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Aggregation on Partially Ordered Sets

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Abstract

In this thesis, we present the results achieved by the author during his PhD studies in the field of aggregation functions. Most of the results focus on associative aggregation functions, with particular emphasis on non-commutative, i.e., non-symmetric, aggregation.

The first part of the thesis is devoted to associative aggregation functions defined on the unit interval. In the second part, we study the extension of these results from the unit interval to bounded lattices and partially ordered sets (posets), which constitute a natural generalization of the original domain. Several similarities, as well as important differences, are identified, analyzed, and discussed.

Keywords:
Aggregation functions, Uninorm, Nullnorm, Finite lattice, Pseudo-t-norm

Abstrakt

V tejto práci prezentujeme výsledky, ktoré autor dosiahol počas svojho doktorandského štúdia v oblasti agregačných funkcií. Väčšina výsledkov sa zameriava na asociatívne agregačné funkcie, pričom osobitný dôraz je kladený na nekomutatívnu, t. j. nesymetrickú, agregáciu.

Prvá časť práce je venovaná asociatívnym agregačným funkciám definovaným na jednotkovom intervale. V druhej časti sa zaoberáme rozšírením týchto výsledkov z jednotkového intervalu na ohraničené zväzy a čiastočne usporiadané množiny (posety), ktoré predstavujú prirodzené zovšeobecnenie pôvodného definičného oboru. Identifikujeme, analyzujeme a diskutujeme viaceré podobnosti, ako aj dôležité rozdiely.

Kľúčové slová:
Agregačné funkcie, Uninorma, Nullnorma, Konečný zväz, Pseudo-t-norma

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1 Introduction

As the title suggests, this dissertation deals with aggregation functions on partially ordered sets. Particular emphasis is placed on associative aggregation functions due to their theoretical and practical significance. First and foremost, associativity is an important theoretical mathematical property that has led to the development of a broad theory of semigroups. From the perspective of aggregation, associativity holds a special position, as it allows for the aggregation of any number of elements by simply repeatedly applying a binary associative aggregation function, which is also of great importance in terms of computational simplicity.

In thesis for dissertation examination the following objectives were formulated:

1. Characterization of non-commutative and associative aggregation functions on the unit interval which are continuous around the main diagonal.

2. Study of the points of non-commutativity and discontinuity of associative aggregation functions.

3. Extension of the study of non-commutative and associative aggregation functions on the unit interval to bounded lattices with special focus on idempotent and later divisible aggregation functions.

4. Examining the connection between various types of associative aggregation functions on a bounded lattice.

5. Investigation of the relation of monotonicity with respect to lattice order of idempotent associative aggregation functions to the induced order.

6. Exploration of the application possibilities of the obtained results.

The respective chapter of the dissertation thesis deals with the corresponding objective. The main results of this work can be divided into two areas based on the domain of the aggregation function. Chapters 3, 4, and 8 present results from aggregation on the unit interval, i.e., in general on a subset of $\mathbb{R}^n$. In the remaining chapters, namely 5, 6, and 7, the focus is on aggregation functions defined on more general structures, specifically sets and partially ordered sets in general.

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Since the thesis is concepted as a collage of author's paper, here is given a list of the pubblications included in the appendix.

In the appendix, the author's most significant papers are included. They are listed in chronological order by publication date as follows.

1. Decomposition of pseudo-uninorms with continuous underlying functions via ordinal sum [9].

2. Idempotent pseudo-uninorms on special bounded lattices [11].

3. Unifunctions and Nullfunctions: A new generalization of overlap and grouping functions for bipolar modeling [6].

4. The structure of pseudo-n-uninorms with continuous underlying functions [12].

5. Self-dual pseudo-uninorms [5].

6. Difference between divisibility and smoothness for pseudo-t-norms on finite lattices (accepted in Studies on Computational Intelligence [8].

7. Non-commutative, associative aggregation functions continuous around the diagonal (under review in Applied and Computational Mathematics) [20].

8. Characterizing functions of pseudo-n-uninorms with continuous underlying functions (under review in IEEE Transactions on Fuzzy Systems) [19].

2 Aggregation on the unit interval

In what follows, we discuss several important associative aggregation functions. To familiarize the reader with these concepts, we provide their definitions.

