

Symmetric lattice identities and medians

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An n -ary lattice term f is symmetric when its value does not change when we permute its variables i.e, $f(x_1, \dots, x_n) = f(x_{\sigma(1)}, \dots, x_{\sigma(n)})$ for all $\sigma \in S_n$. A symmetric lattice identity is an identity where both sides are symmetric terms. An example of a symmetric identity

$$(x \wedge y) \vee (x \wedge z) \vee (y \wedge z) = (x \vee y) \wedge (x \vee z) \wedge (y \vee z),$$

which is famously equivalent to the distributive law (which itself is a nonsymmetric identity).

Conjecture 1 *Each lattice variety can be axiomatized by symmetric identities.*

In this talk we establish that a stronger version of this conjecture is not true: there is a ternary lattice identity which is not equivalent to any system of ternary symmetric identities.

References

- [1] G. GYENIZSE, On the symmetric parts of finitely generated free lattices. Acta Mathematica Hungarica, 169(2), 420-431.
- [2] L. ABURUB, G. GYENIZSE, Outer and inner medians in some small lattices. arXiv preprint arXiv:2603.17958, (2026).