Maximal Set of Non-Isomorphic Lattices Represented by Δ-Incidence Matrices Preserving $\{0, 1\}$-Aggregation

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It is perfectly natural that nowadays we want to perform all computations using computers. However, especially in mathematics, we may encounter structures that are difficult to describe in a way that a computer can understand. One such structure is a lattice, which is an ordered set in which every pair of elements has a well-defined minimum and maximum.

The aim of this paper is therefore to represent lattices using matrices, which computers are very good at working with, and then perform aggregations with them. For example, given two binary matrices of the same dimensions, we define their minimum and maximum as new matrices whose entries are the element-wise minimum or maximum – corresponding to logical conjunction (AND) and disjunction (OR), respectively. We apply such aggregation operations – particularly MIN and MAX – to matrices representing lattices of increasing size, starting with two-element lattices and progressing to larger ones. We will demonstrate that this task is not as straightforward as it might initially seem, as a number of challenges arise that even modern computational tools struggle to overcome. On the other hand, this approach opens up a completely new possibility: until now, we have only been able to perform aggregation within a single, specific lattice. But now, we can aggregate across different lattices themselves. We prove that for both the minimum and the maximum, there exist maximal closed sets of lattices, from which it follows that the "lattice of all equally-sized finite lattices" does not exist.

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