Effect algebras with compression bases

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The current framework for discussing the logical foundations of quantum mechanics is the algebraic structure of an effect algebra, which allows the study of measurements or observables that may be unsharp (see, e.g., [1]). Gudder and Greechie [4] discussed the notion of a sequential effect algebra—an effect algebra on which a "sequential product" is defined. This sequential product satisfies a set of physically motivated axioms as it formalizes the case of sequentially performed measurements. The authors prove that the existence of a sequential product in an effect algebra is a restrictive condition, far from being met by all effect algebras.

Gudder [2] introduced the notion of a compression on an effect algebra and also of a compressible effect algebra. Although the important examples of effect algebras proves to be compressible, examples are also provided of noncompressible effect algebras.

As it turns out, the two notions (sequential effect algebra and compressible effect algebra) are somehow related, since the sequential product with a sharp element (of a sequential effect algebra) defines a compression. Although the restrictions imposed by the existence of a sequential product seem stronger than those determined by compressibility, neither of the two notions is a generalization of the other, as an example of a noncompressible sequential effect algebra shows [2]. However, in a later paper Gudder [3] introduced a common generalization of both sequential effect algebra and compressible effect algebras, namely effect algebras having a compression base.

Tkadlec [5] proved various conditions for an atomic sequential effect algebra or its set of sharp elements to be a Boolean algebra. In this paper we generalize some of these conditions to the case of effect algebras having a compression base, and also present some new ones for this more general framework. The role of the set of sharp elements of the sequential effect algebra will be played by the orthomodular poset of foci (or projections) of the effect algebra's compression base.

We establish some properties of atoms in effect algebras endowed with a compression base, mainly regarding coexistence and centrality. Then we introduce the notion of projection-atomicity which aims to be an analogue, in the framework of effect algebras with a compression base, for the property of an effect algebra of having sharp atoms—used in sequential effect algebras. Consequences of projection-atomicity are studied in general and in a particular case of effect algebras with the maximality property [6, 7]—some of which generalize results obtained in [5]. A few conditions for an atomic compression base effect algebra to be a Boolean algebra are established. Finally, we apply these results to the particular case of sequential effect algebras and find a sufficient condition for them to be Boolean algebras that strengthens previous results by Gudder and Greechie [4] and Tkadlec [5].

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