Universal algebraic geometry of normal bands

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Given an algebra $\mathbb{A} = (A; F)$, we say that a set of n-tuples $S \subseteq A^n$ is an (n-ary) *algebraic set* if S is the set of solutions of a system of equations over A. The collection of all algebraic sets is called the *algebraic geometry* of $\mathbb{A}$ [1]. The algebraic geometry of an algebra only depends on the clone of its term operations. If two clones defined on the same underlying set have the same algebraic geometry, then we say that the two clones are *algebraically equivalent.* For a fixed algebra $\mathbb{A}$, two natural problems arise:

- characterize the algebraic sets over $\mathbb{A}$;
- characterize the clones on the set A that are algebraically equivalent to the clone of term operations of $\mathbb{A}$.

The set of n-ary algebraic sets of $\mathbb{A}$ forms a lattice with respect to inclusion; furthermore, the structure of this lattice is completely determined by the quasivariety generated by $\mathbb{A}$ if it is a normal band. Using this connection, we managed to characterize the lattice of n-ary algebraic sets of every normal band up to isomorphism.

We obtained a similarly complete answer for the second problem as well: except for some special cases, a clone is algebraically equivalent to the clone of a normal band $\mathbb{A}$ if and only if it is the clone of $\mathbb{A}$.

References

[1] B. Plotkin. *Some results and problems related to universal algebraic geometry,* Internat. J. Algebra Comput. **17**, no. 5-6, 1133–1164 (2007)