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# The generalized Choquet integral, comparison with the Choquet integral, and calculation methods

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The Choquet integral is today perhaps the best-known type of integral in the area of nonadditive measures and integrals. It is named after French mathematician Gustave Choquet and since its introduction, several generalizations appeared in the literature [4, 6, 7]. In the contribution, we shall deal with its generalization that is based on the so-called conditional aggregation operators. This construction was developed by Boczek et al. in [2] following the idea of super level measure introduced by Do and Thiele in the paper [5].

The conditional aggregation operator with respect to a set $B \in 2^{[n]} \setminus \{\emptyset\}$, where $[n] = \{1, 2, \ldots, n\}$, $n \in \mathbb{N}$ is a basic set, is a map $\mathsf{A}(\cdot|B)\colon [0,\infty)^{[n]} \to [0,\infty)$ that satisfies the following conditions:

(i) $\mathsf{A}(\mathbf{x}|B) \leq \mathsf{A}(\mathbf{y}|B)$ for any $\mathbf{x}, \mathbf{y} \in [0,\infty)^{[n]}$ such that $x_i \leq y_i$ for any $i \in B$,

(ii) $\mathsf{A}(\mathbf{1}_{B^c}|B) = 0$.

Conditional aggregation operators are a key notion in the concept of a generalized survival function:

$$\mu_{\mathscr{A}}(\mathbf{x}, \alpha) := \min\left\{\mu(E^c) : \mathsf{A}(\mathbf{x}|E) \leq \alpha,\, E \in \mathscr{E}\right\}, \quad \alpha \in [0,\infty),$$

with $\mathscr{A} = \{\mathsf{A}(\cdot|E) : E \in \mathscr{E}\}$ being a family of conditional aggregation operators. It is known that for $\mathscr{A} = \mathscr{A}^{\max}$ with collection $\mathscr{E} = 2^{[n]}$, the standard survival function and the generalized survival function coincide. For general sufficient and necessary conditions under which the above-mentioned survival functions equal we recommend [1].

It is known that the standard Choquet integral is defined as the improper Riemann integral of the survival function. Analogously, the generalized Choquet integral is improper Riemann integral of the generalized survival function. It is therefore a natural question: Which properties do these Choquet integrals have in common and in which do they differ? Directly from the definition of the generalized Choquet integral, it is possible to derive its basic properties such as nondecreasingness with respect to the input vector and monotone measure. Further, the generalized Choquet integral of zero input vector is zero. However, deriving more complex properties based only on the definition is a challenge. In this contribution, we shall discuss further properties of the generalized Choquet integral. As a helpful tool, we use the result presented in [3]. In this paper, the authors have shown that the generalized Choquet integral is the standard Choquet integral on hyperspace, i.e. the following theorem holds: Let $\mathscr{A}$ be a family of conditional aggregation operators with $\mathscr{E} \supseteq \{\emptyset, [n]\}$, $\hat{\mathscr{E}} = \{E^c : E \in \mathscr{E}\}$, $\mu$ be a monotone measure on $\hat{\mathscr{E}}$ and $\mathbf{x} \in [0,\infty)^{[n]}$. Then

$$\mathrm{C}_{\mathscr{A}}(\mathbf{x}, \mu) = \mathrm{Ch}(T_{\mathscr{A},\mathbf{x}}, \mathtt{N}_\mu),$$

where $T_{\mathscr{A},\mathbf{x}}\colon \hat{\mathscr{E}} \to [0,\infty)^{[n]}$ such that $T_{\mathscr{A},\mathbf{x}}(E) = \mathsf{A}(\mathbf{x}|E^c)$ and $\mathtt{N}_\mu\colon 2^{\hat{\mathscr{E}}} \to [0,\infty)$ is a map defined as $\mathtt{N}_\mu(\hat{E}) = \min\{\mu(E) : E \in \hat{\mathscr{E}} \setminus \hat{E}\}$.

It is known that the Choquet integral can be applied in many areas of everyday life thanks to its properties. Among them, the best-known and widely used properties are idempotency or the property of being an averaging type of aggregation. In this contribution, we shall deal with the assumptions (also derived by the transformation theorem) under which the generalized Choquet integral also possesses these properties. We shall also look at e.g. the submodularity of this new integral.

