

On clinker equivalences and their applications

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Dedicated to the memory of Joanna Grygiel

1. Preliminaries

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Pseudocomplemented lattices and related notions

A bounded lattice L is called **pseudocomplemented**, if for any $x \in L$, there exists an element $x^* \in L$ such that for any $z \in L$,

$$x \wedge z = 0 \Leftrightarrow z \leq x^*.$$

$x^* \in L$ is called the **pseudocomplement** of x .

A *Stone algebra* is a pseudocomplemented distributive lattice L such that for every $x \in L$,

$$x^* \vee x^{**} = 1. \quad (S)$$

Regular double Stone algebras

A *double Stone algebra* is a Stone algebra $(L, \leq)$ whose dual $(L, \geq)$ is also a Stone algebra. The dual pseudocomplement of an element $x \in L$ is denoted by x^+ .

A double Stone algebra $(A, \vee, \wedge, *, ^+, 0, 1)$ is called *regular*, if for any $x, y \in L$

$$x^* = y^* \text{ and } x^+ = y^+ \text{ imply } x = y. \quad (M)$$

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- Rough sets were introduced in: Zdzisław Pawlak, *Rough Sets*, Int. J. of Computer and Information Sciences **11** (1982), 341-356.
- His idea was to **approximate** sets in terms of an **information relation** R that describes, for instance, similarity of the objects in a universe U .

Notations

Let R be a binary relation on a set U and $x \in U$; let us denote

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If R is reflexive, then $X_R \subseteq X \subseteq X^R$. The **rough set** of X is the pair

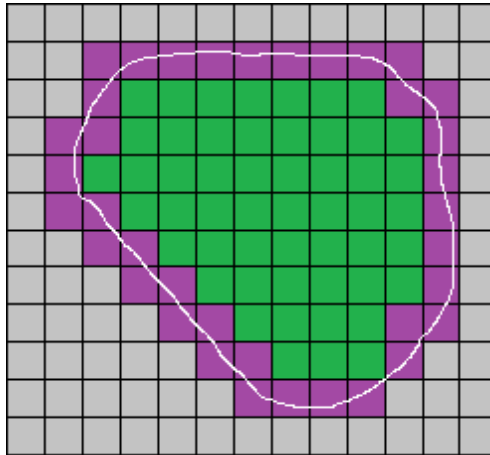
$$\mathcal{A}(X) := (X_R, X^R).$$

The set of rough sets is $RS := \{\mathcal{A}(X) \mid X \subseteq U\}$. This set is ordered by the componentwise order:

$$\mathcal{A}(X) \leq \mathcal{A}(Y) \Leftrightarrow X_R \subseteq Y_R \text{ and } X^R \subseteq Y^R.$$

Visualization of the rough approximations - Figure 1.

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- It was proved that RS is an algebraic, regular double Stone lattice.
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Surprisingly, we found a specific class of reflexive relations, which are neither symmetric nor transitive in general, with the property that the ordered set $(RS, \leq)$ forms an algebraic regular double Stone algebra. These relations, which are more general than the equivalence relations are called **clinker equivalences**.

3. The notion of a clinker equivalence

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Irredundant coverings

A system $\mathcal{H} \subseteq \wp(U)$ of nonempty subsets of U is a *covering* of U iff

$$\bigcup \mathcal{H} = U.$$

The covering $\mathcal{H}$ is *irredundant* if $\mathcal{H} \setminus \{X\}$ is not a covering of U for any $X \in \mathcal{H}$.

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Definition

Let R be a reflexive relation, and let R^{-1} stand for its inverse. We say that R is a **clinker equivalence** if $\{R^{-1}(x) \mid x \in U\}$ is an **irredundant covering** of U .

The term **clinker equivalence** was chosen because the overlapping sets $\{R^{-1}(x) \mid x \in U\}$ resemble the overlapping wooden strakes of a Viking ship's hull, referencing the historical clinker-built method of Norse shipbuilding.

