Quantum Suplattices

Gejza Jenča
Slovak University of Technology

Bert Lindenhovius
Slovak Academy of Sciences

lindenhovius@mat.savba.sk

Discrete quantization is a method of finding noncommutative generalizations of discrete mathematical structures by means of internalizing these structures in a suitable order-enriched dagger compact category **qRel**, whose objects are von Neumann algebras isomorphic to a (possibly infinite) ℓ^∞-sum of matrix algebras called *quantum sets*. The morphisms of **qRel** are noncommutative generalizations of binary relations between sets, called *quantum relations*, and were distilled by Weaver [5] from his work with Kuperberg on the quantization of metric spaces [6]. Other structures that were quantized using discrete quantization include posets [4] and cpos [3].

In this contribution, we apply discrete quantization in order to obtain a noncommutative version of complete lattices (also called suplattices), which we call *quantum suplattices*. We discuss the categorical constructions in the category **Rel** that can be used to define ordinary suplattices, and discuss how to lift these constructions to **qRel** in order to obtain our definition of quantum suplattices. Furthermore, we discuss how classical theorems on suplattices such as the Knaster-Tarski Theorem generalize to the quantum case. We refer to [1] for the concrete constructions of quantum suplattices.

Finally, we discuss how to quantize the concept of a topological space based on the notion of quantum suplattices. Traditionally, C*-algebras form the standard approach to quantum topology, but only generalize locally compact Hausdorff spaces. Our approach is different, and allows for the quantization of specific topological spaces that are not necessarily locally compact or Hausdorff.

References

[1] G. Jenča, B. Lindenhovius, *Quantum Suplattices*, to appear in the Proceedings of the 20th International Conference on Quantum Physics and Logic (QPL 2023).

[2] A. Kornell, *Quantum Sets*, J. Math. Phys. 61, 102202 (2020).

[3] A. Kornell, B. Lindenhovius, M. Mislove, *Quantum CPOs*, Proceedings Quantum Physics and Logic 174--187 (2020).

[4] A. Kornell, B. Lindenhovius, M. Mislove, *A category of quantum posets*, Indagationes Mathematicae, Volume 33, Issue 6, 1137--1171 (2022).

[5] N. Weaver, *Quantum Relations*, Memoirs of the American Mathematical Society, Vol. 215, No. 1010 (2012).

[6] G. Kuperberg, N. Weaver, *A von Neumann algebra Approach to Quantum Metrics*, Memoirs of the AMS, Vol. 215, No. 1010 (2012).