

**MAXIMAL SET OF
NON-ISOMORPHIC LATTICES
REPRESENTED BY
 Δ -INCIDENCE MATRICES
PRESERVING
 $\{0, 1\}$ -AGGREGATION**

SSAOS 2026



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2. MINIMUM AND MAXIMUM OF TWO LATTICES
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4. THE NUMBER OF ALL n -ELEMENT LATTICES

Δ -INCIDENCE MATRIX

DEFINITION

Let L be a finite lattice with elements $x_1, x_2, \dots, x_n$, where x_1 is the least element of L and x_n is the greatest element of L . We define the square matrix $I_L = (a_{ij})$ as follows:

$$a_{ij} = \begin{cases} 1 & ; x_i \leq x_j \\ 0 & ; \text{otherwise,} \end{cases}$$

for all $i, j \in \{1, \dots, n\}$. We refer to I_L as *the incidence matrix of the lattice L* .

$$I_L = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & & & 1 \\ \vdots & & \ddots & & \vdots \\ 0 & & & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

- First row: $x_1 \leq x_j$
- Last column: $x_i \leq x_n$
- First column: $x_i \not\leq x_1$
- Last row: $x_n \not\leq x_j$
- Main diagonal: $x_i \leq x_i$

Δ -INCIDENCE MATRIX

Δ -INCIDENCE MATRIX:

$$\Delta_{\mathbf{L}} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 0 & 1 & \bullet & \cdots & \bullet & \bullet & 1 \\ 0 & \mathbf{0} & \ddots & \ddots & & \bullet & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & \mathbf{0} & & \ddots & 1 & \bullet & 1 \\ 0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix}$$

$$\bullet \in \{0, 1\}$$

UPPER Δ -MATRIX:

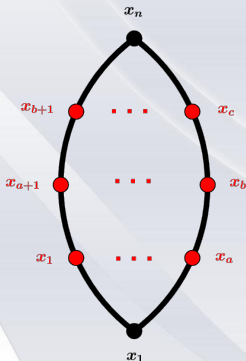
$$\uparrow \Delta_{\mathbf{L}} = \begin{pmatrix} \bullet & \cdots & \bullet \\ & \ddots & \vdots \\ & & \bullet \end{pmatrix}$$

UNIQUE Δ -INCIDENCE MATRIX

PROPOSITION

Let L be a finite lattice with elements $x_1, x_2, \dots, x_n$. Then it is possible to label the elements of L in such a way that the condition $x_i < x_j$ holds whenever $i < j$. This implies that for every finite lattice L , there exists a unique Δ -incidence matrix Δ_L .

Proof:



TWO-ELEMENT LATTICE

FOR EXAMPLE

1. $2 = \{x_1, x_2\}$



$$\Delta_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

THREE-ELEMENT LATTICE

FOR EXAMPLE

2. $\mathbf{3} = \{x_1, x_2, x_3\}$

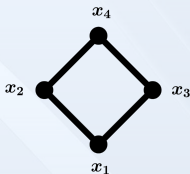


$$\Delta_{\mathbf{3}} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

FOUR-ELEMENT LATTICES

FOR EXAMPLE

3. $4_D = \{x_1, x_2, x_3, x_4\}$ and $4_C = \{x_1, x_2, x_3, x_4\}$



$$\Delta_{4_D} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{0} & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\uparrow \Delta_{4_D} = (0)$$



$$\Delta_{4_C} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{1} & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\uparrow \Delta_{4_C} = (1)$$

QUESTION 1

$\mathcal{L}$... the set of all finite lattices

$\mathcal{I}_\Delta$... the set of all Δ -incidence matrices

$$f : \mathcal{L} \rightarrow \mathcal{I}_\Delta$$

IS THE MAPPING f A BIJECTION?

One-to-one correspondence between all finite lattices and all Δ -incidence matrices.

FIVE-ELEMENT LATTICES

FOR EXAMPLE

4. Every five-element lattice **5** can be represented by a Δ -incidence matrix Δ_5 in the form:

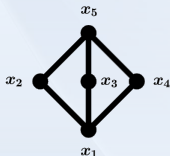
$$\Delta_5 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \bullet & \bullet & 1 \\ 0 & 0 & 1 & \bullet & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

where $\bullet \in \{0, 1\}$.