From the application point of view, it is also important to know the formulas for calculating the generalized Choquet integral. In the talk, we shall point out two possible approaches to its calculation (via aggregation operators and via monotone measure). We shall also present pseudo-algorithms for their software implementation and we indicate advantages and disadvantages of both approaches.

**Acknowledgement.**
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## Aggregation functions in optimization of ensemble classifiers

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In this contribution, combination of classifiers (cf. [2, 5]) is considered in the case of high dimensional datasets. As a base classifier we consider decision tree with diverse parameters. Aggregation functions are applied to fuse the output values of the individual classifiers. Known families of aggregation functions [1] such as quasi-arithmetic means, ordered weighted averages [4], convex combinations of means and examples of pre-aggregation functions [3] are studied and compared concerning their usefulness in the given classification method. Based on the performed experiments the ranking of aggregations was determined with respect to each parameter of an aggregation. Moreover, several statistical tests were performed to study the groups of aggregations (without distinguishing their parameters) concerning the statistical significance of the obtained results. The presented algorithm is focused on a non public-available dataset called Asthma which was gathered as a result of the scientific project dedicated to the disease of asthma which was conducted together with medical specialists (cf. [6]). However, other well-known in literature microarray datasets were also taken into account in order to compare the obtained results between the new datasets and some existing ones.

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# On a fuzzy implication and its natural negation

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Fuzzy implications are one of the main operations in fuzzy logic. For this reason many families of these connectives are examined. We present a construction that allows us to obtain a new fuzzy implication from a family of fuzzy connectives of this type. It is a modification of the one introduced in Lima, Bedregal and Mezzomo (2000). We examine its properties depending on the properties of its generators. Moreover, we consider the relationship between the construction and a general structure presented in Zhou (2021).

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## On OWA operators based on generalized Choquet integrals

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The ordered weighted averaging operator (OWA), a very simple but powerful aggregation tool, was introduced by R. R. Yager [1]. Since its introduction in 1988, it obtained a great attention from the scientific community and it has been applied in various areas, for example, in decision making, data mining, image recognition, etc. As it is well-known, OWA operators are a special case of the Choquet integral with respect to a symmetric measure. Nowadays, there are many extensions and generalizations of OWA operators, see the review paper [3].

Recently, Boczek et al. [2] introduced a conditional aggregation operator which plays an important role in defining a generalized level measure. The new approach through conditional aggregation operators and generalized level measures provides a wide range of options for defining generalized integrals based on generalized level measures, for example, the generalized Choquet integral. In our talk, we propose a generalized OWA operator based on the generalized Choquet integral with respect to a symmetric measure and we examine some of its basic properties and applications.

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# On maximal chain-based Choquet-like integrals

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By detailed studying the basic properties (e.g. transitivity) of the Choquet integral relation established in [2], we noticed a very interesting formula for computing "discrete Choquet-like integrals" derived from the original Choquet integral. Obviously, in definition of the Choquet integral of a vector $\mathbf{x} \in [0,\infty[^n$ with respect to a monotone measure $\mu$, the permutation $\sigma : [n] \to [n]$ plays an ordering role in order to reorder $x_i$ according to their magnitudes, so the discrete Choquet integral can be written as

$$\mathrm{nCh}(\mathbf{x},\mu) = x_{x_{\sigma(1)}}\mu\left(A_{\mathbf{x},\sigma(1)}\right) + \sum_{i=2}^{n} x_{x_{\sigma(i)}}\left(\mu\left(A_{\mathbf{x},\sigma(i)}\right) - \mu\left(A_{\mathbf{x},\sigma(i-1)}\right)\right)$$

with $x_{x_{\sigma(i)}}$ indicating the $i$-th largest element of $\mathbf{x}$ and $A_{\mathbf{x},\sigma(i)} = \{\sigma(1),\dots,\sigma(i)\}$. Then it is natural to rearrange the vector $\mathbf{x}$ according to a different vector $\mathbf{y}$. Indeed, following [2, equation (A.10)] for $\mathbf{x},\mathbf{y} \in [0,\infty[^n$ we put

