SUBDIRECTLY IRREDUCIBLE DOUBLE BOOLEAN ALGEBRAS

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☞Introduction
☞Congruence on double Boolean algebra
☞Subdirectly irreducible double Boolean algebra
☞Conclusion and perspectives

Introduction

The definition and investigation of double Boolean algebras (dBas) arise from the development of Contextual Logic at TU Darmstadt during the past years.
Contextual Logic is intended to be a mathematization of the traditional philisophical logic with its doctrines of concepts,jugements and conclusions.
A survey of the basic ideas and results of this approach can be found in [Wi00], for more detailed information see [Pr98], [GW99], [Wi00] and [Vor05].

Definition 1 ([GW99])

1. *A **formal context** is a triple $K = (G; M; I)$, where G is the set of objects, M is the set of properties and $I \subseteq G \times M$.*

   For any $X \subseteq G$; $Y \subseteq M$ the following sets are defined :
   $Y' = \{g \in G : gIm, \forall m \in Y\}$, $X' = \{m \in M : gIm, \forall g \in X\}$.

2. *A **formal concept** is a pair (A, B) with $A \subseteq G$, $B \subseteq M$ such that $A' = B$ and $B' = A$; A and B are called the extent and the intent of the formal concept (A, B), respectively.*

The set of all concepts of a context K is denoted by $\mathcal{B}(K)$. An order relation is defined on $\mathcal{B}(K)$ as follows :

$$(A, B) \leq (C, D) \iff A \subseteq C (\iff D \subseteq B). \quad (1)$$

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The basic theorem on concept lattice states that:

Theorem 1 ([GW99])

The partial ordered set $(\mathcal{B}(K), \leq)$ forms a complete lattice called the ***concept lattice*** *of the context K and conversely, each complete lattice is isomorphic to a concept lattice of a suitable context K.*

To facilitate the description of concepts, the notion of "concept" has been successively generalized to that of "semiconcept" and "protoconcept".

Definition 2 ([GW99])

The pair (A, B) is called a ***semiconcept*** *if $A' = B$ or $B' = A$.*
The pair (A, B) is called a ***protoconcept*** *if and only if $A'' = B'$. The set of all protoconcepts of a context K is denoted by $\mathfrak{P}(K)$.*

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The following operations are defined on $\mathfrak{P}(K)$. For (A,B) and (C,D) in $\mathfrak{P}(K)$,

$$
\begin{array}{llcl}
\text{meet:} & (A,B)\sqcap(C,D) & := & (A\cap C,(A\cap C)') \\
\text{join:} & (A,B)\sqcup(C,D) & := & ((B\cap D)',B\cap D) \\
\text{negation} & \neg(A,B) & := & (G\backslash A,(G\backslash A)') \\
\text{opposition} & \lrcorner(A,B) & := & ((M\backslash B)',M\backslash B) \\
\text{nothing} & \bot & := & (\emptyset,M) \\
\text{all} & \top & := & (G,\emptyset)
\end{array}
$$

$(\mathfrak{P}(K),\sqcap,\sqcup,\neg,\lrcorner,\bot,\top)$ is an algebra of type $(2,2,1,1,0,0)$ called the protoconcept algebra of the context K.

Theorem 2 ([Wil00])

The following equations hold in the algebra of protoconcepts.