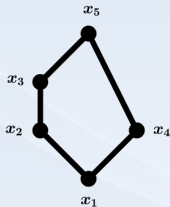
- The total number of Δ -incidence matrices Δ_5 is $2^3 = 8$.

FIVE-ELEMENT LATTICES

$$\Delta_{5_1} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{0} & \mathbf{0} & 1 \\ 0 & 0 & 1 & \mathbf{0} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

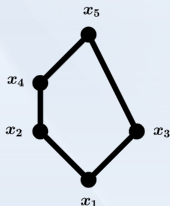


$$\Delta_{5_2} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{1} & \mathbf{0} & 1 \\ 0 & 0 & 1 & \mathbf{0} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

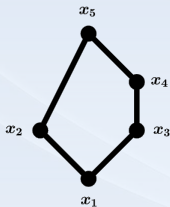


FIVE-ELEMENT LATTICES

$$\Delta_{5_3} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{0} & \mathbf{1} & 1 \\ 0 & 0 & 1 & \mathbf{0} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

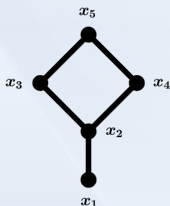


$$\Delta_{5_4} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{0} & \mathbf{0} & 1 \\ 0 & 0 & 1 & \mathbf{1} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

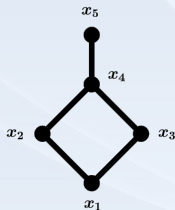


FIVE-ELEMENT LATTICES

$$\Delta_{5_5} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{1} & \mathbf{1} & 1 \\ 0 & 0 & 1 & \mathbf{0} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$



$$\Delta_{5_7} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{0} & \mathbf{1} & 1 \\ 0 & 0 & 1 & \mathbf{1} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$



FIVE-ELEMENT LATTICES

$$\Delta_{5_8} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{1} & \mathbf{1} & 1 \\ 0 & 0 & 1 & \mathbf{1} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$



FIVE-ELEMENT LATTICES

$$\Delta_{5_6} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \mathbf{1} & \mathbf{0} & 1 \\ 0 & 0 & 1 & \mathbf{1} & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

- The Δ -incidence matrix Δ_{5_6} does **not correspond to any Hasse diagram**.
- It **violates the transitivity** of the order relation:

$$x_2 \leq x_3 \quad \text{and} \quad x_3 \leq x_4 \quad \text{and} \quad x_2 \not\leq x_4$$

ANSWER 1

$$f : \mathcal{L} \rightarrow \mathcal{I}_\Delta$$

IS THE MAPPING f A BIJECTION? **NO**

REASONS:

1. There exist Δ -incidence matrices that **violate the transitivity of the order relation** in a lattice.
2. There exist distinct Δ -incidence matrices that **represent the same lattice** – in the sense of isomorphism.

$$\Delta_5^* = \{\Delta_{5_1}, \Delta_{5_3}, \Delta_{5_5}, \Delta_{5_7}, \Delta_{5_8}\}$$

QUESTION 2

DO A MINIMUM AND A MAXIMUM EXIST FOR TWO LATTICES WITH THE SAME NUMBER OF ELEMENTS?

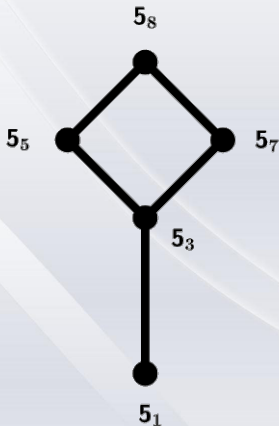
The first indication that aggregation operations can be performed directly on lattices themselves.

DEFINITION

We say that for two Δ -incidence matrices $\Delta_{\mathbf{A}} = \{a_{ij}\}$ and $\Delta_{\mathbf{B}} = \{b_{ij}\}$, the relation $\Delta_{\mathbf{A}} \leq \Delta_{\mathbf{B}}$ holds if and only if $a_{ij} \leq b_{ij}$ for every $i, j \in \{1, \dots, n\}$.

LATTICE OF FIVE ELEMENT LATTICES

- All five-element non-isomorphic lattices from the set Δ_5^* form a lattice.



QUESTION 3

DO ALL NON-ISOMORPHIC LATTICES WITH THE SAME NUMBER OF ELEMENTS FORM A LATTICE?

Existence of the lattice of all n -element lattices.

