An Extension of a State from Interval Valued Fuzzy Sets to MV-Algebra

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The probability theory on MV-algebras was built almost ten years ago by Riečan and Mundici (see [4]). We work with other quantum structure - interval valued fuzzy sets. In the paper we take an MV-algebra, which covers the system of interval valued fuzzy sets and extend the state (equivalent of a probability function in the classical probability theory) to this MV-algebra. This way, the result of the probability theory on MV-algebras (see e. g. [1]) can be used for interval valued fuzzy sets as well. Research on similar topic - intuitionistic fuzzy sets and MV-algebras, was done by Riečan (see [2]). Moreover, there has been shown, that intuitionistic fuzzy sets and interval valued fuzzy sets are isomorphic ([3]).

May Ω be a nonempty set. By interval valued fuzzy set (or IVF-set) we mean each pair

$$A = (\mu_A, \nu_A),$$

where $\mu_A,\ \nu_A:\ \Omega \to [0,1]$ and there holds:

$$\mu_A \leq \nu_A.$$

Let's denote the family of all IVF-sets by $\mathcal{V}$. We will use Łukasiewicz connectives for $A, B \in \mathcal{V}$:

$$A \oplus B = ((\mu_A + \mu_B) \wedge 1, (\nu_A + \nu_B) \wedge 1),$$
$$A \odot B = ((\mu_A + \mu_B - 1) \vee 0, (\nu_A + \nu_B - 1) \vee 0).$$

Definition 1 *A mapping $m : \mathcal{V} \to [0,1]$ is called a state if the following properties are satisfied:*

1. $m((1_\Omega, 1_\Omega)) = 1,\ m((0_\Omega, 0_\Omega)) = 0,$
2. $A \odot B = (0_\Omega, 0_\Omega) \Rightarrow m(A \oplus B) = m(A) + m(B),$
3. $A_n \nearrow A \Rightarrow m(A_n) \nearrow m(A),$

$\forall A, B, A_i \in \mathcal{V}\ (i = 1, \ldots, n).$

The definition of a state on MV-algebra is formally the same as the Definition 1.

The main results of the paper are summarized in the following two theorems:

Theorem 1 *May* $\mathcal{G} = \{A = (\mu_A, \nu_A); \mu_A, \nu_A : \Omega \to R\}$ *and the summation be defined by the formula*

$$A + B = (\mu_A + \mu_B, \nu_A + \nu_B)\ \forall A, B \in \mathcal{G}.$$

May the partial ordering on $\mathcal{G}$ *be given by the formula*

$$A \leq B \ \Leftrightarrow\ \mu_A \leq \mu_B \ \wedge\ \nu_A \leq \nu_B$$

and - denotes the inverse operation to +, $0_{\mathcal{G}} = (0_\Omega, 0_\Omega)$ *is the neutral element of +,* $1_{\mathcal{G}} = (1_\Omega, 1_\Omega)$*. May* $\mathcal{M}$ *be an interval in* $\mathcal{G}$*,* $\mathcal{M} = [0_{\mathcal{G}}, 1_{\mathcal{G}}]$ *with the operations*

$$A \oplus B = ((\mu_A + \mu_B) \wedge 1, (\nu_A + \nu_B) \wedge 1),$$

$$A \odot B = ((\mu_A + \mu_B - 1) \vee 0, (\nu_A + \nu_B - 1) \vee 0).$$

Then the system $(\mathcal{M}, \oplus, \odot, \leq, 0_{\mathcal{G}}, 1_{\mathcal{G}})$ *is a MV-algebra and* $\mathcal{V} \subset \mathcal{M}$*.*

Theorem 2 *May* $\bar{m} : \mathcal{M} \to [0, 1]$ *be defined by the formula*

$$\bar{m}(A) = \bar{m}(\mu_A, \nu_A) = m(\mu_A, 1) - m(0, 1 - \nu_A),$$

where $m : \mathcal{V} \to [0, 1]$ *is a state on* $\mathcal{V}$*. Then*

1. $\bar{m}(A) = m(A)\ \forall A \in \mathcal{V}$*,*
2. $\bar{m}$ *is a state on* $\mathcal{M}$*.*

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References

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