$$
\begin{array}{llll}
(1a) & (x \sqcap x) \sqcap y = x \sqcap y & (1b) & (x \sqcup x) \sqcup y = x \sqcup y \\
(2a) & x \sqcap y = y \sqcap x & (2b) & x \sqcup y = y \sqcup x \\
(3a) & x \sqcap (y \sqcap z) = (x \sqcap y) \sqcap z & (3b) & x \sqcup (y \sqcup z) = (x \sqcup y) \sqcup z \\
(4a) & x \sqcap (x \sqcup y) = x \sqcap x & (4b) & x \sqcup (x \sqcap y) = x \sqcup x \\
(5a) & x \sqcap (x \vee y) = x \sqcap x & (5b) & x \sqcup (x \wedge y) = x \sqcup x \\
(6a) & x \sqcap (y \vee z) = (x \sqcap y) \vee (x \sqcap z) & (6b) & x \sqcup (y \wedge z) = (x \sqcup y) \wedge (x \sqcup z) \\
(7a) & \neg\neg(x \sqcap y) = x \sqcap y & (7b) & \lrcorner\lrcorner(x \sqcup y) = x \sqcup y \\
(8a) & \neg(x \sqcap x) = \neg x & (8b) & \lrcorner(x \sqcup x) = \lrcorner x \\
(9a) & (x \sqcap \neg x) = \bot & (9b) & (x \sqcup \lrcorner x) = \top \\
(10a) & \neg\bot = \top \sqcap \top & (10b) & \lrcorner\top = \bot \sqcup \bot \\
(11a) & \neg\top = \bot & (11b) & \lrcorner\bot = \top
\end{array}
$$

$$(12)\ (x \sqcap x) \sqcup (x \sqcap x) = (x \sqcup x) \sqcap (x \sqcup x)$$

where $x \vee y = \neg(\neg x \sqcap \neg y)$ *and* $x \wedge y = \lrcorner(\lrcorner x \sqcup \lrcorner y)$

Introduction

Definition 3 ([VW05])

The structure $\underline{D} = (D; \sqcap, \sqcup, \neg, \lrcorner, \bot, \top)$ of type $(2, 2, 1, 1, 0, 0)$ is called ***double Boolean algebra (dBa)*** *if it satisfies the 23 equations of Theorem 2.*

On a dBa $\underline{D} = (D; \sqcap, \sqcup, \neg, \lrcorner, \bot, \top)$ the relation $\sqsubseteq$ is defined as follows:

$$x \sqsubseteq y \Longleftrightarrow x \sqcap y = x \sqcap x \text{ and } x \sqcup y = y \sqcup y \tag{2}$$

"$\sqsubseteq$" is a quasi-order on D.
We set $D_{\sqcap} = \{x \in D : x \sqcap x = x\}$, $D_{\sqcup} = \{x \in D : x \sqcup x = x\}$ and $D_p = D_{\sqcap} \cup D_{\sqcup}$.

Definition 4 ([Wil00],[Kwu07])

A dBa $\underline{D}$ is called :

1. ***Pure*** *if for all $x \in D$, either $x \sqcap x = x$ or $x \sqcup x = x$.*
2. ***Trivial*** *if $\top \sqcap \top = \bot \sqcup \bot$.*
3. ***Regular*** *if "$\sqsubseteq$" is an order.*

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Example 1

We consider the context describe by the following cartesian table:

	a	b	c
1	×		×
2		×	×
3	×	×	×

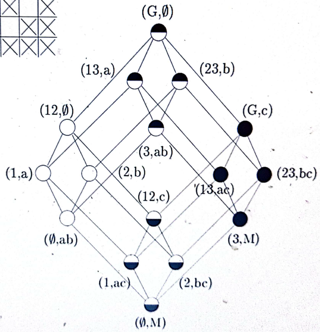
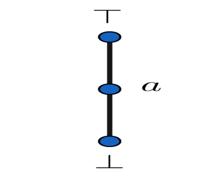


Fig. 1. A context and its protoconcept algebra

Introduction

Example 2

The algebra $\underline{D}_{3,I} = (D_{3,I} = \{\bot, a, \top\}; \sqcup, \sqcap, \neg, \lrcorner, \bot, \top)$ *with* $D_{3_\sqcap} = \{\bot, a\}$ *and* $D_{3_\sqcup} = \{a, \top\}$ *is a* ***pure, trivial and regular dBa*** *with the Cayley tabular given below [Kwu07] :*



$$D_{3_\sqcap} = \{\bot, a\} \text{ et } D_{3_\sqcup} = \{a, \top\}.$$

$\sqcap$	$\bot$	a	$\top$
$\bot$	$\bot$	$\bot$	$\bot$
a	$\bot$	a	a
$\top$	$\bot$	a	a

