The Greechie diagrams of Lattice-ordered Effect Algebras

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The aims of this paper are to present the techniques of constructing a lattice-ordered effect algebra with the given family of MV-algebras. First, the definitions of Greechie diagrams of finite lattice-ordered effect algebras are introduced. As applications of the Greechie diagrams of lattice-ordered effect algbras, the relationships between the blocks of finite lattice-ordered effect algbras are studied. Then we prove that any finite lattice-ordered effect algebra L satisfying the condition $A(S(L)) = \{\imath(a)a | a \in A(L)\}$ can be gotten by substituting the atoms of the orthomodular lattice $S(L)$ with lattice-ordered effect algebras which are the horizontal sums of chains. At same time, we give one kind of technique to construct a lattice-ordered effect algebra with the given family of MV-algebras. At last, we prove the "Loop lemma" of MV-algebras pasting.

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