Indecomposable involutive 2-permutable solutions of Yang-Baxter equation

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The Yang-Baxter equation is a fundamental equation occurring in integrable models in statistical mechanics and quantum field theory. Let V be a vector space. A solution of the Yang–Baxter equation is a linear mapping $r : V \otimes V \to V \otimes V$ such that

$$(id \otimes r)(r \otimes id)(id \otimes r) = (r \otimes id)(id \otimes r)(r \otimes id).$$

Description of all possible solutions seems to be extremely difficult and therefore there were some simplifications introduced. Let X be a basis of the space V and let $\sigma : X^2 \to X$ and $\tau : X^2 \to X$ be two mappings. We say that (X, σ, τ) is a set-theoretical solution of the Yang–Baxter equation if the mapping $x \otimes y \to \sigma(x, y) \otimes \tau(x, y)$ extends to a solution of the Yang–Baxter equation. It means that $r : X^2 \to X^2$, where $r = (\sigma, \tau)$ satisfies the braid relation:

$$(id \times r)(r \times id)(id \times r) = (r \times id)(id \times r)(r \times id).$$

A solution is called non-degenerate if the mappings $\sigma_x = \sigma(x, \_)$ and $\tau_y = \tau(\_, y)$ are bijections, for all $x, y \in X$. A solution (X, σ, τ) is involutive if $r^2 = id_{X^2}$.

It can be proved that being an involutive solution reduces to the following equation:

$$\sigma_x \sigma_y = \sigma_{\sigma_x(y)} \sigma_{\tau_y(x)}.$$

Moreover, in this case we have $t_y(x) = \sigma^{-1}_{\sigma_x(y)}(x)$ and we need to consider one binary operation only.

An involutive solution is called 2-permutable if it satisfies

$$\sigma_{\sigma_x(z)} = \sigma_{\sigma_y(z)},$$

for all $x, y, z \in X$. An involutive solution is called indecomposable if the permutation group generated by $\{\sigma_x \mid x \in X\}$ acts transitively on X. In our talk we shall speak about a generic construction of indecomposable 2-permutable involutive solutions.