Representations of distributive residuated lattice expansions

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These two talks will present joint and ongoing work with Claudette Robinson (University of Johannesburg), Peter Jipsen (Chapman University), Wilmari Morton (Wits University), and Jaš Šemrl (University of the West of Scotland).

Over the past few years we have been studying a class of residuated lattices expansions known as quasi relation algebras. These were first described by Galatos and Jipsen [6]. In particular, we are interested in *distributive* quasi relation algebras (i.e. where the lattice reduct is distributive). Our focus has been on obtaining representations of distributive quasi relation algebras (DqRAs) as algebras of binary relations.

In the first talk, we will give some background, describe our main representation construction [4], and give some related results that will be needed later [3]. Then we will show how pregroups (a class of expansions of partially ordered monoids, first described by Lambek [9]) can be used to find representable algebras [5].

In the second talk, we will describe dual frames developed for DqRAs [1, 2]. While these dual representations are themselves interesting, our main focus will be on how they are used for *representation games* for the algebras. Inspired by previous game theoretic approaches to finding representations [7, 8], we present two representation games, to be played on finite DqRAs and their dual frames. These methods yield previously inaccessible representation results.

During the talks we will also highlight some interesting open problems.

References

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