Unbounded linear operators in effect algebras

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Let $\mathcal{H}$ be a complex infinite-dimensional Hilbert space and let $L(\mathcal{H})$ and $B(\mathcal{H})$ be the set of (possibly unbounded) linear operators and the set of all bounded linear operators in $\mathcal{H}$, respectively. In 1994 D.J. Foulis and M.K. Bennet [1] introduced the abstract notion of effect algebra for modeling of unsharp measurements in quantum mechanics. The prototype of this notion was the set

$$E(\mathcal{H}) = [0, I] = \{A \in B(\mathcal{H})\, 0 \leq (Ah, h) \leq (h, h) \text{ for all } h \in \mathcal{H}\}\,.$$

However, in quantum physics, most quantities are represented by unbounded operators. Recently (see, [2], [3]) some examples of effect algebras and generalized effect algebras of unbounded operators were introduced and studied.

First, we review some properties of unbounded linear operators. Then we show that there is no unbounded extension of a bounded positive linear operator and consequently that the sum of positive unbounded linear operators cannot be bounded. We give rather trivial proofs based only on polarization identity.

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References

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