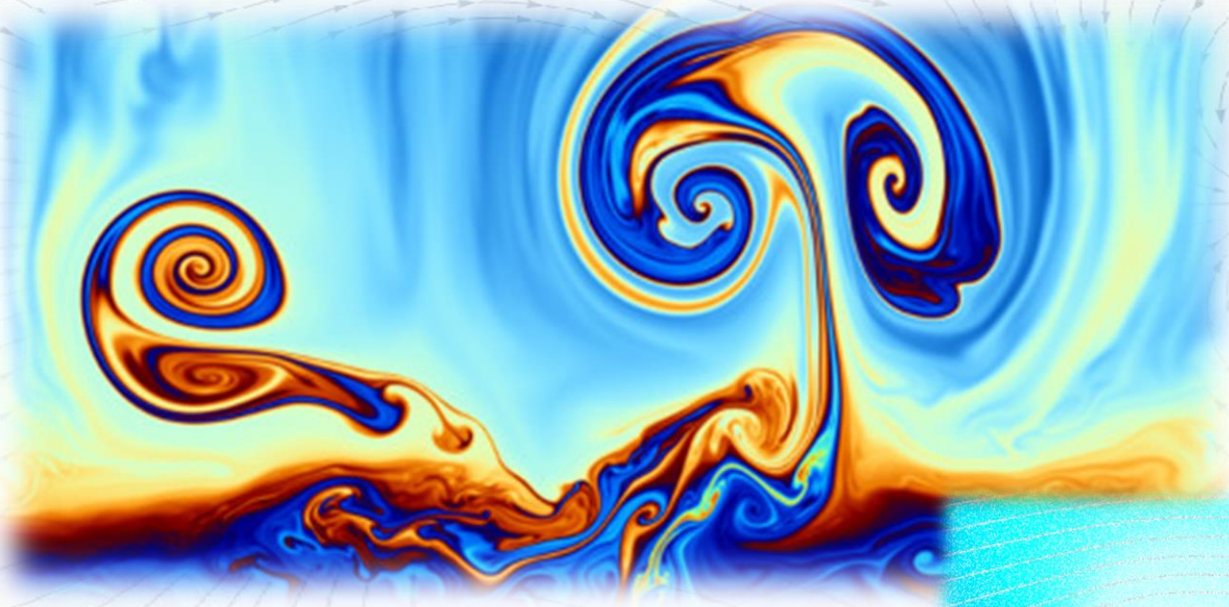
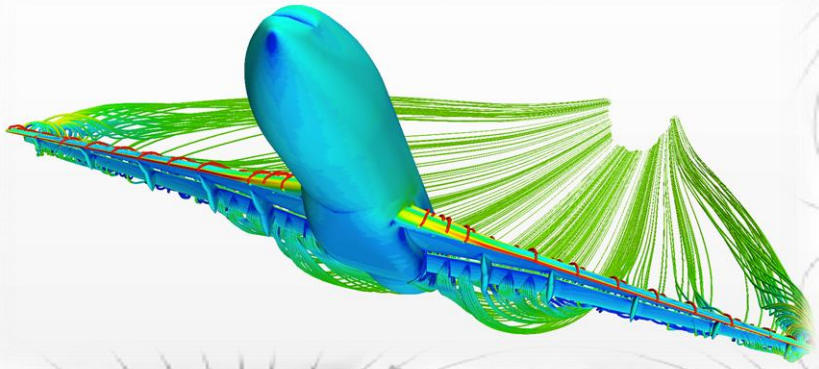
Modelovanie prúdenia tekutín

MARTIN ČAVARGA

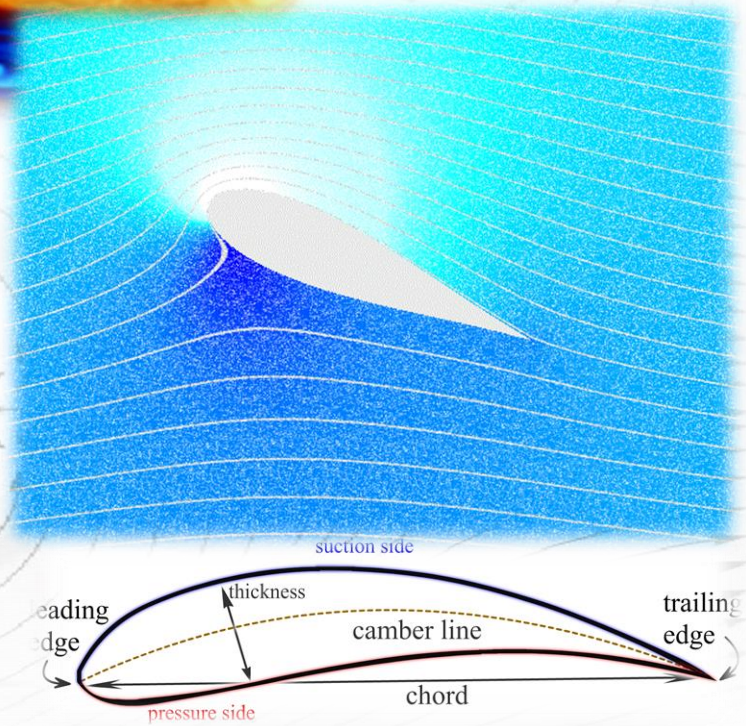
------- MODELOVANIE PRÚDENIA -------

- Čo sú to tekutiny a aké majú vlastnosti?
- Čo znamená modelovať prúdenie?