SIX-ELEMENT LATTICES

FOR EXAMPLE

5. Every six-element lattice **6** can be represented by a Δ -incidence matrix Δ_6 in the form:

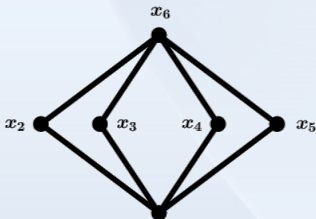
$$\Delta_6 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & \bullet & \bullet & \bullet & 1 \\ 0 & 0 & 1 & \bullet & \bullet & 1 \\ 0 & 0 & 0 & 1 & \bullet & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

where $\bullet \in \{0, 1\}$.

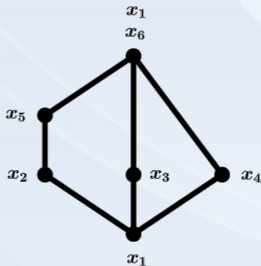
- The total number of Δ -incidence matrices Δ_6 is $2^6 = 64$.

SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

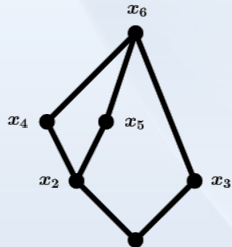


$$\uparrow\Delta_{6_2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

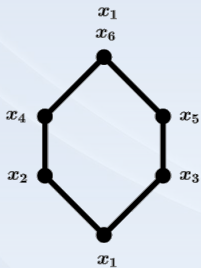


SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_3} = \begin{pmatrix} 0 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$$

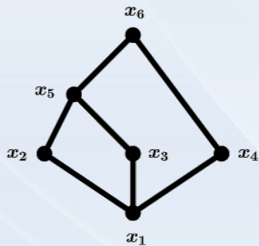


$$\uparrow\Delta_{6_4} = \begin{pmatrix} 0 & 1 & 0 \\ & 0 & 1 \\ & & 0 \end{pmatrix}$$



SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_5} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



SIX-ELEMENT LATTICES

- For instance, the following upper Δ -matrix

$$\uparrow\Delta'_{6_2} = \begin{pmatrix} 1 & 0 & 0 \\ & 0 & 0 \\ & & 0 \end{pmatrix} \cong \begin{pmatrix} 0 & 0 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix} = \uparrow\Delta_{6_2}$$

represents the same (isomorphic) lattice as the upper Δ -matrix $\uparrow\Delta_{6_2}$.

- The following upper Δ -matrix

$$\uparrow\Delta'_{6_5} = \begin{pmatrix} 0 & 0 & 1 \\ & 0 & 0 \\ & & 1 \end{pmatrix} \cong \begin{pmatrix} 0 & 0 & 1 \\ & 0 & 1 \\ & & 0 \end{pmatrix} = \uparrow\Delta_{6_5}$$

represents the same (isomorphic) lattice as the upper Δ -matrix $\uparrow\Delta_{6_5}$.

SIX-ELEMENT LATTICES

- Next upper Δ -matrices:

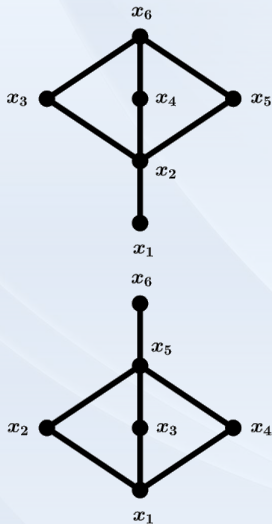
$$\begin{aligned}\uparrow\Delta_{6_6} &= \begin{pmatrix} 1 & \mathbf{0} & 0 \\ & 1 & 0 \\ & & 0 \end{pmatrix} & \uparrow\Delta_{6_7} &= \begin{pmatrix} 0 & 0 & 0 \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix} & \uparrow\Delta_{6_8} &= \begin{pmatrix} 1 & 0 & \mathbf{0} \\ & 0 & 1 \\ & & 0 \end{pmatrix} \\ & & \uparrow\Delta_{6_9} &= \begin{pmatrix} 0 & 1 & \mathbf{0} \\ & 0 & 0 \\ & & 1 \end{pmatrix}\end{aligned}$$

- These matrices **violate the transitivity of the order relation** and therefore do not represent any lattice.