$$\mathrm{nCh}_{(\mathbf{y})}(\mathbf{x},\mu) := x_{y_{\sigma(1)}}\mu\left(A_{\mathbf{y},\sigma(1)}\right) + \sum_{i=2}^{n} x_{y_{\sigma(i)}}\left(\mu\left(A_{\mathbf{y},\sigma(i)}\right) - \mu\left(A_{\mathbf{y},\sigma(i-1)}\right)\right), \tag{1}$$

where the permutation $\sigma$ rearranges the vector $\mathbf{y}$ (not vector $\mathbf{x}$!). On the one hand, we provide a generalization of (1), known as the induced Choquet integral [4], in two ways. On the other hand, the idea of extending this formula for arbitrary vectors $\mathbf{x}$ defined on $]-\infty,\infty[^n$ presented by Honda et al., leads to interesting results and maybe, it can be understood as an extension of the induced Choquet integral.

As the name of the new class of integrals suggests, the main ingredient in its definition is a maximal chain $\mathcal{C} = \{C_0, C_1, \dots, C_\ell\}$ in a $\sigma$-algebra $\Sigma$ of subsets of $X = [n]$. Considering the concept of conditional aggregation operator $\mathsf{A}$, see [1], and nonnegative inputs $\mathbf{x} \in [0,\infty[^n$ we introduce the functional

$$\Sigma\text{-}\mathrm{MaxCh}_{\mathcal{C},\mathsf{A}}(\mathbf{x},\mu) = \sum_{k=1}^{\ell} \mathsf{A}(\mathbf{x}|C_k \setminus C_{k-1}) \otimes (\mu(C_k) - \mu(C_{k-1})), \tag{2}$$

where $\otimes\colon [0,\infty[^2 \to [0,\infty[$ is some binary relation. The idea of using the maximal chains on $\Sigma = 2^{[n]}$ and $\otimes = \cdot$ can be identified with the idea of reordering some input according to other vector as in the case (1). However, the case $\Sigma \subset 2^{[n]}$ seems to be more interesting. We will show that formula (2) covers for instance IOWA operators, but surprisingly it can be computed from the vector $\mathbf{x}$ which is not measurable with respect to $\Sigma$.

Except of this interesting point, we will show an equivalent representation of MCC-integral using Möbius transform. As indicated above, the study of MCC-integrals for real-valued inputs is also very interesting. The idea is analogous to idea of extending the Choquet integral in the symmetric and asymmetric way. In fact, writing a new functional as a difference of two MCC-integrals, we have 64 integrals that can be reduced to four classes of functionals somehow connected with the symmetric and asymmetric Choquet integral and their complementary versions, too, i.e., a quadruplet of Choquet integral, see [3].

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# Modifiers preserving monotony

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A fuzzy set can represent a linguistic variable, like dark, young, fast, etc. Such a variable can be altered using a modifier, like very, little, somewhat, etc. We deal with a question under which conditions a modified fuzzy set preserves the monotony of the original one. In this research we focus on premodifiers and postmodifiers with values in a bounded lattice and with relational modifiers, that represent the case when a particular relation among the elements of the domain is known. In both cases we provide a complete description of such modifiers.

If $\mathcal{F}(X)$ denotes the collection of all fuzzy subsets of $X$, then by a fuzzy modifier we understand any mapping $m : \mathcal{F}(X) \to \mathcal{F}(X)$ (see [1]). A detailed study of modifiers can be found in [2] or [3]. The authors in [1] distinguish between modifiers with pure premodification (a shift in the direction of $X$) and modifiers with pure postmodification (a shift in the direction of the unit interval). More exactly:

**Definition 1** *[1] Let $m$ be a modifier on $\mathcal{F}(X)$. If there is a mapping $t : X \to X$ such that for each $A \in \mathcal{F}(X)$ there is $m(A) = A \circ t$, then $m$ is a modifier with pure premodification.*

**Definition 2** *[1] Let $m$ be a modifier on $\mathcal{F}(X)$. If there is a mapping $r : [0, 1] \to [0, 1]$ such that for each $A \in \mathcal{F}(X)$ there is $m(A) = r \circ A$, then $m$ is a modifier with pure postmodification.*

