Quantum logics and two-dimensional states

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In our contribution we will deal with the characterization of orthomodular lattices by means of special types of two-dimensional states G. Under special marginal conditions a two-dimensional state G can operate as, for example, infimum measure, supremum measure or symmetric difference measure for two elements of an orthomodular lattice [2, 3, 5].

To model noncompatible events, a quantum logic was chosen among various algebraic structures as the suitable one. We can give a characterization of a center in various types of quantum logics by means of special two-dimensional states defined on them. Any quantum logic can be described as a union of blocks (a block in a given quantum logic $\mathcal{L}$ is the maximal Boolean subalgebra of $\mathcal{L}$) [6]. Center $C(\mathcal{L})$ of a quantum logic $\mathcal{L}$ is its Boolean subalgebra of elements compatible with all other elements of L. Each quantum logic $\mathcal{L}$ has a center that can be taken as a common part of its blocks. We study three types of quantum logics:

(T1) a quantum logic $\mathcal{L}$ as a horizontal sum of k maximal Boolean algebras (blocks);

(T2) a quantum logic $\mathcal{L}$ created from two blocks with non trivial center;

(T3) a quantum logic $\mathcal{L}$ with nontrivial center as a union of k blocks $\mathcal{B}_i, i \leq k$, where $\mathcal{B}_i \cap \mathcal{B}_j \subset C(\mathcal{L})$ for $i \neq j$.

Let us realize the process of investigation of two events A, B, each of them expressed as $A = \{a_1, \dots, a_n\}$, $B = \{b_1, \dots, b_k\}$, according to its organization. How to face the situation, when simple events a_i, b_j cannot be verified simultaneously, but, despite this fact, we are able to obtain some information about a_i while one of b_j does not come into being? Videlicet, how to deal with $f(a_i|b_j^\perp)$ or $f(b_j|a_i^\perp)$? For that reason we effort to find a basic structure created by these observations (e.g. whether some "property levels" of A, B are the same). More precisely let A, B be orthogonal partitions of unit 1_L. Let us denote $B^\perp = \{b_1^\perp, \dots, b_k^\perp\}$ and $A^\perp = \{a_1^\perp, \dots, a_n^\perp\}$. Then

$$P(A, B^\perp) = \begin{pmatrix} p(a_1, b_1^\perp) & \cdots & p(a_1, b_k^\perp) \\ \vdots & \ddots & \vdots \\ p(a_n, b_1^\perp) & \cdots & p(a_n, b_k^\perp) \end{pmatrix},$$

where $p(a_i, b_j^\perp) = m(b_j)m(a_i|b_j^\perp)$, $i = 1, \ldots, n$ and $j = 1, \ldots, k$. By analogy we get $P(B, A^\perp)$. Let us denote $p_s = 0.5(p(a,b) + p(b,a))$. In [4], inter alia, it has been proved that p_s is an s-map and $p_s(a,b) = p_s(b,a)$ for any $a, b \in L$ and, moreover, $p(a,a) = p_s(a,a)$ for each $a \in L$. As $d_p(a_i, b_j) = p(a_i, b_j^\perp) + p(a_i^\perp, b_j)$, the matrix

$$D_{p_s}(A,B) = \frac{1}{2}(P(A, B^\perp) + P(B, A^\perp)^T)$$

is the matrix for the function d_{p_s}. The sum of d_{p_s} throughout all levels gives us basic information about given structure [1]. For example if $S_{d_{p_s}}$ is not integer number then A, B do not create one Boolean algebra. Conversely if $S_{d_{p_s}} \in N$, it does not mean then A and B belong to one Boolean algebra.

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References

[1] Drobná, E., Nánásiová O., Valášková Ľ., Quantum logics and bivariable functions, **Kybernetika**, 46 (6), 2010, 982-995

[2] Nánásiová O., Principle Conditioning, **Int. Journ. Of Theor. Physc.**, Vol. 43, 2004, 1383-1395

[3] Nánásiová O., Map for Simultaneus Measurements for a Quantum Logic, **Int. J. Theor. Phys.**, 42, 2003, 1889-1903

[4] Nánásiová O., Trokanová K., Žembery I., Commutative and non commutative s-maps. **Forum Statisticum Slovacum** 2, 2007, 172-177

[5] Nánásiová O., Valášková Ľ., Maps on a quantum logic. **Soft Computing** Vol. 14, 2010, 1047-1052

[6] Pták P., Pulmannová S., Quantum Logics Kluwer Acad. Press, Bratislava 1991.