SIX-ELEMENT LATTICES

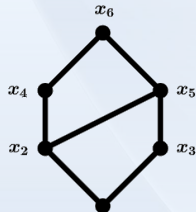
$$\uparrow\Delta_{6_{10}} = \begin{pmatrix} 1 & 1 & 1 \\ & 0 & 0 \\ & & 0 \end{pmatrix}$$

$$\uparrow\Delta_{6_{11}} = \begin{pmatrix} 0 & 0 & 1 \\ & 0 & 1 \\ & & 1 \end{pmatrix}$$

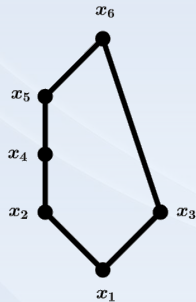


SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_{12}} = \begin{pmatrix} 0 & 1 & 1 \\ & 0 & 1 \\ & & 0 \end{pmatrix}$$



$$\uparrow\Delta_{6_{13}} = \begin{pmatrix} 0 & 1 & 1 \\ & 0 & 0 \\ & & 1 \end{pmatrix}$$



SIX-ELEMENT LATTICES

- For instance, the following upper Δ -matrix

$$\uparrow\Delta'_{\mathbf{6}_{12}} = \begin{pmatrix} 1 & 0 & 1 \\ & 0 & 0 \\ & & 1 \end{pmatrix} \cong \begin{pmatrix} 0 & 1 & 1 \\ & 0 & 1 \\ & & 0 \end{pmatrix} = \uparrow\Delta_{\mathbf{6}_{12}}$$

$$\uparrow\Delta'_{\mathbf{6}_{13}} = \begin{pmatrix} 0 & 0 & 0 \\ & 1 & 1 \\ & & 1 \end{pmatrix} \cong \begin{pmatrix} 0 & 1 & 1 \\ & 0 & 0 \\ & & 1 \end{pmatrix} = \uparrow\Delta_{\mathbf{6}_{13}}$$

- The lattices $\mathbf{6}_{10}$ and $\mathbf{6}_{11}$ are represented by exactly one Δ -incidence matrix.

There exists no other Δ -incidence matrix that represents the lattices $\mathbf{6}_{10}$ and $\mathbf{6}_{11}$.

SIX-ELEMENT LATTICES

- Next upper Δ -matrices, that **violate the transitivity**.

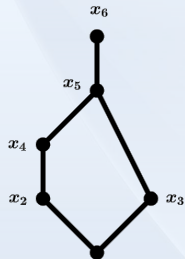
$$\uparrow\Delta_{6_{14}} = \begin{pmatrix} 1 & 1 & \mathbf{0} \\ & 0 & 1 \\ & & 0 \end{pmatrix} \quad \uparrow\Delta_{6_{15}} = \begin{pmatrix} 1 & 1 & \mathbf{0} \\ & 0 & 0 \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{16}} = \begin{pmatrix} 1 & \mathbf{0} & 0 \\ & 1 & 0 \\ & & 1 \end{pmatrix}$$

$$\uparrow\Delta_{6_{17}} = \begin{pmatrix} 1 & 0 & \mathbf{0} \\ & 0 & 1 \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{18}} = \begin{pmatrix} 0 & 1 & \mathbf{0} \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{19}} = \begin{pmatrix} 0 & 1 & \mathbf{0} \\ & 0 & 1 \\ & & 1 \end{pmatrix}$$

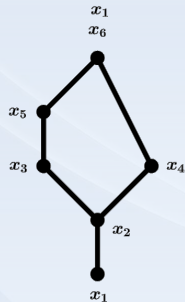
$$\uparrow\Delta_{6_{20}} = \begin{pmatrix} 0 & 0 & 1 \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{21}} = \begin{pmatrix} 1 & \mathbf{0} & \mathbf{0} \\ & 1 & 1 \\ & & 0 \end{pmatrix} \quad \uparrow\Delta_{6_{22}} = \begin{pmatrix} 1 & \mathbf{0} & 1 \\ & 1 & 0 \\ & & 0 \end{pmatrix}$$

SIX-ELEMENT LATTICES

$$\uparrow\Delta_{\mathbf{6}_{23}} = \begin{pmatrix} 0 & 1 & 1 \\ & 0 & 1 \\ & & 1 \end{pmatrix}$$



$$\uparrow\Delta_{\mathbf{6}_{24}} = \begin{pmatrix} 1 & 1 & 1 \\ & 0 & 1 \\ & & 0 \end{pmatrix}$$



SIX-ELEMENT LATTICES

- Next upper Δ -matrices, that **violate the transitivity**.