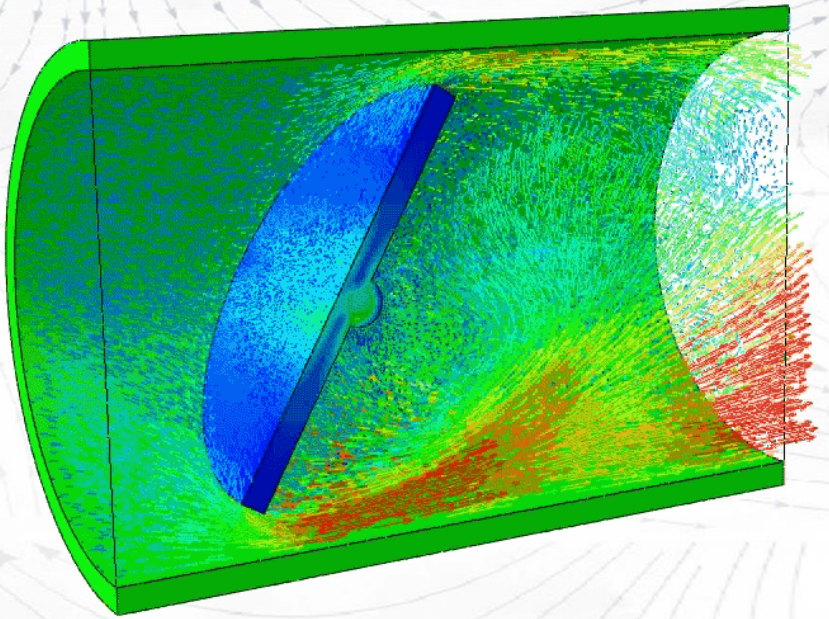




- Aplikácie
- Zjednodušený model prúdenia a dizajn profilov krídiel



------- ČO ZNAMENÁ MODELOVAŤ PRÚDENIE? -------



$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$-\frac{1}{\rho} p + \mathbf{f} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}$$

$$-\nabla p + \rho \mathbf{f} + \nabla \cdot (\lambda \nabla \cdot \mathbf{v} \mathbf{I}) + \nabla \cdot (\mu (\nabla \mathbf{v} + \nabla \mathbf{v}^\top)) =$$

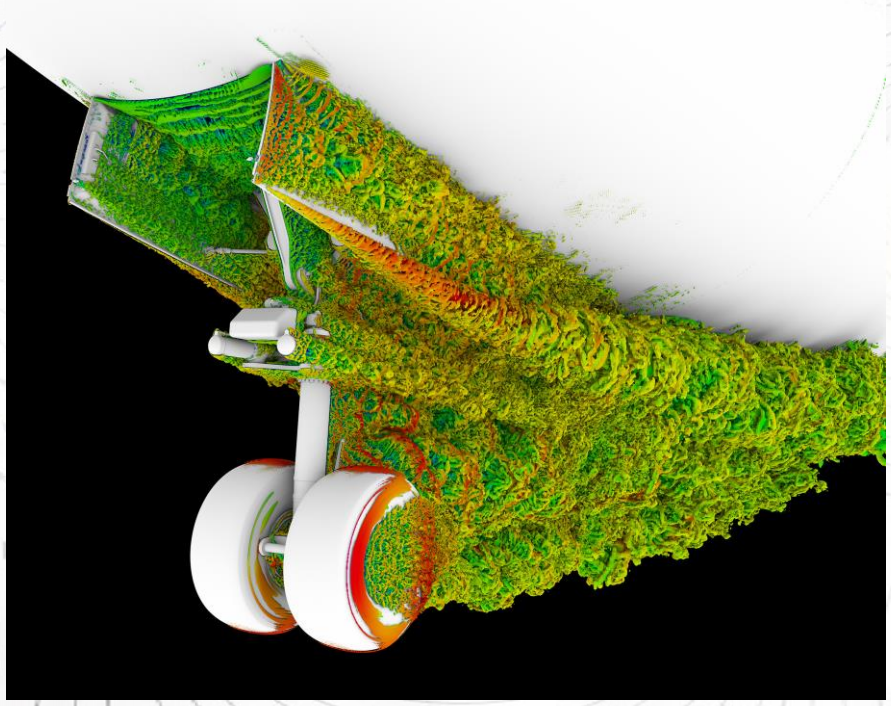
$$= \frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v})$$

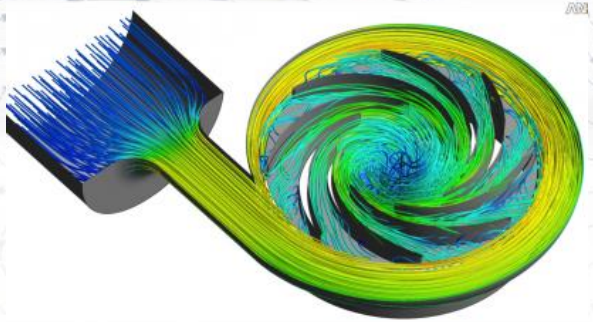
- Veličiny: $p,\ \rho,\ \mathbf{v} = (v_1, v_2, v_3),\ T,\ u$
- Zachovanie hmotnosti
- Zachovanie hybnosti (Newtonov 2. p. zákon)
- Vnútorné trenie a vedenie tepla

