Abstract commutator theory in concrete classes

David Stanovský
Charles University, Prague
stanovsk@karlin.mff.cuni.cz

The aim of the series is to introduce the abstract commutator theory of universal algebra and to show how it applies in various concrete varieties, such as groups, loops, semigroups or quandles.

(1) *Solvability and nilpotence: the beginnings and motivation.* I will start with the original historical motivation: the Galois theory of field extensions. Perhaps surprisingly, the same concept of solvability appears naturally in other situations, for instance, in the complexity analysis of the identity checking and equation solving problem. Is solvability and nilpotence a common phenomenon throughout the world of algebraic structures?

(2) *Abelianness and centrality.* What makes an algebraic object abelian? One possible answer is that such an object admits some sort of module structure. I will show an abstract syntactic condition that defines abelianness in universal algebra, and I will discuss how it relates to a module representation. In the second part of the lecture, I will discuss the concept of the center. How does it generalize from groups to loops (that is, when losing associativity) or inverse semigroups (that is, when losing inverses)?

(3) *The commutator.* Finally, I will define the commutator of two congruences, and the derived concepts of solvability and nilpotence. I will show how the theory can be adapted in concrete classes, for instance loops or quandles, and how to apply these concepts to obtain conceptually simple inductive proofs. I also plan to briefly discuss central extensions.