$\sqcup$	$\bot$	a	$\top$
$\bot$	a	a	$\top$
a	a	a	$\top$
$\top$	$\top$	$\top$	$\top$

x	$\bot$	a	$\top$
$\neg x$	a	$\bot$	$\bot$
$\lrcorner x$	$\top$	$\top$	a

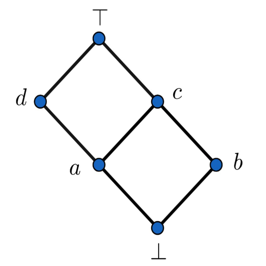
Example 3

The algebra $\underline{D}_6 = (D_6 = \{\bot, a, b, c, d, \top\}, \sqcup, \sqcap, \neg, \lrcorner, \bot, \top)$ *with* $D_{6_\sqcap} = \{\bot, a, b, c\}$ *and* $D_{6_\sqcup} = \{a, c, d, \top\}$ *is a* ***pure and regular dBa which is not trivial*** *with the Cayley tabular given below :*

$\sqcap$	$\bot$	a	b	c	d	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
a	$\bot$	a	$\bot$	a	a	a
b	$\bot$	$\bot$	b	b	$\bot$	b
c	$\bot$	a	b	c	a	c
d	$\bot$	a	$\bot$	a	a	a
$\top$	$\bot$	a	b	c	a	c

$\sqcup$	$\bot$	a	b	c	d	$\top$
$\bot$	a	a	c	c	d	$\top$
a	a	a	c	c	d	$\top$
b	c	c	c	c	$\top$	$\top$
c	c	c	c	c	$\top$	$\top$
d	d	d	$\top$	$\top$	d	$\top$
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$

x	$\bot$	a	b	c	d	$\top$
$\neg x$	c	b	a	$\bot$	b	$\bot$
$\lrcorner x$	$\top$	$\top$	d	d	c	a



Definition 5 ([BS81])

Let $\underline{B} = (B, \vee, \wedge, {}', 0, 1)$ *be a Boolean algebra.*

(a) *A subset* I *of* B *is called an* ***ideal*** *if it satisfies:*

  (1) $0 \in I$
  (2) $\forall x, y \in B,\ x, y \in I \Rightarrow x \vee y \in I$
  (3) $\forall x, y \in B,\ y \in I,\ x \leq y \Rightarrow x \in I$

(b) *A subset* F *of* B *is called a* ***filter*** *if it satisfies:*

  (1) $1 \in F$
  (2) $\forall x, y \in B,\ x, y \in F \Rightarrow x \wedge y \in F$
  (3) $\forall x, y \in B,\ x \in D,\ x \in F,\ x \leq y \Rightarrow y \in F$

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Definition 6

Let $\underline{D} = (D; \sqcup, \sqcap, \neg, \lrcorner, \bot, \top)$ *be a dBa. An equivalence relation* θ *on* D *is called* ***Congruence*** *if it is compatible with* $\sqcap$, $\sqcup$, $\neg$ *and* $\lrcorner$, *that is for any* $(a, b), (c, d) \in \theta$, $(a \sqcap c, b \sqcap d), (a \sqcup c, b \sqcup d), (\neg a, \neg b), (\lrcorner a, \lrcorner b) \in \theta$.

Lemma 3

Let $\underline{D} = (D; \sqcup, \sqcap, \neg, \lrcorner, \bot, \top)$ *be a dBa and* θ *be a congruence relation on* $\underline{D}$. *Then*

1. $[\bot]_\theta \cap D_\sqcap$ *is an ideal of* $\underline{D}_\sqcap$.
2. $[\top]_\theta \cap D_\sqcup$ *is a filter of* $\underline{D}_\sqcup$.