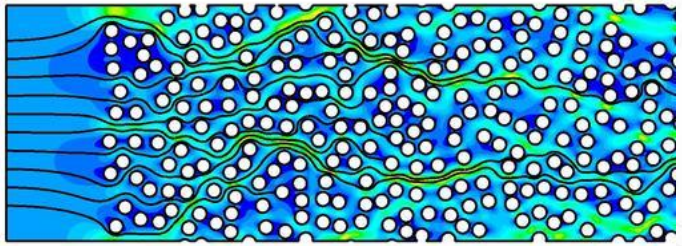
$$\rho \mathbf{f} \cdot \mathbf{v} + \nabla \cdot (\frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^\top) \otimes \mathbf{v}) + \rho(-k \nabla T) - \nabla \cdot (-k \nabla T) =$$

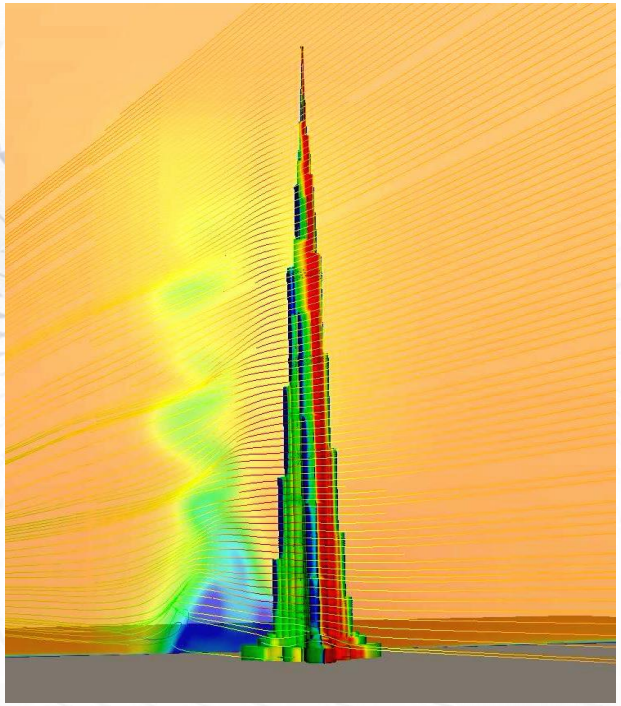
$$= \frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\rho E \mathbf{v})$$