Definition 2.1 ([21])
An associative, binary function, which is non-decreasing in all coordinates (or simply non-decreasing)

(1) $PT : [0,1]^2 \to [0,1]$ which has the neutral element 1 is called a pseudo-triangular norm (pseudo-t-norm).

(2) $T : [0,1]^2 \to [0,1]$ which is commutative and has the neutral element 1 is called a triangular norm (t-norm).

(3) $PS : [0,1]^2 \to [0,1]$ which has the neutral element 0 is called a pseudo-triangular conorm (pseudo-t-conorm).

(4) $S : [0,1]^2 \to [0,1]$ which is commutative and has the neutral element 0 is called a triangular conorm (t-conorm).

(5) $P : [0,1]^2 \to [0,1]$ which has the neutral element $e \in [0,1]$ is called a pseudo-uninorm.

(6) $U : [0,1]^2 \to [0,1]$ which is commutative and has a neutral element $e \in [0,1]$ is called a uninorm.

(7) $V : [0,1]^2 \to [0,1]$ which is commutative and has an annihilator $a \in [0,1]$ such that 0 acts as the local neutral element on $[0,a]$ and 1 acts as the local neutral element on $[a,1]$ is called a nullnorm.

In the articles [5, 9, 18], the author deals with the characterization and study of important classes of pseudo-uninorms. In [9], a characterization of pseudo-uninorms with continuous underlying functions was carried out. It was proven that every such pseudo-uninorm can be expressed as a Clifford ordinal sum of trivial semigroups, projective semigroups and specific semigroups given in the following definition.

Definition 2.2 ([14])
Let $0 \le a < b \le c \le d < f$. Then

1. a semigroup $(]a, b[\cup\{c\}\cup]d, f[, *)$ is called a *representable semigroup* if $*$ is isomorphic to a restriction of a representable uninorm to $]0, 1[^2$.

2. a semigroup $(]a, b[, *)$ is called a *t-strict semigroup* if $*$ is isomorphic to a restriction of a strict t-norm to $]0, 1[^2$.

3. a semigroup $(]d, f[, *)$ is called a *s-strict semigroup* if $*$ is isomorphic to a restriction of a strict t-conorm to $]0, 1[^2$.

4. a semigroup $([a, b[, *)$ is called a *t-nilpotent semigroup* if $*$ is isomorphic to a restriction of a nilpotent t-norm to $[0, 1[^2$.

5. a semigroup $(]d, f], *)$ is called *s-nilpotent semigroup* if $*$ is isomorphic to a restriction of a nilpotent t-conorm to $]0, 1]^2$.

This result not only extended the characterizations of other classes of pseudo-uninorms [16, 18] and uninorms in general [14], but also pointed out that the structure of these non-commutative aggregation functions is not fundamentally different from their commutative versions. Based on the results obtained in [9], the author characterized all self-dual pseudo-uninorms by proving that each self-dual pseudo-uninorm has continuous underlying functions [5] and thus showed that pseudo-uninorms are also suitable for modeling dual and bipolar processes.

Unlike pseudo-uninorms, where the characterization was analogous to that of uninorms, in [10], the author, together with Mesiarová-Zemánková, introduced a new class of aggregation functions, which they called pseudo-n-uninorms and which arose by removing the commutativity axiom from the definition of n-uninorms.

Definition 2.3 ([10])
Let $n \in \mathbb{N}$. Let $P^n : [0,1]^2 \to [0,1]$ be a non-decreasing, associative binary function with n-neutral element $\{e_1, \ldots, e_n\}_{z_1,\ldots,z_{n-1}}$. Then P^n is called a pseudo-n-uninorm.

At the same time, the structure of pseudo-n-uninorms reveals the limitations of commutative constructions based on mutually disjoint summands. These findings later led to the introduction of a new construction method called the non-commutative ordinal sum [17]. We later used this method to characterize pseudo-n-uninorms with continuous associated functions [12] and all associative non-decreasing (non-commutative) functions that are continuous around the main diagonal [20]. We present the detailed results in Chapter 3.

