any hereditary proper subclass of permutation graphs has bounded twin-width [1], while class of all permutation graphs does not.

We studied superclass of permutation graphs called circle graphs (intersection graph of chords of a circle), and we concluded that a subclass of circle graphs has bounded twin-width if and only if some permutation graph is forbidden as induced subgraph.

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Uniquely 3-edge-colourable graphs with cyclic connectivity 4

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The study of uniquely k-edge-colourable graphs was initiated by Greenwell and Kronk in 1973 [3]. Thomason [4] proved that for each $k \geq 4$, each uniquely k-edge-colourable graph is isomorphic to the star $K_{1,k}$. The class of cubic uniquely 3-edge-colourable graphs, denoted by UEC_3, remains uncharacterised. Although there is an infinite family of graphs from UEC_3, even triangle-free [1], the only known cyclically 4-edge-connected such graph is the generalised Petersen graph $P(9, 2)$ of order 18, whose cyclic edge-connectivity is 5.

Let G be an uniquely 3-edge-colourable cubic graph with cyclic connectivity 4 and a 4-edge-cut separating G into two cyclic components G_1 and G_2. We show that each of G_1 and G_2 can be completed to a graph from UEC_3 by adding two adjacent vertices. This result is similar to Golberg's result that each snark with cyclic connectivity 4 can be decomposed into two snarks in a similar manner [2]. Also, we prove several other structural results about graphs from UEC_3 with cyclic connectivity 4. Our results are useful for a computer search for small graphs from UEC_3 and perhaps, for proving that no graphs from UEC_3 with cyclic connectivity 4 exist.

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Classification of finite group actions on orientable surfaces

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The problem of classifying all finite group actions on orientable surfaces of low genera is considered. Our aim is to extend the existing classifications (genera 2 and 3 by Broughton (1990), genus 4 by Bogopolsky (1991) and genus 5 by Kuribayashi and Kimura (1990)) to higher genera using computer algebra systems. On the other hand, Marston Conder classified actions of all "large groups" on orientable surfaces of genera up to 101 (late 2000's). The latter catalogue is not satisfactory because of two reasons:

1. The groups considered in Conder's list have orders at least $4(g-1)$, given genus g, and
2. The equivalence of the group actions on Conder's list is different of the equivalence considered in earlier works.

We will mainly discuss the problem of transferring (so-called) topological equivalence of actions into purely algebraic form and further, the problem of implementation of such test. As follows, the problem of equivalence of group actions involves (at least partial) knowledge of group of automorphisms of given Fuchsian group — a deep and interesting problem itself.

Interval vertex coloring of trees

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A vertex k-coloring (not necessarily proper) is an *open (closed) interval k-coloring* if for every vertex v the set of colors used in the open (closed) neighborhood of v forms an interval of integers. The largest k for which there exists an open (closed) interval k-coloring of G is called *open (closed) interval chromatic number* of G, and we denote it by $\chi_{io}(G)$ ($\chi_{ic}(G)$).

There was a lot of attention given to the edge version of such a coloring (the existence of an interval edge coloring of given graph and the value of the corresponding chromatic index - see e.g. [1,2,3]), but the vertex version of this coloring was (as far as we know) not investigated. In the talk we present algorithms for finding the precise value of χ_{io} and χ_{ic} for trees.

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Classification of orientably-regular maps of genus $p+1$ for prime p without computational assistance

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An orientably-regular map is an embedding of a connected graph on an compact orientable surface such that the group all orientation-preserving automorphisms of the embeddings is transitive on the vertex-edge incident pairs. In 2010 M. Conder, J. Š. and T. Tucker classified orientably-regular maps on surfaces of genus $p+1$ for prime p. However, a part of this classification – for $p \leq 83$ and dividing the order of the automorphism group – relied on computational results of M. Conder. In the talk we will indicate how this computational assistance can be avoided.

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Determining the perfect matching index of a snark is NP-complete

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A conjecture of Berge predicts that every bridgeless cubic graph can have its edges covered with at most five perfect matchings. If the graph in question has no 3-edge-colouring, then at least four perfect matchings are necessary. It was proved by Esperet and Mazzuoccolo [1] that it is NP-complete to decide whether four perfect matchings are enough to cover the edges of a bridgeless cubic graph. A disadvantage of the proof (noted by the authors) is that the constructed graphs have 2-cuts.

In this talk we show that small cuts can be altogether avoided and that the problem remains NP-complete even for nontrivial snarks – that is, cyclically 4-edge-connected cubic graphs with no 3-edge-colouring. As a by-product, we provide a rich family of nontrivial snarks that cannot be covered with four perfect matchings. The methods rely on the theory of tetrahedral flows developed by Máčajová and Škoviera in [2].

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Hamilton-connected {claw,net}-free graphs

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We complete the characterization of forbidden generalized nets implying Hamilton-connectedness of a 3-connected claw-free graph (where the *generalized net* is the graph $N_{i,j,k}$ obtained by attaching endvertices of three paths of lengths i, j, k to a triangle). We first present the necessary techniques that allow to handle the problem, namely:

- we strengthen the closure concept for Hamilton-connectedness in claw-free graphs, introduced by the second and third authors, such that not only the line graph preimage of a closure, but also its core has certain strong structural properties,
- we prove a special version of the "nine-point-theorem" by Holton et al. that allows to handle Hamilton-connectedness of "small" $\{$claw, $N_{i,j,k}\}$-free graphs,
- by combination of these techniques, as an application, we prove that for $X \in \{N_{1,3,3}, N_{1,1,5}, N_{2,2,3}\}$, every 3-connected $\{$claw, $X\}$-free graph is Hamilton-connected.

All the results on Hamilton-connectedness are sharp and answer a long-standing open question.

Scheduling of incomplete tournaments

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Tournament in which n participants play each other in $n-1$ rounds is called a round robin tournament. Such a tournament is quite fair for each participant but for instance in lots of chess tournaments the number of participants is too large to play all $n-1$ rounds. So, how can we schedule a tournament with n participants into r, $r < n-1$ rounds if we want the tournament to be as fair as possible?
The Swiss system is often used, but it cannot be used when we need to schedule the whole tournament ahead. We can try to use graph theory to find the best possible schedule. We show for which number of vertices n a connected handicap cubic graph with proper 3-edge-coloring exists and how to use it to create a different graph with higher regularity r which is again handicap. We also show that if there is an advantage (e.g. the advantage of first move in a chess game), we can always schedule the r-round tournament in which each participant plays in each round, so that for each participant the difference between the number of games with and without his advantage is exactly 1 if r is odd and 0 if r is even. Moreover, we can ensure that no participant plays three or more consecutive games with the advantage nor without the advantage during the tournament.

Testing isomorphism of chordal graphs of bounded leafage is in FPT

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It is known that testing isomorphism of chordal graphs is as hard as the general graph isomorphism problem. Every chordal graph can be represented as the intersection graph of some subtrees of a tree. The leafage of a chordal graph, is the minimum number of leaves in the representing tree. We construct a fixed-parameter tractable algorithm testing isomorphism of chordal graphs with bounded leafage. The key point is a fixed-parameter tractable algorithm finding the automorphism group of a colored 3-order hypergraph with bounded sizes of color classes of vertices.

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