Remark

For any irredundant covering $\mathcal{H}$ of the universe U there exists a clinker equivalence on $R \subseteq U \times U$ such that $\{R^{-1}(x) \mid x \in U\} = \mathcal{H}$.

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Another approach to clinker equivalences is by Formal Concept Analysis.

4. Some notions from FCA

Formal Contexts

A *formal context* $\mathcal{K} = (G, M, I)$ consists of two sets G and M and a binary relation $I \subseteq G \times M$. The elements of G are called *objects* and the elements of M are called *attributes* of the context; in order to express that the object $g \in G$ has the attribute $m \in M$ we write $(g, m) \in I$. for all subsets $A \subseteq G$ and $B \subseteq M$ we define:

$$A' := \{m \in M \mid (g, m) \in I, \text{ for all } g \in A\}, \text{ and}$$

$${}^{\prime}B := \{g \in G \mid (g, m) \in I, \text{ for all } m \in B\}$$

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A *formal concept* of the context $\mathcal{K}$ is a pair $(A, B) \in \wp(G) \times \wp(M)$ with $A' = B$ and ${}^{\prime}B = A$. The set A is called the *extent* and B is called the *intent* of the concept (A, B) .

Concept lattices

The set of all concepts of the context (G, M, I) is denoted by $\mathcal{L}(G, M, I)$ and $\mathcal{E}(G, M, I)$ stands for the set of all extents of (G, M, I) . Ordering $\mathcal{L}(G, M, I)$ as follows

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$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \Leftrightarrow B_1 \supseteq B_2,$$

we obtain a complete lattice, the *concept lattice of the context* $\mathcal{K} = (G, M, I)$. Similarly, ordering the extents of $\mathcal{K}$ by $\subseteq$, we get a complete lattice $(\mathcal{E}(G, M, I), \subseteq)$ isomorphic to the concept lattice $\mathcal{L}(G, M, I)$.

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It is an interesting question to find such relations $I \subseteq U \times U$, for which $\mathcal{L}(U, U, I)$ is a **Boolean lattice**? A well-known example for an atomic Boolean concept lattice is the concept lattice of the context $(U, U, \neq)$.

For any relation $R \subseteq U \times U$ we introduce the notations:

$$\wp(U)_R := \{X_R \mid X \subseteq U\}, \quad \wp(U)^R := \{X^R \mid X \subseteq U\}.$$

It is known that $(\wp(U)_R, \subseteq)$ and $(\wp(U)^R, \subseteq)$ are dually isomorphic complete lattices, called *rough approximation lattices* corresponding to R .

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Now, consider the **context** $\mathcal{K} = (U, U, I)$, where $I := R^c$. Then for any set $X \subseteq U$ it can be easily shown that

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Theorem 1.

- (1) For any relation $R \subseteq U \times U$ we have $\wp(U)_R = \mathcal{E}(U, U, R^c)$.
- (2) If $R \subseteq U \times U$ is a clinker equivalence then $\mathcal{E}(U, U, R^c)$ and $\mathcal{L}(U, U, R^c)$ are atomic Boolean lattices.

5. Characterizations of clinker equivalences

An other characterization of clinker equivalences is given in the following

Let us define a relation $\kappa(R^{-1})$ on U by setting

$$\kappa(R^{-1}) := \{(x, y) \mid R^{-1}(x) = R^{-1}(y)\}.$$

Then $\kappa(R^{-1})$ is an equivalence (called the *kernel equivalence* of R^{-1}), because it is the kernel of the map $x \mapsto R^{-1}(x)$, $x \in U$.

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Theorem 2.

Let R be a reflexive relation on U . The following are equivalent:

- (i) R is a clinker equivalence.
- (ii) For each $\kappa(R^{-1})$ -class C there is an $x \in C$ such that $C = R(x)$.
- (iii) There exists an equivalence E on U such that $R \circ E = R$ and each E -class C equals $R(x)$ for some $x \in C$.