From an application perspective, continuity is a key property because, on the one hand, it allows us to model real-world problems and, on the other hand, ensures the stability of the calculations. Particularly because of this second reason, it is important when applying aggregation functions to know where discontinuities occur, because in individual calculations, significant and unwanted deviations may occur precisely in the regions of discontinuity, which is important to know in advance. Whenever a continuous associative aggregation function is different from semi-t-operator, some points of discontinuity will inevitably occur there. Therefore, we have set as a further goal of our work to characterize the points of discontinuity of certain types of associative aggregation functions. Moreover, since we are specifically focused on aggregation functions that need not necessarily be commutative, we were also interested in determining in which regions of the unit can lead to non-commutativity.

In the case of pseudo-uninorms with continuous underlying functions, we addressed these questions in [9], where we showed that all points of discontinuity lie on the characterizing function of the given pseudo-uninorm. The characterizing function of a pseudo-uninorm divides the unit square into two parts in such a way that the pseudo-uninorm takes values less than the neutral element e at points lying below the characterizing function, and conversely, values greater than e at points above the characterizing function. An interesting observation is also that that if the pseudo-uninorm with continuous associated functions P does not commute at the point (x, y), i.e., $P(x, y) \neq P(y, x)$, then this is a point of discontinuity, and thus it lies on the characterizing function of the pseudo-uninorm.

In the case of pseudo-n-uniform norms with continuous underlying functions, the situation is in some respects similar and in others fundamentally different. As for points of discontinuity, just as in the previous case, in this situation as well the point of discontinuity lies on one of the characterizing functions of the pseudo-n-uninorm [19]. In the case of non-commutativity points, the situation is somewhat more complicated. In this case, all non-commutativity points lie between some characterizing function of the pseudo-n-uninorm and its characterizing cofunction [19]. By characterizing cofunction, we mean the corresponding characterizing function of the pseudo-n-uninorm, which is obtained by interchanging the coordinates of the original pseudo-n-uninorm. For more details, we refer to Chapter 4.

As we have already noted above, for some associative functions we are unable to ensure continuity over the entire domain. Since no uninorm is continuous on the entire unit square, apart from trivial cases, this idea led the author, together with Kalina, to introduce a new type of aggregation functions, which they called unifunctions [6].

Definition 2.4 ([7])
Let $e \in [0, 1]$. An n-ary function $UF_e : [0, 1]^n \to [0, 1]$ is called a unifunction if it is non-decreasing in all variables, commutative, continuous, and the following conditions hold.

1. For $(x_1, \ldots, x_n) \in [0, e]^n$, $UF_e(x_1, \ldots, x_n) = 0$ if and only if $x_i = 0$ for some $i = 1, \ldots, n$.

2. For $(x_1, \ldots, x_n) \in [e, 1]^n$, $UF_e(x_1, \ldots, x_n) = 1$ if and only if $x_i = 1$ for some $i = 1, \ldots, n$.

3. $UF_e(x_1, \ldots, x_n) = e$ implies $(x_1, \ldots, x_n) \notin ([0, e]^n \cup [e, 1]^n) \setminus \{(e, \ldots, e)\}$.

Their structure is based on the structure of uninorms, but unlike uninorms, which preserve associativity at the expense of continuity, the opposite is true for unifunctions. These are thus continuous aggregation functions, which are, however, necessarily non-associative if $e \notin \{0, 1\}$. It should also be noted that just as uninorms are a generalization of triangular norms and triangular conorms, unifunctions are a generalization of overlap functions [2] and grouping functions [3] in a very similar manner. A summary of results from the theory of unifunctions is presented in Chapter 8.

3 Aggregation on partially ordered sets

In general, unit intervals and linear orderings are not always sufficient for an adequate description of various processes and structures encountered in applications. For this reason, the theory of aggregation functions defined on more general settings, such as sets and partially ordered sets, has been gaining increasing attention. In particular, these frameworks allow for a more flexible modelling of aggregation phenomena beyond the classical real-valued case.

Since associativity itself does not impose additional structural requirements on the underlying domain, it is possible to extend and redefine associative aggregation functions in a natural and straightforward way directly on arbitrary sets.

The original definition of aggregation functions on the unit interval is based only on the boundary conditions and the existence of order. Therefore, bounded posets (partially ordered sets) [1] forms a suitable domain for further generalization of aggregation functions. On the one hand, this approach enables the possibility to model incomparable choices, which are common in human decision making and thus better reflect the real world. On the other hand moving from the unit interval as a subdomain of the real line, results in loss of several important properties, such as topology, existence of well defined addition, multiplication and many other issues. Nevertheless, all of the mentioned makes the research on aggregation functions on posets and lattices very interesting and open topic.