The mapping $t$ is called a premodifier and the mapping $r$ is called a postmodifier. For these we have the following results, both are valid for lattice-valued fuzzy sets:

**Proposition 1** *A postmodifier $r$ preserves convexity if and only if it is a meet homomorphism on $L$, i.e. for each $\alpha, \beta \in L$ there is $r(\alpha \wedge \beta) = r(\alpha) \wedge r(\beta)$.*

**Proposition 2** *The premodifier $t$ preserves convexity if and only if $t$ is monotone.*

We also study relational modifiers, defined in [1]. The first of the modifier classes is based on a given relation and a conjunctor.

**Definition 3** *[1] Let $X$ be a universe equipped with a fuzzy relation $R : X^2 \to [0, 1]$. Let $A : X \to [0, 1]$ and $C$ be a conjunctor on the unit interval. Then the mapping $R_C(A) : X \to [0, 1]$ for which*

$$R_C(A)(x) = \sup\{C(R(t, x), A(t)), t \in X\}$$

*is a fuzzy modifier of $A$ under the relation $R$ and the conjunctor $C$.*

The second class of modifiers is based on a given relation and an implicator.

**Definition 4** *[1] Let $X$ be a universe equipped with a fuzzy relation $R : X^2 \to [0,1]$. Let $A : X \to [0,1]$ and $I$ be an implicator on the unit interval. Then the mapping $R_I(A) : X \to [0,1]$ for which*

$$R_I(A)(x) = \inf\{I(R(t,x), A(t)), t \in X\}$$

*is a fuzzy modifier of $A$ under the relation $R$ and the implicator $I$.*

For these modifiers we have the following results, in which by a relation compatible with the order in $X$ we understand a relation $R$, such that $R(x,z) \leq R(x,y)$ for $x < y < z$ and by a seminorm we understand a conjunctor with the neutral element 1.

**Proposition 3** *The modifier $R_C$ preserves monotony of $A$ for each $A : X \to [0,1]$ if and only if $R$ is a reflexive relation compatible with the order on $X$, and $C$ is a seminorm.*

**Proposition 4** *Let $I$ be an implicator. Then $R_I$ preserves monotony of $A$ for each $A$ if and only if $R$ is a reflexive relation compatible with the order on $X$.*

As convex fuzzy sets on a totally ordered space consist of an increasing and a decreasing part, monotony in both previous statements can be replaced by convexity.

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## Discussion on idempotent associative monotone binary operations on posets

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Uninorms, nullnorms and other binary associative aggregation functions are usually studied on bounded lattices. In [2], we generalized this approach and studied uninorms and nullnorms on bounded posets. Since [2] was written during the CoVID pandemic, we have decided to come back to that paper.

In our contribution we will recall some sufficient and some necessary conditions set on a poset under which it is possible to construct idempotent associative and monotone binary operations defined on such a poset. Our attention will be focused mainly on uninorms and nullnorms.

We will show also first results of our new study. This new part concerns 2-uninorms (see [1]). The extension of our study is quite natural since proper nullnorms are special cases of proper 2-uninorms. In our interpretation, a 2-uninorm is proper if it is not a uninorm, this means, if the operation has no neutral element. We will discuss also several types of 2-uninorms on posets.

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# Special cases of aggregation functions on bounded lattices

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The map $Ag : \bigcup_{n \in \mathbb{N}} [0,1]^n \to [0,1]$, for which the monotonicity and boundary conditions are satisfied, is said to be an aggregation function. Obviously, the family of all triangular norms (t-norms) and triangular conorms (t-conorms), respectively, is the special case of the aggregation functions. For instance, an aggregation function with a special position with respect to the family of all triangular norms (conorms), is an arithmetic mean (average), denoted by $Avg$, where $T \leq T_M \leq Avg \leq S_M \leq S$ and $T(S)$ is an arbitrary t-norm (t-conorm) and $T_M(S_M)$ denotes the minimum (maximum).