$$\uparrow\Delta_{6_{25}} = \begin{pmatrix} 1 & \mathbf{0} & \mathbf{0} \\ & 1 & 1 \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{26}} = \begin{pmatrix} 1 & 1 & \mathbf{0} \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{27}} = \begin{pmatrix} 1 & 1 & \mathbf{0} \\ & 0 & 1 \\ & & 1 \end{pmatrix}$$

$$\uparrow\Delta_{6_{28}} = \begin{pmatrix} 0 & 1 & 1 \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{6_{29}} = \begin{pmatrix} 1 & \mathbf{0} & 1 \\ & 1 & 1 \\ & & 0 \end{pmatrix} \quad \uparrow\Delta_{6_{30}} = \begin{pmatrix} 0 & 1 & \mathbf{0} \\ & 1 & 1 \\ & & 1 \end{pmatrix}$$

$$\uparrow\Delta_{6_{31}} = \begin{pmatrix} 1 & 1 & \mathbf{0} \\ & 1 & 1 \\ & & 0 \end{pmatrix} \quad \uparrow\Delta_{6_{32}} = \begin{pmatrix} 1 & \mathbf{0} & 1 \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix}$$

SIX-ELEMENT LATTICES

THIRD POSSIBLE ISSUE:

$$\uparrow \Delta_{633} = \begin{pmatrix} 0 & \mathbf{1} & \mathbf{1} \\ & \mathbf{1} & \mathbf{1} \\ & & 0 \end{pmatrix}$$

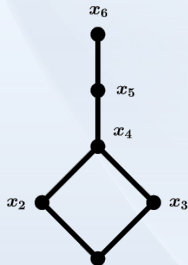
- Satisfy the transitivity of the order relation.
- Represents **only a partially ordered set, not a lattice**, because:

$$\mathbf{x_2 \leq x_4, x_5} \quad \text{and} \quad \mathbf{x_3 \leq x_4, x_5}$$

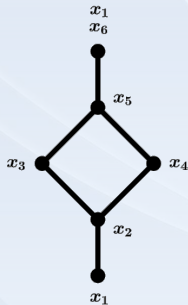
$\Rightarrow$ The elements x_2 and x_3 do not have a least upper bound.

SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_{34}} = \begin{pmatrix} 0 & 1 & 1 \\ & 1 & 1 \\ & & 1 \end{pmatrix}$$

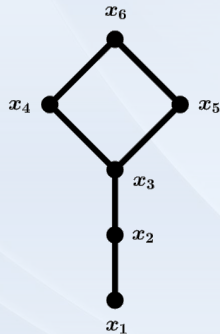


$$\uparrow\Delta_{6_{35}} = \begin{pmatrix} 1 & 1 & 1 \\ & 0 & 1 \\ & & 1 \end{pmatrix}$$



SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_{36}} = \begin{pmatrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 0 \end{pmatrix}$$



SIX-ELEMENT LATTICES

- Next upper Δ -matrices, that **violate the transitivity**.

$$\uparrow\Delta_{637} = \begin{pmatrix} 1 & \mathbf{0} & 1 \\ & 1 & 1 \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{638} = \begin{pmatrix} 1 & 1 & \mathbf{0} \\ & 1 & 1 \\ & & 1 \end{pmatrix} \quad \uparrow\Delta_{639} = \begin{pmatrix} 1 & 1 & 1 \\ & 1 & \mathbf{0} \\ & & 1 \end{pmatrix}$$

SIX-ELEMENT LATTICES

$$\uparrow\Delta_{6_{40}} = \begin{pmatrix} 1 & 1 & 1 \\ & 1 & 1 \\ & & 1 \end{pmatrix}$$



ANSWER 1

- The entire set $\mathcal{I}_\Delta$ of all Δ -incidence matrices contains:
 1. matrices that violate the transitivity of the order relation.
 2. matrices that represent only a partially ordered set (but not a lattice).
 3. matrices that represent a valid lattice (or an isomorphic lattice).
- The resulting set of non-isomorphic six-element lattices is:

$$\mathcal{L}_6 = \{\mathbf{6}_1, \dots, \mathbf{6}_5, \mathbf{6}_{10}, \dots, \mathbf{6}_{13}, \mathbf{6}_{23}, \mathbf{6}_{24}, \mathbf{6}_{34}, \mathbf{6}_{35}, \mathbf{6}_{36}, \mathbf{6}_{40}\}$$

$\Rightarrow$ The set $\mathcal{L}_6$ contains 15 “unique” six-element lattices.