First and foremost, when aggregating over partially ordered sets, it is important to realize that many properties known from the unit interval cannot be applied to a general partially ordered set. At the same time, since every interval is also a sequence, we know that two elements are always comparable, which, however, does not hold in general. All these considerations significantly influence the specificity of associative aggregation functions on partially ordered sets. Observe that the exchange a domain from the unit interval to a bounded posets in Definition 2.1 leads to immediate redefinition of the concepts to any bounded poset.

In [11], we studied idempotent pseudo-uninorms on a special type of sets. We assumed that the identity element of the pseudo-uninorm is comparable to all elements of the set. In this special case, we succeeded in showing that every such pseudo-uninorm has an ordinal character, where the individual summands are either lower or upper semilattices, or possibly a projective semigroup.

The definition of continuity adopted on the finite lattice is not suitable for practical applications, since in this sense even a drastic function is a continuous triangular norm on such a set. Therefore, for finite chains with triangular norms, divisibility, smoothness, or the mean value property [13] have been used instead of classical continuity. In [8], we introduced these concepts for finite lattices as well. Their definitions are as follows. Note that $\prec$ denotes here a cover relation, i.e., $x \prec y$ if and only if $x < y$ and there is no $y \in P$ such that $x < z < y$.

Definition 3.1 ([8])
Let L be a finite lattice and $f : L \to L$ a non-decreasing function. Then, we say that

1. f is smooth if for any $x, y \in L$ such that $x \prec y$ then $f(x) \preceq f(y)$ holds.

2. f has the intermediate value property if for all $x, y, t \in L$ such that $f(x) \leq t \leq f(y)$, there exists $z \in [x, y]$ such $f(z) = t$,

3. f is 1-Lipschitz if for any interval $[x, y]$ of length n, the sublattice $[f(x), f(y)]$ is of length n or less.

Definition 3.2 ([8])
Let L be a finite lattice and $T : L^2 \to L$ be a pseudo-t-norm. Then, we say that

1. T is smooth if T is smooth in both variables.

2. T has the intermediate value property if T has the intermediate value property in both variables.

3. T is 1-Lipschitz if T is 1-Lipschitz in both variables.

4. T is called divisible if for all $x \leq y$ there exists $z_1, z_2 \in L$ with $T(y, z_1) = T(z_2, y) = x$.

At the same time, we showed that, unlike in chains, in the case of triangular norms on lattices, these are generally distinct properties. We were also interested in the relationship between these properties and the possible commutativity of the pseudo-triangular norm. It turned out that on general spaces there exist divisible pseudo-triangular norms as well as smooth pseudo-triangular norms, which need not be triangular norms. However, if a pseudo-triangular norm is both divisible and smooth, then it is necessarily commutative as well.

In [11], it was proven that a non-trivial pseudo-uninorm on a lattice can also be a semi-t-operator (for the definition of semi-t-operator on bounded lattices see [4]).

Moreover, we are currently preparing a publication in which we show that each smooth idempotent uninorm on an arbitrary lattice is a nullnorm.

Finally, we know that every associative, idempotent, and commutative function induces an order [15], so it is interesting to investigate the relationship between the original order and the order induced by such an aggregate function. It can be shown that, thanks to this order, we can always decompose a space with an idempotent nullnorm into a system of mutually disjoint sets that are closed under the given nullnorm.

4 Conclusions

The results presented here highlight the fact that the theory of associative aggregation functions remains a dynamically evolving field of research in which algebraic, order-theoretic, and analytical approaches are naturally intertwined. The present dissertation extends several classical results from the unit interval to the non-commutative setting, as well as to the setting of lattices and partially ordered sets. At the same time, it introduces new classes of aggregation functions, new construction methods, and new characterizations, thereby contributing to a deeper understanding of the structure of associative aggregation functions in a general setting.

The results obtained open up opportunities for further research, particularly in the area of classifying aggregation functions on general ordered structures, further investigation of the relationship between algebraic properties and order, as well as in applications in decision-making processes, fuzzy logic, and the modeling of artificial intelligence systems. It can therefore be expected that the presented results will provide a suitable foundation for further theoretical and applied developments in this field.

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