

Solving lattice-valued relational equations and inequations

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We will present our investigation on the existence of optimal solutions to lattice-valued relational equations and inequations, extending the classical notions of greatest and least solutions to maximal and minimal ones [2, 3, 4, 5]. A key result establishes that infinite distributivity of infimum relative to supremum in the codomain lattice is both necessary and sufficient for typical inequations to admit a greatest solution. We show that minimal solutions may fail to exist even under strong lattice assumptions, while meet-continuity, though sufficient for maximal solutions, is not always necessary.

A distinctive aspect of our work is the connection between approximate solutions and factor algebras, where weak congruences provide the structural framework for transferring solvability properties [1]. This perspective links the study of relational systems with algebraic structures, clarifying how approximate solutions can be systematically understood through weak-congruence theory.

Connected to this are quotient structures of lattice-valued matrix spaces, constructive algorithms for least solutions that terminate after countably many steps, and refined lattice-theoretic conditions for solution existence. Altogether, the results advance a connection of the theory of lattice-valued equations and inequations with the theory of weak congruences (solutions of equations on factor subalgebras).

This research was supported by the Science Fund of the Republic of Serbia, # Grant no 6565, Advanced Techniques of Mathematical Aggregation and Approximative Equations Solving in Digital Operational Research-AT-MATADOR.

References

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