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We also have proved the following properties:

Proposition 1.

Let R be a clinker equivalence on U .

- (a) If R is symmetric, then R is an equivalence.
- (b) If R is transitive, then R is an equivalence.
- (c) If R^{-1} is also a clinker equivalence, then $R = R^{-1}$ is an equivalence.

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A reflexive relation ρ on U is called a *representative relation* for the equivalence E , if each class C of E has an element $c \in C$ with $\rho(c) \subseteq C$.

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Proposition 2.

- ① If ρ is a representative relation for an equivalence E on U , then $R = \rho \circ E$ is a clinker equivalence.
- ② Let R be a reflexive relation on U and let $\mathcal{K}$ denote the set of all $\kappa(R^{-1})$ -classes. For each $C \in \mathcal{K}$, set an element $a_C \in C$ and define $\delta_C := \{(a_C, b) \in R \mid b \notin C\} \subseteq U \times U$. Then, the relation

$$S := R \setminus \left(\bigcup_{C \in \mathcal{K}} \delta_C \right)$$

is a clinker equivalence.

Figure 2

Examples: Set $U = \{1, 2, 3, 4, 5\}$. (a) Define a relation $\rho \subseteq U \times U$ by
 $\rho(1) = \{1, 2\}$, $\rho(2) = \{2, 3\}$, $\rho(3) = \{3\}$, $\rho(4) = \{4, 5\}$, $\rho(5) = \{1, 4, 5\}$.

ρ is a representative relation for the equivalence E having as classes $\{1, 2\}$, $\{3\}$, $\{4, 5\}$. By Proposition 2(i), $R = \rho \circ E$ is a clinker equivalence. We get:

$R = \{(1, 2), (2, 3), (2, 4), (2, 5), (3, 3), (4, 5), (5, 1), (5, 2), (5, 4)\} \cup \Delta$,
where: $\Delta = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$.

Figure 2

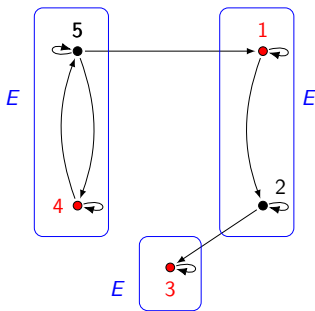
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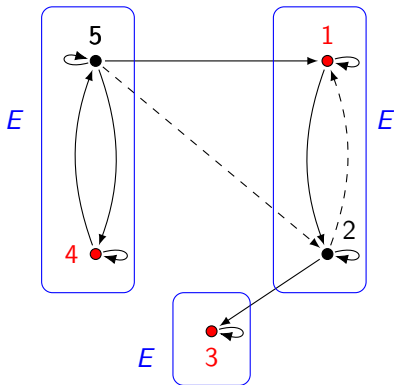
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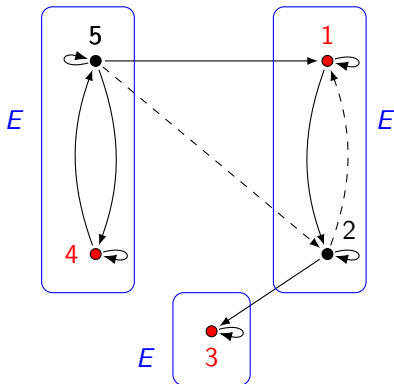
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(b) The relation $R = \{(1, 2), (1, 3), (1, 5), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 5), (4, 1), (4, 3)\} \cup \Delta$ is not a clinker equivalence, because the R^{-1} -neighborhoods

$$R^{-1}(1) = R^{-1}(2) = R^{-1}(3) = \{1, 2, 3, 4\}, \quad R^{-1}(4) = \{4\}, \quad R^{-1}(5) = \{1, 3, 5\}$$

do not form an irredundant covering. The $\kappa(R^{-1})$ classes are:
 $C = \{1, 2, 3\}$, $D = \{4\}$, $F = \{5\}$.