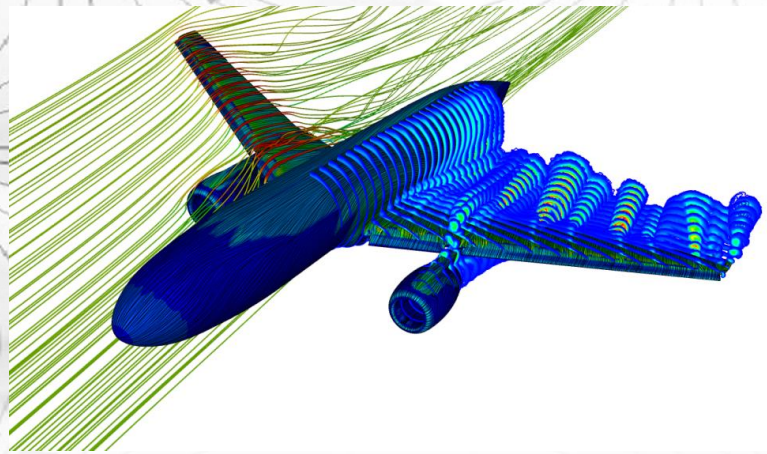
------- APLIKÁCIE -------

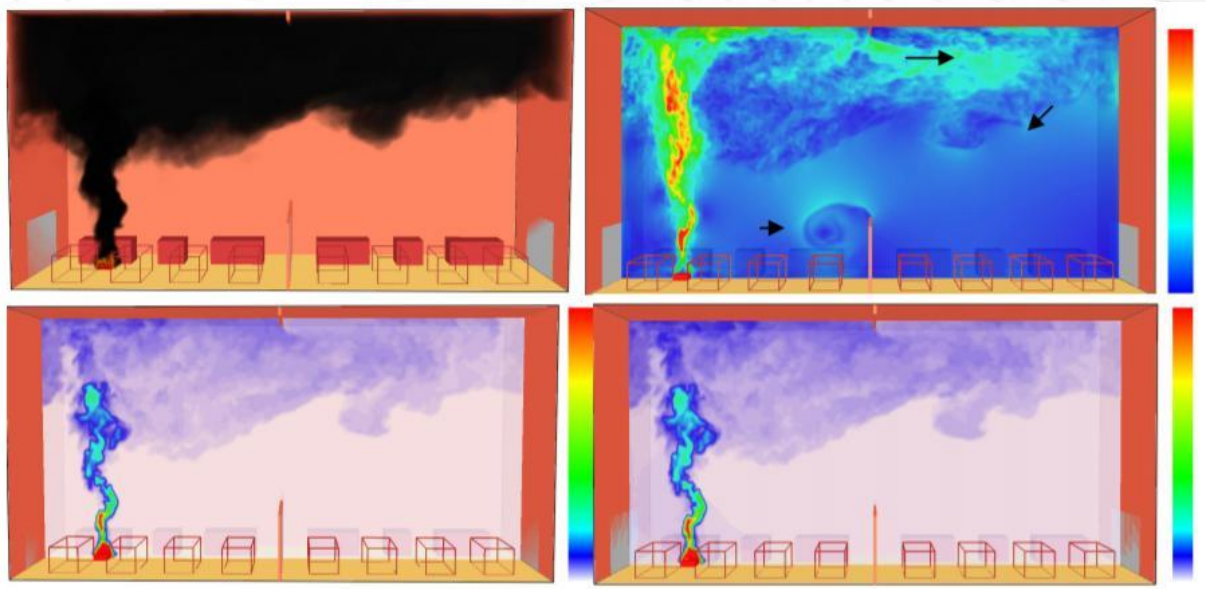








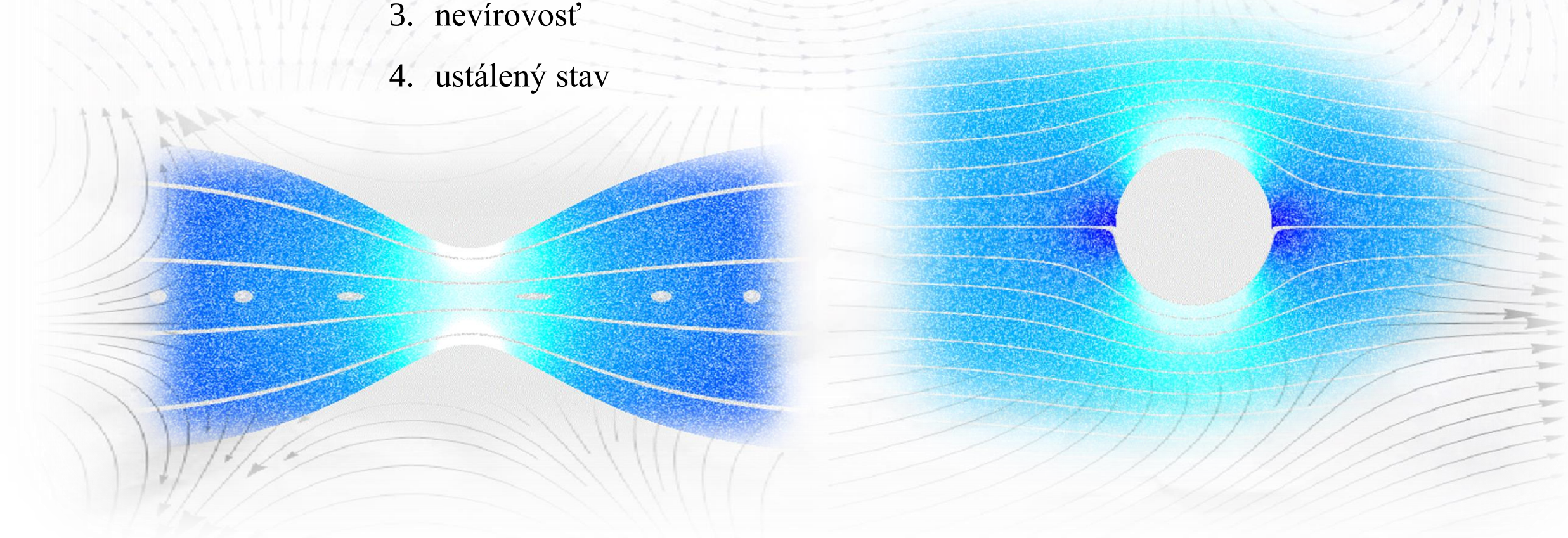




------- IDEÁLNE PRÚDENIE -------

Budeme pracovať s prúdením, ktoré spĺňa nasledujúce podmienky:

1. nulová viskozita (vnútorné trenie)
2. nestlačiteľnosť
3. nevírovosť
4. ustálený stav



