Clinker equivalences

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We study the maximal relations $\varrho \subseteq U \times U$ that define atomic Boolean concept lattices $\mathfrak{V}(U, U, \varrho)$. The complement $R = (U \times U) \setminus \varrho$ of such a relation ϱ can be characterized as a reflexive relation with the property that the sets $R^{-1}(x) := \{y \in U \mid (y, x) \in R\}$ form an irredundant covering of U. It was proved by J. Järvinen and S. Radeleczki that the rough sets induced by a relation R with this property form an algebraic regular double Stone lattice, i.e. they have the same structure as in the case of an equivalence relation. Therefore, such a reflexive relation R can be viewed as a new generalization of the notion of an equivalence relation, however, in the case when R is symmetric or transitive, it becomes an ordinary equivalence. The term "clinker equivalence" was chosen as its name, because the geometric form of the covering $U = \{R^{-1}(x) \mid x \in U\}$ resembles the sail of Viking ship, specifically referencing the historical clinker-built method of Norse shipbuilding. Here, we present some examples and applications of these relations. We show that they can be interpreted as special directed similarity relations, and can serve as useful tools in classification problems and concept approximations.

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