

# On algebras with easy direct limits

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The direct limit construction belongs to basic tools used in Universal algebra. Moreover, G. Grätzer [1], page 161 specified problems for the direct limit operator that have not yet been solved. The question of what we can get by a direct limit construction from one algebra is very basic in this context and little explored.

Let  $\mathcal{A}$  be an algebra. We denote by  $\underline{\mathbf{L}}\mathcal{A}$  the class of all isomorphic copies of direct limits which can be obtained from  $\mathcal{A}$  and we denote by  $\mathbf{R}\mathcal{A}$  the set of all retracts of  $\mathcal{A}$ . Then  $\mathbf{R}\mathcal{A} \subseteq \underline{\mathbf{L}}\mathcal{A}$ . In general, an algebra from  $\underline{\mathbf{L}}\mathcal{A}$  need not be a retract of  $\mathcal{A}$ , [2].

We say that  $\mathcal{A}$  is *with easy direct limits* if every algebra from  $\underline{\mathbf{L}}\mathcal{A}$  is isomorphic to a retract of  $\mathcal{A}$ . Retracts of an algebra with easy direct limits form a direct limit closed class and it is the next motivation to deal with direct limit families in which only one algebra occurs.

The goal of the talk is to give a results overview on algebras with easy direct limits. General algebras as well as several specific algebraic structures as monounary algebras, vector spaces and groups will be mentioned.

## References

- [1] Grätzer G. Universal algebra, 2nd edition with updates, Springer Science+Business Media, LLC, 2008.
- [2] Halušková E. *Monounary algebras with easy direct limits*. Miskolc Math. Notes 2019, vol. 19, no.1, 291–302.