Dual spaces of ortholattices

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We describe digraphs with topology, which give dual representations of ortholattices. This is done via so-called dual Ploščica spaces of lattices. We define the dual space of a general ortholattice as the dual Ploščica space of the lattice reduct of the ortholattice equipped with a map representing the orthocomplement operation. We introduce an abstract ortho-Ploščica space capturing the properties of the dual space of an ortholattice, and we present dual representation theorems between general ortholattices and the ortho-Ploščica spaces. We illustrate our dual representations by examples and present an open problem.

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