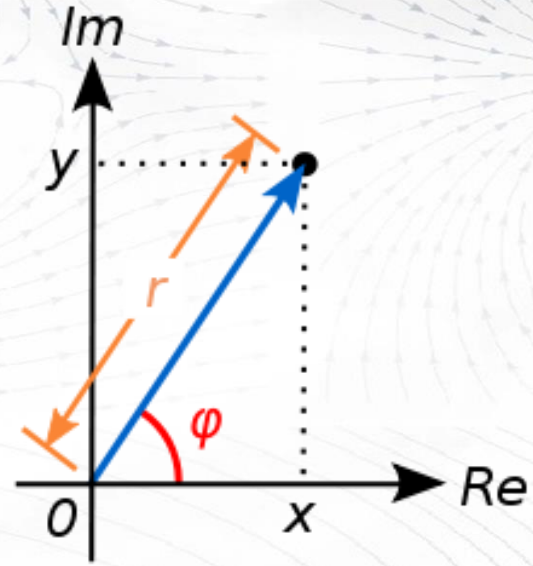
------- KOMPLEXNÉ ČÍSLA -------

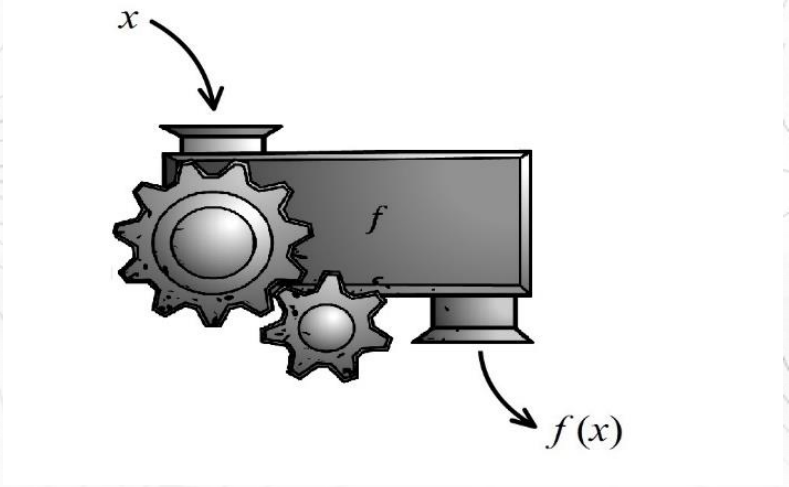
- Niektoré rovnice nemajú riešenie medzi reálnymi číslami $x^2 + 1 = 0$

