# Completely hereditarily atomic OMLs

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This talk has two parts. The first part is somewhat tutorial in nature. We review aspects of orthomodular lattices (OMLs) and two important methods to construct them, the Kalmbach construction and the constructions of Keller, Gross and Kunzi via orthomodular spaces. In the second part of the talk we introduce the main topic, that of algebraic OMLs and their weakening to completely hereditarily atomic OMLs. Completely hereditarily atomic OMLs can be described in various ways, one of which is that they are complete OMLs whose blocks are all atomic. Motivated by issues in extending quantum set theory to the infinite-dimensional setting, we consider the relationship between algebraicity and the covering property, and their weaker forms, for OMLs. We recall that the covering property says that if a is an atom that does not lie beneath $x$, then $a \vee x$ covers $x$.

**Theorem 1** *An OML is algebraic and has the covering property iff it is a direct product of finite-height modular ortholattices.*

So to move past the finite-dimensional setting, one cannot retain both algebraicity and the covering property. We use the two construction techniques discussed in the tutorial to provide somewhat involved examples of directly irreducible OMLs of infinite height, one that is algebraic with the 2-covering property, and one that is completely hereditarily atomic with the covering property.