Exploration of non-classical negation axioms

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We explore the implicational interconnections between different types of lattice based non-classical logic algebras. That is, we consider structures of the form $\mathbf{L} = \langle L; \wedge, \vee, 0, 1, {}^{-} \rangle$ with the background assumption that $\langle L; \wedge, \vee, 0, 1 \rangle$ is a bounded lattice. Our investigation focusses on the additional axioms the 'negation' (or 'complementation') operator $x \mapsto \overline{x}$ may fulfil, and on the implications between those properties. In this respect we consider several generalisations of orthocomplemented lattices (as used in quantum logic) and of pseudocomplemented lattices (as appearing, *e.g.*, in Heyting algebras when dealing with intuitionistic logic). The properties of negation we study are mainly taken from the ones surveyed in [3], without any claims to represent the available literature with any degree of completeness. A different set of negation axioms was examined in [2], but the final picture we get of the situation is richer than both of these works and more along the lines of [1], where again other properties were looked at.

References

[1] F. Dau. *Implications of properties concerning complementation in finite lattices,* In Contributions to General Algebra **12**, (Vienna, 1999), 145–154. Heyn, Klagenfurt, 2000.

[2] J. M. Dunn and Ch. Zhou. *Negation in the context of gaggle theory,* Studia Logica **80** 235–264 (2005) doi:10.1007/s11225-005-8470-y

[3] W. H. Holliday. *A fundamental non-classical logic,* Logics **1** 36–79 (2023) doi:10.3390/logics1010004