Our study is divided in two directions. In the first step, we study t-norms as a part of the family of aggregation functions bounded by the drastic t-norm and the minimum t-norm, respectively. T-norms are a kind of binary operators widely used in fuzzy set theory and fuzzy logic (see [1, 2]). They were originally introduced in the unit interval $[0,1]$ by Menger (1942) in his study of statistical metric spaces. Many authors have extended the t-norms into a wider environment, specifically, into the bounded lattices (see [3]). Any bounded lattice with a t-norm forms a partially ordered commutative monoid with neutral element 1. We have focused on tools which allow us to obtain t-norms on special families of bounded lattices. Some general results for the construction of t-norms on bounded lattices are presented. We would like to determine the number of t-norms on these algebraic objects. In our contribution are presented some examples of "small" lattices, i.e. the lattices with number of elements up to six. For each of them we know the complete collection of t-norms.

Analogously, we consider the means as an another interesting part of the aggregation functions. The means are bounded by the minimum and the maximum operator. We have chosen both groups for deeper study since many applications of these groups are known. In similar way, we present the means on some examples of lattices.

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# Risk measures from the point of view of Choquet integral quadruplet

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Choquet integral quadruplet consists of four integrals of Choquet type, namely asymmetric and symmetric Choquet integrals as well as their complementary versions, which were introduced in [1]. They all are direct generalizations of Lebesgue integral, from which possible applications arise. One of them is considered in risk theory and finance, particularly in defining risk measures used to assess how to invest or diversify one's portfolio. On the basis of a specific risk measure called distortion risk measure given as

$$\rho_m(\xi) = \int_0^\infty m(\mathcal{S}_{P,\xi}(x))\,\mathrm{d}x,$$

with $m : [0,1] \to [0,1]$ being a so-called distortion function where $m(0) = 0$ and $m(1) = 1$, a new risk measure is proposed with respect to the whole Choquet integral quadruplet. Its basic properties relevant to the application are studied. In addition, an implementation with real-life data representing the S&P 500 index of currently most productive corporations in the US is performed.

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# Possible and necessary similarities measures of interval-valued fuzzy sets used to selection of relevant features in diagnostic problem

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This presentation will be concentrated on the application of possible (optimistic) and necessary (pessimistic) similarity measures of interval-valued fuzzy sets to the problem of selecting relevant attributes as input to medical diagnostic by the classification algorithm. The proposed measures are based on the measures of comparability in the sense of possible (optimistic) and necessary (pessimistic) studies in [2]. The paper presents a modified IV-Relief algorithm using mentioned above similarity measures adequate to the epistemic issues. We study the effectiveness of the proposed algorithm on a well-known breast cancer diagnostic data set. The proposed techniques extend existing classification and measure methods so that they work on uncertain data and develop methods from the papers [1, 3].

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## Fuzzy local contrast measures based on aggregation functions

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In this contribution the concept of a local contrast of a fuzzy relation [4] with the use of a consensus measure (cf. [2]) is introduced. A construction method of such local contrast using aggregation functions and fuzzy implications is considered. Other construction methods are also pointed out (cf. [3]). Several examples of local contrasts are provided. In image processing, a grayscale image of $N \times M$ pixels may be interpreted as a collection of $N \times M$ elements arranged in rows and columns. A numerical value representing intensity, chosen from the set $\{0, 1, 2, ..., L - 1\}$, is assigned to each element. An image $Q$ is just a matrix so it may be represented as a fuzzy relation $R$ on a finite set [6] such that the membership degree of each element (pixel) is its intensity divided by $L - 1$. For a color image, its channels can be independently represented in the same way. The salient region detection problem [1, 5] relies heavily on the concept of image contrast and has commonly accepted measures of solution quality. Therefore we propose to evaluate introduced fuzzy contrasts on real-world data by applying them in a salient region detection problem. We believe that evaluating the quality of obtained saliency maps will give us a clue about the usefulness of local contrast definitions and also possibility to compare multiple ones.

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# A big data classification in survival analysis using dependent competing risks

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Recently, as production of data is vastly increasing, massive data sets are collected from different sources, e.g. results of sensors in an industrial process or in healthcare devices. These data sets are often used to predicting/classifying one continues/dichotomous response variable by all the other explanatory variables. The concordance probability index, also called the C-index, is a known as a popular measure to asses the discriminatory ability of a classification model [1]. In this work, we introduce a copula-based classification approach in survival analyses when the competing risks are dependent.

