

Stability of Mapping Spaces in ε -Simplicial Sets

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A familiar stability phenomenon in simplicial homotopy theory is that horn-filling conditions characterizing categories and groupoids are preserved under mapping-space constructions. For instance, simplicial sets satisfying the inner horn-filling conditions retain this property after passing to suitable mapping spaces, and an analogous statement holds for Kan complexes.

In this talk, I will discuss a related phenomenon for ε -simplicial sets. These were introduced in the simplicial study of Frobenius monoids in the category **Rel** of sets and relations [1], [2]. Within this framework, we consider a distinguished class of ε -fibrant objects, characterized by appropriate filling conditions.

A key ingredient is the close relationship between ε -simplicial sets and paracyclic sets. This connection provides a useful way to reinterpret certain simplicial filling problems and plays an essential role in the analysis of ε -fibrancy.

The main result is a stability theorem showing that, under suitable hypotheses, the relevant ε -fibrancy conditions are preserved under mapping-space constructions. I will explain the analogy with the classical cases of categories and groupoids, the role of paracyclic structures, and the main ideas of the proof.

References

- [1] D. Lachman. *Simplicial approach to Frobenius algebras in the category of relations*, Appl. Categ. Structures **34**, 25 (2026).
- [2] D. Lachman. *Simplicial perspectives on (pseudo) effect algebras*, arXiv:2510.02748 (2025).