Critical elements in algebras of numerical events

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The probability $p(s)$ of the occurrence of an event pertaining to a physical system which is observed in different states s determines a function p from the set S of states of the system to $[0,1]$. The function p is called a numerical event or more precisely an S-probability. When appropriately structured, sets P of numerical events form so-called algebras of S-probabilities, which are orthomodular posets that can serve as quantum logics. If one deals with a classical physical system, then P will be a Boolean algebra. Starting with a supposed quantum logic or logics obtained by individual measurements, newly added numerical events may turn out to be crucial for the assumption of classicality or non-classicality of the physical system. We will call those numerical events critical and will study their impacts among various classes of algebras of numerical events. Moreover, we will consider the situation of enlarging S by new states and will describe how elements of an algebra of S-probabilities contribute to the character of a logic when there exists a certain relation between them and the element that is newly added.

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