Consider a data set $\mathbf{D} = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_n, y_n)\}$, where $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{id})$, $\mathbf{x}_i \in \mathcal{X}$ is the i-th vector of inputs or observations for $i = 1, 2, \ldots, n$, and $y_i \in \mathcal{Y}$ is the corresponding outcome variable. In a classification approach, the major goal is constructing (or finding) a separating hyper plane classifier to learn a map $f : \mathcal{X} \to \mathcal{Y}$ which takes the features $\mathbf{x} \in \mathbf{X}$ of a data point as its input and outputs a predicted label. The special case of the outcome variable is a class label $y_i \in \{-1, 1\}$, i.e., it has two possible values such as: negative/positive, pathogenic/benign, patient/normal, etc.

The concordance probability corresponds to the probability that a randomly selected subject with outcome $Y = 1$ has a higher predicted probability $\pi(\mathbf{X}) = \mathrm{P}(Y = 1|\mathbf{X})$ than a randomly selected subject with outcome $Y = 0$, where $\mathbf{X}$ corresponds to the vector of variables [3]:

$$C = \mathrm{P}(\pi(\mathbf{X}_i) > \pi(\mathbf{X}_j)|Y_i = 1, Y_j = 0). \tag{1}$$

A pair of an observation with its prediction that satisfies the above condition is called a concordant pair. Hence, the concordance probability can also be defined as the probability that a randomly selected comparable pair of observations with their predictions and its normal ranges between 0.5 and 1, and the closer it is to 1, the better is its discriminatory ability. If its value drops below 0.5, the predictions are consistently inconsistent. As a special case of 1, the Receiver Operating Characteristic curve (ROC) has been suggested by Bamber (1975) [3] and the Area Under the ROC Curve (AUC) is a well known index of the discriminatory ability of a classifier. It is known that, in a binary outcome and in the absence of ties in the predictions, the concordance probability, C, equals the AUC.

In this work we consider that the explanatory random variables $X_1, X_2, \ldots, X_d$ are related using a copula, i.e., there exist a grounded, d-increasing and uniformly distributed marginal function $C : [0,1]^d \to [0,1]$, in which it joins the multivariate distribution function $F_{X_1,X_2,\ldots,X_d}$ to its marginals $F_{X_i}, i = 1, 2;, \ldots, d$, using

$$F_{X_1,X_2,\ldots,X_d}(x_1, x_2, \ldots, x_d) = C(F_{X_1}(x_1), F_{X_2}(x_2), \ldots, F_{X_d}(x_d)). \tag{2}$$

Using a machine learning approach, we first train associated copula as well as the concordance probability index in the "train set" and implementing the results of the train set on the "test set", helps us to identify the most accurate classification rule. In this regards, we asses our approach in a simulated data set and present its application in a real life data set. In the simulation study, we consider three copulas: Gaussian, t and Gumbel. We generate $n = 5000$ random samples from a trivariate Gaussian copula, a trivariate t copula and a bivariate Gumbel copula in such a way that there are some linear as well as some nonlinear relations between the generated variables. Our criterions in this study are three most important evaluation measures: Sensitivity, Specificity and Accuracy [4]. A cross-validation analysis shows that implementing the dependency between the explanatory variables significantly improves the performance of the classification regarding to these three criteria. Also, considering the German breast cancer data set, and assuming that the main predictors of interest are: hormonal therapy (yes/no), tumor size, tumor grade (levels: I to III), number of lymph node, progesterone receptor concentration, estrogen receptor concentration, and the outcome is the binary variable: whether the disease reoccured/not, we found the outperform of our algorithm.

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## Data analysis with using generalized additive models

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Smoothing models, and in particular, generalized additive models (GAM) are a collection of non-parametric regression techniques that attempt to estimate complex relationships between the response variable and the covariates. The advantage of GAMs in respect to other models is that the shape of the response curves reflecting the relationships between dependent and continuous independent variables are data driven, instead of being predefined by parametric forms.