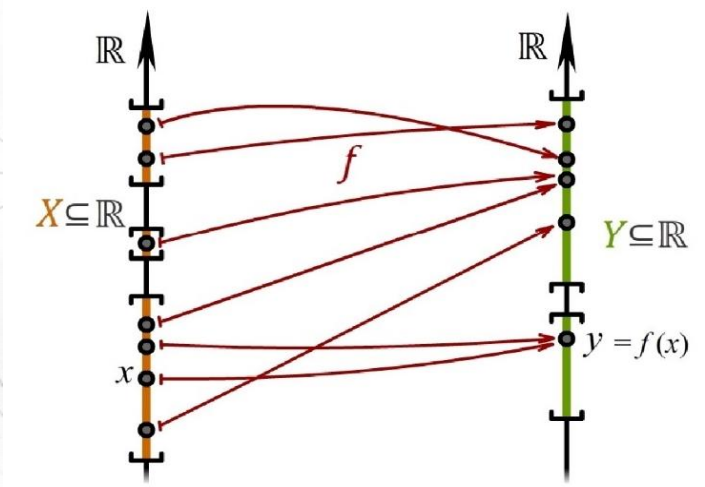
$$x^3 + ax^2 + bx + c = 0$$

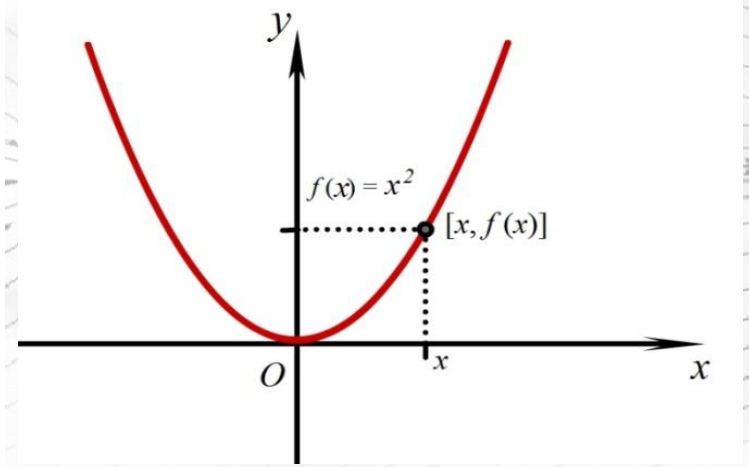
- uvádzame $i = \sqrt{-1}$

- všeobecne $z = x + iy$

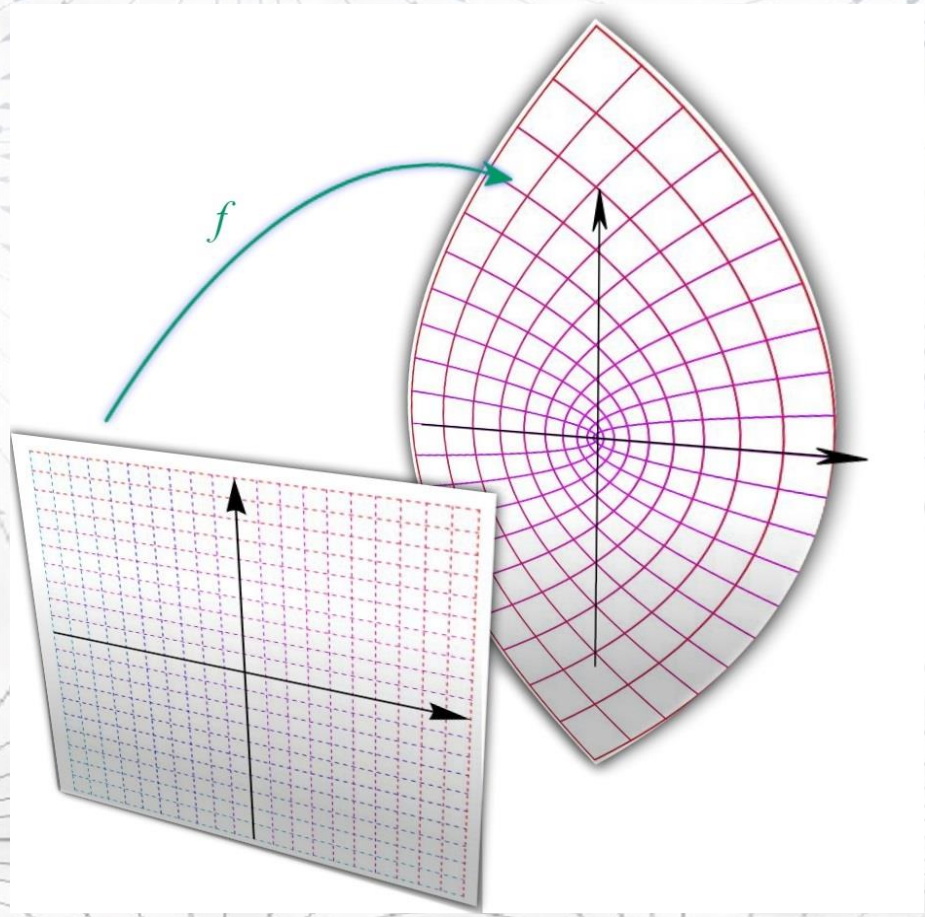


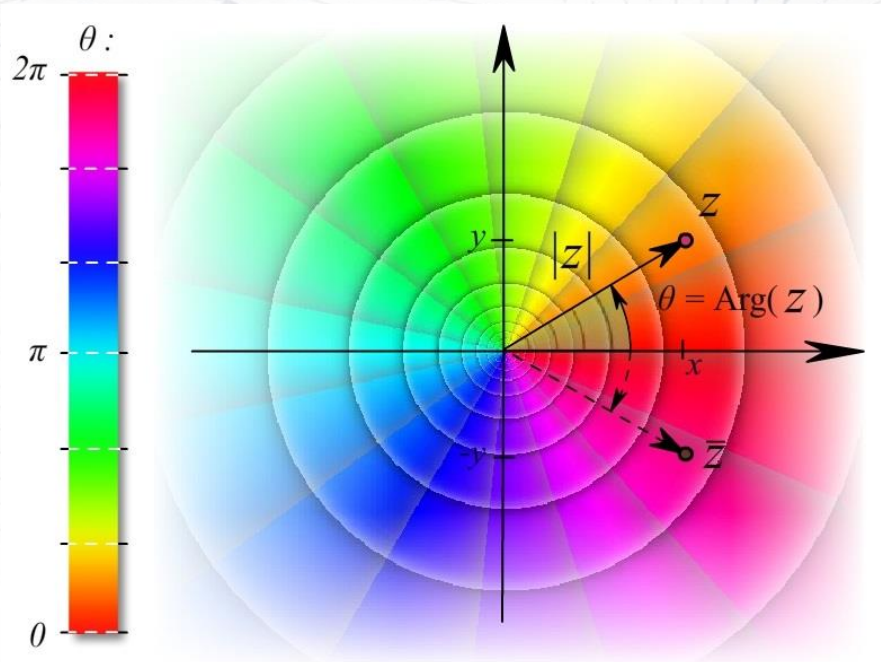