ANSWER 2

DO A MINIMUM AND A MAXIMUM EXIST FOR TWO LATTICES WITH THE SAME NUMBER OF ELEMENTS?

YES, BUT ...

- We prove whether the set $\mathcal{L}_6$ is closed under meet and join operations.

Proof:

$$\mathbf{6}_{34} \wedge \mathbf{6}_{36} = \min(\Delta_{\mathbf{6}_{34}}, \Delta_{\mathbf{6}_{36}}) = \Delta_{\mathbf{6}_{33}} \rightarrow \mathbf{6}_{33} ?$$

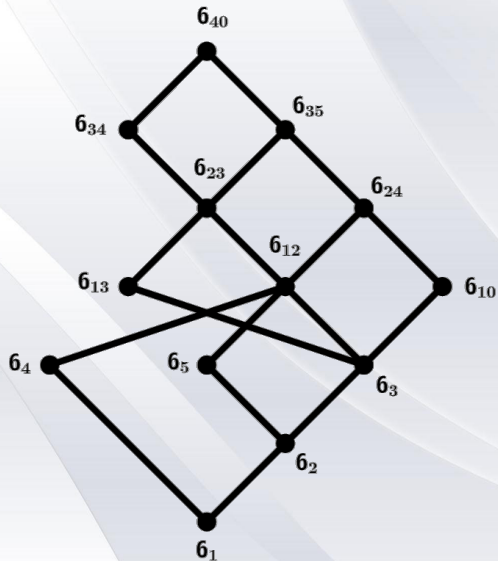
QUESTION 4

WHAT IS THE LARGEST SUBSET (IN TERMS OF INCLUSION) OF $\mathcal{L}_6$ THAT IS CLOSED UNDER MEET AND JOIN OPERATIONS?

- All six-element lattices (including isomorphic ones) have 2^{36} subsets.
- We identifying 13 non-isomorphic lattices that do form a lattice.

$\{\mathbf{6}_1, \mathbf{6}_2, \mathbf{6}_3, \mathbf{6}_4, \mathbf{6}_5, \mathbf{6}_{10}, \mathbf{6}_{12}, \mathbf{6}_{13}, \mathbf{6}_{23}, \mathbf{6}_{24}, \mathbf{6}_{34}, \mathbf{6}_{35}, \mathbf{6}_{40}\}$

QUESTION 4



ANSWER 3

DO ALL NON-ISOMORPHIC LATTICES WITH THE SAME NUMBER OF ELEMENTS FORM A LATTICE?

NO

DO NON-ISOMORPHIC LATTICES WITH THE SAME NUMBER OF ELEMENTS FORM A LATTICE?

YES

THE NUMBER OF ALL n -ELEMENT LATTICES

PROPOSITION

The number of all n -element lattices, for $n \geq 4$, is bounded above by 2^k , where $k = \frac{1}{2}(n-3)(n-2)$, and bounded below by 2. That is, for the set of all finite n -element lattices $\mathcal{L}_n$, we have:

$$2 \leq |\mathcal{L}_n| \leq \frac{1}{2}(n-3)(n-2)$$

- $n = 7$: 1 024 all Δ -incidence matrices.
 - 357 matrices represent (isomorphic) lattices or posets.
- $n = 8$: 32 768 all Δ -incidence matrices.
 - 4 824 matrices represent (isomorphic) lattices or posets.
- $n = 9$: 2 097 152 all Δ -incidence matrices.
 - 96 428 matrices represent (isomorphic) lattices or posets.
- $n = 10$: 268 435 456 all Δ -incidence matrices.
 - 2 800 472 matrices represent (isomorphic) lattices or posets.

THE NUMBER OF ALL n -ELEMENT LATTICES

HYPOTHESIS

The number of all n -element lattices, for $n \geq 11$, is bounded above by: $0.01 \cdot 2^k$, where $k = \frac{1}{2}(n-3)(n-2)$.

WHAT WILL YOU SEE IN THE FUTURE?

1. MEET AND JOIN MATRICES
2. PERIPHERAL Δ -CONSTRUCTION
3. $\{0, 1\}$ -AGGREGATION OF FINITE LATTICES

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THANK YOU VERY
MUCH FOR YOUR
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