State operators on Boolean algebras

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State MV-algebras, i.e. MV-algebras with an internal state that were introduced by T. Flaminio and F. Montagna in [8, 9] were extended for even more general structures like BL-algebras in [3] and non-commutative versions of MV-algebras (pseudo MV-algebras, GMV-algebras) in [10], pseudo BL-algebras and lastly for bounded pseudo-hoops by L. C. Ciungu in [2]. Moreover, several types of so called state operators were studied including endomorphisms h that satisfy condition $h^2 = h$, i.e. $h(h(x)) = h(x)$. Such morphisms were called state-morphisms and for MV-algebras they were mainly studied by A. Di Nola and A. Dvurečenskij (later also by A. Lettieri) [4, 5, 6].

This short contribution tries to recall some stages of development of the notion of state algebras and present some results that are common for various structures. Moreover, for the case of Boolean algebras it is shown that if we apply original definition of T. Flaminio and F. Montagna then the only state operators are precisely endomorphisms h satisfying $h^2 = h$ property. This may shed light on the reason for introducing the notion of state-morphisms.

Recent direction of investigation (not mentioned in this contribution) is a characterization of subdirectly irreducible elements of state structures, e.g. for MV-algebras [7] and for BL-algebras (and more general structures) [1].

References

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