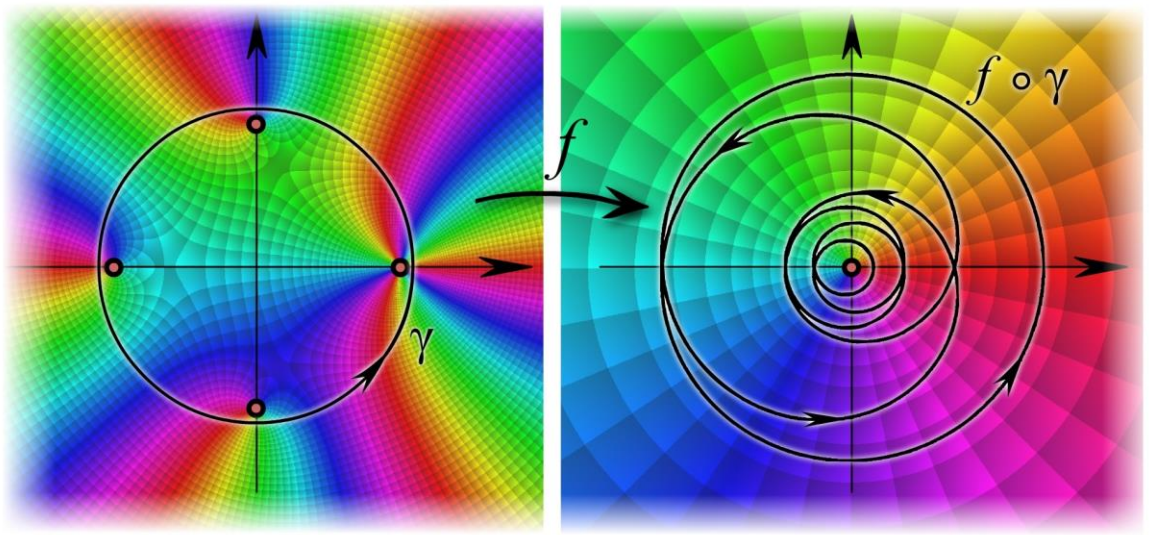


------- KOMPLEXNÉ FUNKCIE -------

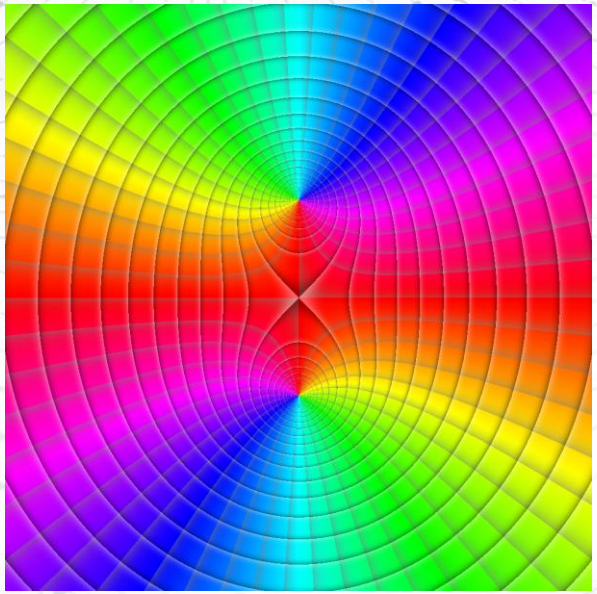


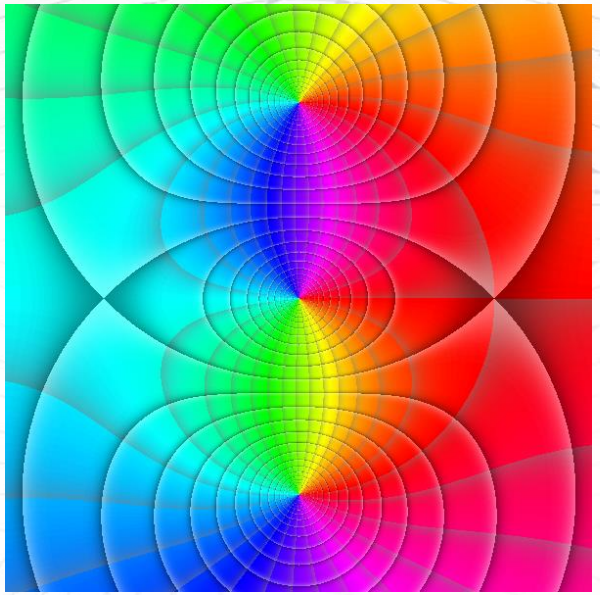


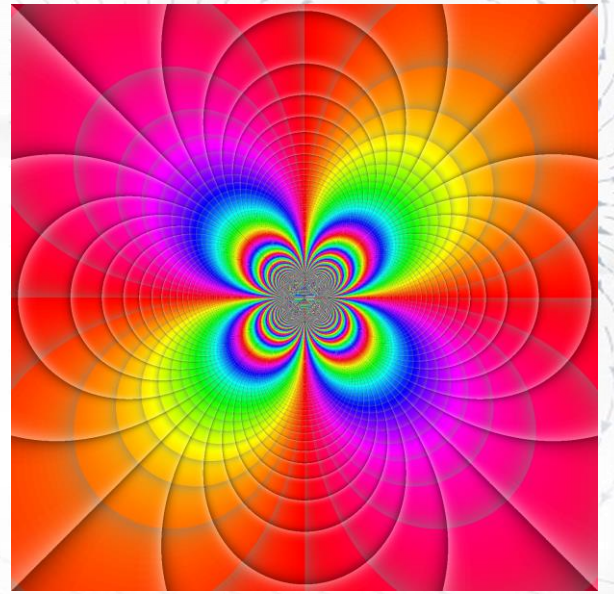
$$f(z) = z^2$$

