S-preclones and the Galois connection $^S\mathrm{Pol} - {}^S\mathrm{Inv}$

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We consider so-called *S-operations* $f : A^n \to A$ for which each variable gets a *signum* $s \in S$ representing ``properties'' like, e.g., order preserving or order reversing with respect to a partial order on A. The set S of such properties has the structure of a monoid reflecting the behaviour of composition of S-operations (e.g., order reversing composed with order reversing is order preserving). The collection of all operations with prescibed properties for their signed variables is not a clone (since it is not closed under arbitrary identification of variables), but it is a preclone with special properties what leads to the notion of *S-preclone*. We introduce *S-relations* $\varrho = (\varrho_s)_{s \in S}$, *S-relational clones* and a preservation property $(f \overset{S}{\triangleright} \varrho)$, and consider the induced Galois connection $^S\mathrm{Pol} - {}^S\mathrm{Inv}$. The S-preclones turn out to be just the Galois closures. Moreover we can characterize the Galois closures on the relational side as *S-relational clones*.