Figure 3

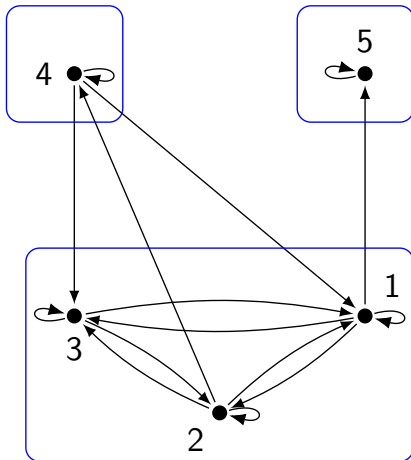
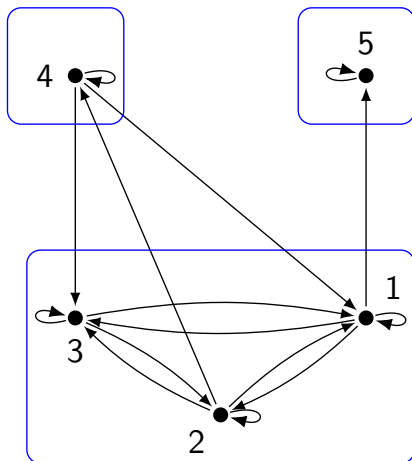
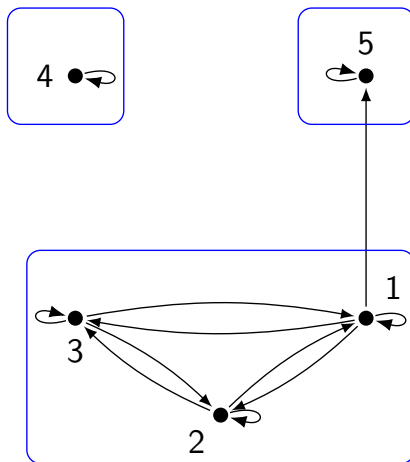


Figure 3



Selecting from these classes the elements $a_C = 2$, $a_D = 4$, and $a_F = 5$, we get $\delta_C = \{(2, 4)\}$, $\delta_D = \{(4, 1), (4, 3)\}$, $\delta_F = \emptyset$.

By Proposition 2(ii) relation $S := R \setminus (\delta_C \cup \delta_D \cup \delta_F) = \{(1, 2), (1, 3), (1, 5), (2, 1), (2, 3), (3, 1), (3, 2), (3, 5)\} \cup \Delta$ is a clinker equivalence.



Indeed, the sets below form an irredundant covering of U .

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Clinker equivalences can be interpreted as special **directed similarity relations** (in the sense of Tversky). We pointed out several applications of them:

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Finally, we present a representation result for clinker equivalences:

Representation Theorem

(i) Let $(P, \leq)$ be a partially ordered set, $U \neq \emptyset$ a universe, and $f, \varphi: U \rightarrow P$, $g: U \rightarrow U$ some mappings satisfying the conditions below:

(1) $\text{Im} f := \{f(x) \mid x \in U\}$ is an antichain in $(P, \leq)$;

(2) $f(x) \leq \varphi(x)$, for all $x \in U$;

(3) $f(x) = \varphi(g(x))$, for each $x \in U$.

Then the relation R defined by the formula

$$(x, y) \in R \Leftrightarrow f(y) \leq \varphi(x), \text{ for any } x, y \in U \quad (*)$$

is a **clinker equivalence** on U .

(ii) If $R \subseteq U \times U$ is a **clinker equivalence**, then there exists some maps $f, \varphi: U \rightarrow P$, $g: U \rightarrow U$ satisfying conditions (1),(2) and (3) and the equivalence $(*)$.

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