Before statistical modelling the dataset were inspected for identifying outliers in the data, homogeneity and zero inflation in the dependent variable, collinearity between explanatory covariates and the nature of relationships between response and independent variables. To understand the influence of different predictors on parasite abundance, a GAM with the NBD andlogarithmic link function were applied. The final models included host length and water temperature (model 1) or months (model 2) as predictor variables. The response curves in the GAM showed that relationships between the abundance of parasite and the three variables selected were not linear. The probability of having parasites increases with longer fish length, low temperature and cold season.

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# Positively homogeneous and super-/sub-additive aggregation functions and properties of their generalized opposite diagonal cuts

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An aggregation function is any non-decreasing function $A\colon [0,\infty[^n \to [0,\infty[$ satisfying the condition $A(\mathbf{0}) = 0$, where $n \in \mathbb{N}$ is some natural number. Let

$$S = \big\{\mathbf{x} \in [0,1]^{n-1} \colon \Sigma(\mathbf{x}) \leq 1\big\}.$$

A generalized opposite diagonal (GOD) function of the aggregation function $A$ is a mapping $\delta_A\colon S \to [0,\infty[$ given by $\delta_A(\mathbf{x}) = A\big(\mathbf{x}, 1-\Sigma(\mathbf{x})\big)$, where $\Sigma(\mathbf{x})$ denotes the sum of the coordinates of the vector $\mathbf{x}$.

An aggregation function $A$ is called super-additive if and only if

$$A(\mathbf{x}+\mathbf{y}) \geq A(\mathbf{x}) + A(\mathbf{y})$$

and sub-additive if and only if

$$A(\mathbf{x}+\mathbf{y}) \leq A(\mathbf{x}) + A(\mathbf{y})$$

for every $\mathbf{x}, \mathbf{y} \in [0,\infty[^n$.

We have obtained the following result [1] relating positive homogeneity of super-additive and sub-additive aggregation functions to concavity and convexity of their GOD functions, respectively.

**Theorem 1.** *A positively homogeneous aggregation function $A$ is super-additive (sub-additive) if and only if its generalized opposite diagonal cut $\delta_A$ is concave (convex).*

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# Open problems of the construction of moderate deviation functions

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Aggregation functions are used to evaluate the data, the resulting values of which should characterize the set of input data as accurately as possible and thus significantly help in decision-making. Construction of specific means using the so-called deviation function was introduced by Z. Daróczy [2]. These means are based on the deviation between two real values and can be used to merge a set of input values into one aggregated output value. However, Daróczy means in some cases do not meet the properties of aggregation functions, since they need not be monotone.

In [3–5] the so-called moderate deviation function is introduced, which ensures that functions based on moderate deviation functions satisfy all properties of the aggregation functions. At present, research offers various constructions of aggregation functions, which are based on the use of the mentioned moderate deviation functions [1, 3–6].

The contribution introduces new constructions of the so-called moderate deviation functions using an automorphism $\phi$ in the symmetric interval $[-1, 1]$ and a negation, which is generated using the automorphism in the form $N(x) = \phi^{-1}(-\phi(x))$. It also proposes construction methods for a moderate deviation function in a general closed interval $[a, b] \subset ]-\infty, \infty[$ by isomorphism from the interval $[a, b]$ to $[-1, 1]$. The aggregation functions constructed this way meet all the required properties of the aggregation functions.

The main part of the contribution is the solution of open problems related to the generation of such nontrivial constructions of moderate deviation functions that will show the full essence of aggregation functions based on moderate deviation functions.

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# Ordering intervals by degree of totalness

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In the domain of intervals, the attempt to extend aggregation functions that require ordering of all the elements in question, such as the Choquet integral, generates an additional problem since the ordering of a finite number of intervals is not possible by means of a partial order. The simplest solution is based on Aumann's approach of set-valued functions integration [1]. However, it does not consider the initial structure of the elements to be aggregated, but works separately with the upper and lower bounds. In order to take into account the ordinal structure of the intervals, an approach was proposed in [2] by means of admissible orders [3]. But such a proposal raises the problem of selecting an appropriate admissible order. In order to try to take into account the ordinal structure of the data, the concept of admissible permutation is introduced in [4]. This concept allows us to determine the number of possible orderings that can be obtained by admissible orders. This work proposes the aggregation by consensus of all Choquet integral fused values that can be obtained by all admissible permutations.