Definition 7 ([Vor05])

For a dBa $\underline{D} = (D; \sqcup, \sqcap, \neg, \lrcorner, \bot, \top)$ *we define two functions* $\cdot : D \times D \longrightarrow D$ *and* $+ : D \times D \longrightarrow D$ *by:*

$$x \cdot y := (x \sqcup \lrcorner y) \wedge (\lrcorner x \sqcup y),$$

$$x + y := (x \sqcap \neg y) \vee (\neg x \sqcap y).$$

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Congruence on double Boolean algebra

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Notation 1.1

Let $\underline{D}$ be a dBa. For a congruence generating pair (I, F) we denote by $\theta_{I,F}$ the congruence relation on $\underline{D}$ generated by (I, F) and by $\mathfrak{C}(\underline{D})$ the set of all congruence generating pair of $\underline{D}$.

We have $a\theta_{I,F}$ iff $a + b \in I$ and $a.b \in F$. On the set $\mathfrak{C}(\underline{D})$ of all congruences generating pair of a given dBa $\underline{D}$ we define an order relation by :

$$(I, F) \leq (G, H) \iff I \subseteq G \text{ and } F \subseteq H \text{ (See[Vor05])}.$$

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Subdirectly irreducible dBa

Theorem 4 ([Vor05])

Let $\underline{D} = (D; \sqcup, \sqcap, \neg, \lrcorner, \bot, \top)$ *be a double Boolean algebra. The map*

$$\begin{array}{rccc} \phi : & Con(\underline{D}) & \longrightarrow & \mathfrak{C}(\underline{D}) \\ & \theta & \longmapsto & ([\bot]_\theta \cap D_\sqcap,\ [\top]_\theta \cap D_\sqcup) \end{array}$$

is an isomorphism betwen the lattice of congruences on $\underline{D}$ *and the ordered set of all congruences generating pair of* $\underline{D}$*.*

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An algebra $\underline{A}$ *is* ***subdirectly irreducible*** *if its congruences lattice has one atom.*

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Proposition 1

Let $\underline{D}$ be a double Boolean algebra.

1. *The map f defined from $Con(\underline{D_p})$ to $Con(\underline{D})$ by $\theta \mapsto f(\theta) = \theta \cup \Delta_D$ is an embedding.*
2. *If $\underline{D}$ is regular and $card(D_p) > 1$, then* $\bigcap(Con(\underline{D})\backslash\{\Delta_D\}) = \bigcap(Con(\underline{D_p})\backslash\{\Delta_{D_p}\}) \cup \Delta_D$.
3. *If $\underline{D}$ is not regular and $card(D_p) > 1$, then* $\bigcap(Con(\underline{D})\backslash\{\Delta_D\}) = \Delta_D$

Corollary 5

A regular double Boolean algebra $\underline{D}$ such that D_p is nontrivial is subdirectly irreducible if and only if D_p is subdirectly irreducible.

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Subdirectly irreducible dBa

Corollary 6

Let $\underline{D}$ be a double Boolean algebra not regular. $\underline{D}$ is subdirectly irreducible if and only if $Card(D) \leq 2$.

Proposition 2

Let $\underline{D}$ be a double Boolean algebra such that for every $x \in D$, $\neg_\lrcorner x \sqsubseteq x$ and $x \sqsubseteq_\lrcorner \neg x$. The double Boolean algebra $\underline{D}$ is subdirectly irreducible if and only if $card(D_\sqcap) \leq 2$ and $card(D_\sqcup) \leq 2$.

Subdirectly irreducible dBa

Corollary 6

Let $\underline{D}$ be a double Boolean algebra not regular. $\underline{D}$ is subdirectly irreducible if and only if $Card(D) \leq 2$.

Proposition 2

Let $\underline{D}$ be a double Boolean algebra such that for every $x \in D$, $\neg\lrcorner x \sqsubseteq x$ and $x \sqsubseteq \lrcorner\neg x$. The double Boolean algebra $\underline{D}$ is subdirectly irreducible if and only if $card(D_{\sqcap}) \leq 2$ and $card(D_{\sqcup}) \leq 2$.

Conclusion

In this work we have shown that the study of the subdirectly irreducibility of a double Boolean algebra is focused on his pure part and we hve characterized some specific double Boolean algebras subdirecly irreducibles.

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THE END !!!

THANK FOR

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