However, the problem of choosing the best admissible permutation remains open. To solve this problem we introduce the concept of degree of totalness. Starting from the assumption that the partial order is a crisp relation, the intention of our work is to fuzzify this idea, so that we can obtain a degree to which this relation is held. Then, the Choquet integral for intervals with respect to the best admissible permutation is introduced.

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# A general framework for selecting and ranking records from heterogeneous datasets based on interval-valued fuzzy sets and admissible orders

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Retrieving records from a dataset is an important task in nowadays data intense evaluations and computer applications. However, in real-word data collections, data can be heterogeneous showing a mixture of numerical, categorical, fuzzy, binary or other data types. Moreover, users' preferences can be expressed in various ways, which do not always match the types of attributes' values in a dataset or may be complex (e.g., different interest among preferences, constraints, wishes, etc.) [1]. In practice, heterogeneity is considered as a special type of contamination in the process of retrieving records. The main challenge in searching heterogeneous datasets is processing heterogeneity and uncertainty in the data [2]. Although some existing approaches can handle these issues in querying by using available similarity measures[3], the most of them can only be applied to one type of data. These methods may be still improved by eliminating the inconsistency in computing the similarity measures. Efficient selection and ranking of heterogeneous records require a flexible mechanism that can be applied to any combination of data types. Our idea is to construct a universal framework for handling data heterogeneity and uncertainty in building a robust query engine. Consequently, we transform data and user preferences into fuzzy domain and define a conformance measure for querying heterogeneous objects described by mixed attributes. Fuzzy set theory can deal with imprecision and heterogeneity, but with mitigated efficacy when using ordinal (type-1) fuzzy sets[4], in particular for record ranking. In order to overcome this shortcoming, we study interval-valued fuzzy sets [5] for better representation of uncertainty and heterogeneity as well as interval-valued aggregation functions. To achieve the appropriate ranking of the heterogeneous records, it will be necessary to compare the resulting intervals[6]. We assume that the proposed approach can benefit in a wide range of heterogeneous data processing tasks, such as classification, data mining, recommendation, data analysis, decision-making, etc.

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# Biological data: image processing, segmentation analyses, grab cut, maschine learning and other techniques

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This contribution deals with the proceeding of medical and biological data. Mathematical modeling is a very good and strong tool for the analysis and modeling of this data. Consequently, mathematical modelling brings a good analytical, enumeration, and visualization perspective if representing biological and medical data for users. We discuss some methods for medical image analyses. One point brings perspective from discrete mathematics and discrete algorithms, dealing with graph theory, statistics, data anlyses, image processing and other one perspective brings maschine learning, We focus on the analyses of the techniques, using image processing and segmnetation of the objects, Our segmentation process is based on graph cuts, grab cuts, machine learning, and other types of techniques.

A machine learning-based segmentation technique is required to get good performance to deal with biological images. Through semantic segmentation, regions of interest can be identified for cell assessment. Clinicians can use segmentation results to identify abnormal cell and improve therapy planning. The creation of high-quality labelled and annotated datasets is a critical part of achieving the algorithmic goal of automated medical image segmentation. It is often difficult to collect clean annotations for cell segmentation at pixel level. To overcome the pixel level segmentation approach, semantic segmentation is one of the most important steps in the computer-aided diagnosis process. In this work, we propose a semantic segmentation framework which is based on the U-net architecture. This framework consists of an encoder-decoder network and a skip connection. The encoder-decoder networks share a common characteristic of combining coarse-grained, shallow, low-level semantic features of the encoder sub-network with fine-grained features of the decoder sub-network.

In this work we have a cooperation with Medical Faculty of Commenius University in Bratislava. We cooperate with the Institute of Imunology, the Institute of Anatomy, the Institute of Medical Physics, Biophysics, Informatics and Telemedicine. Image data comes from Comenius University, from Medical Faculty.

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