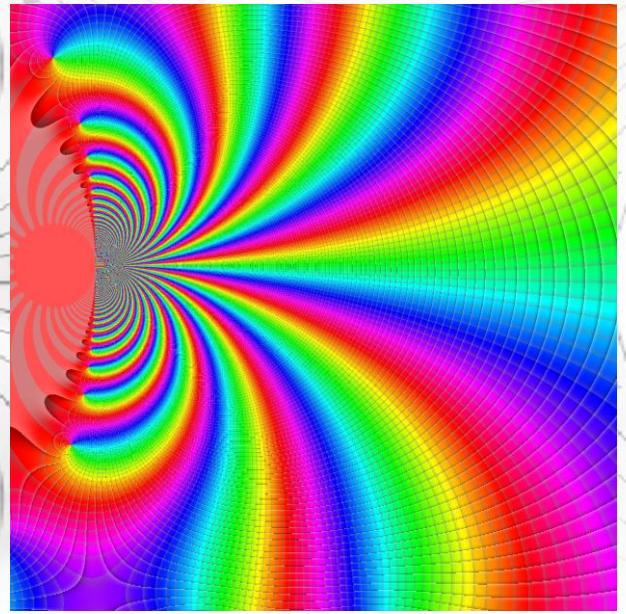


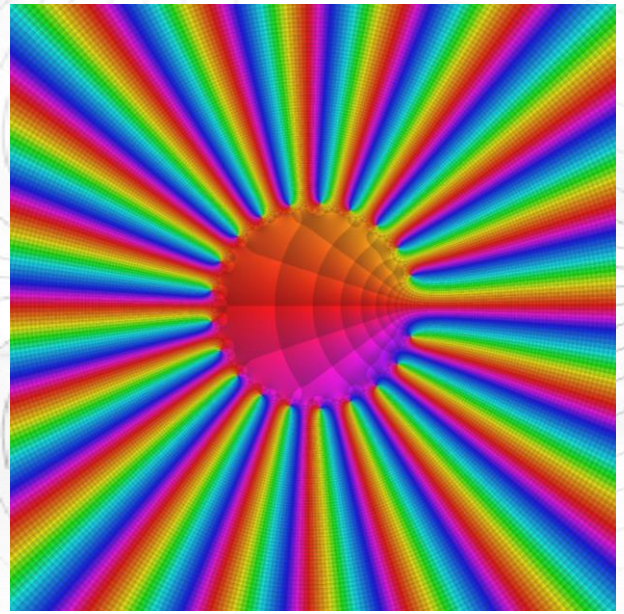
------- KOMPLEXNÉ FUNKCIE -------

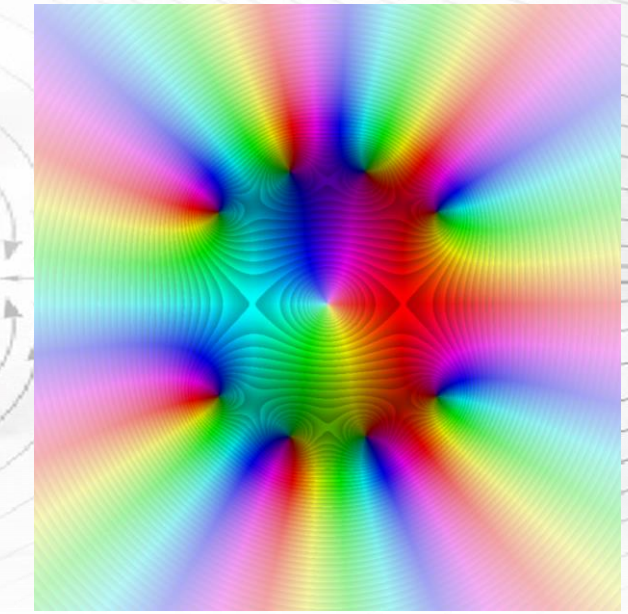




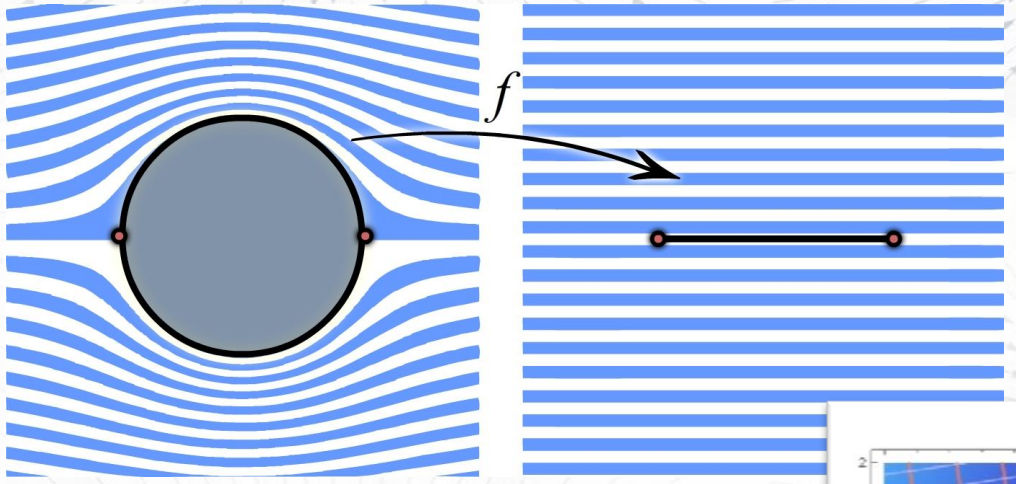






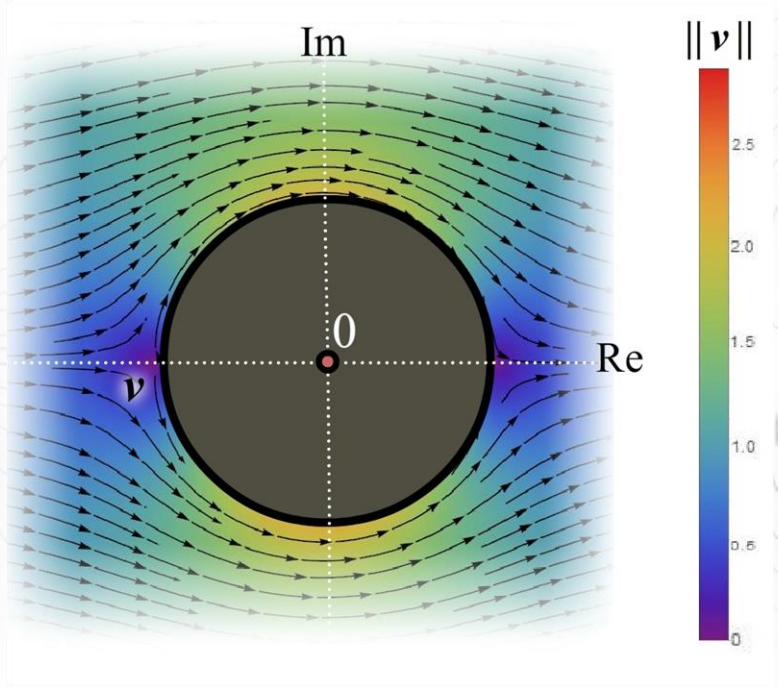


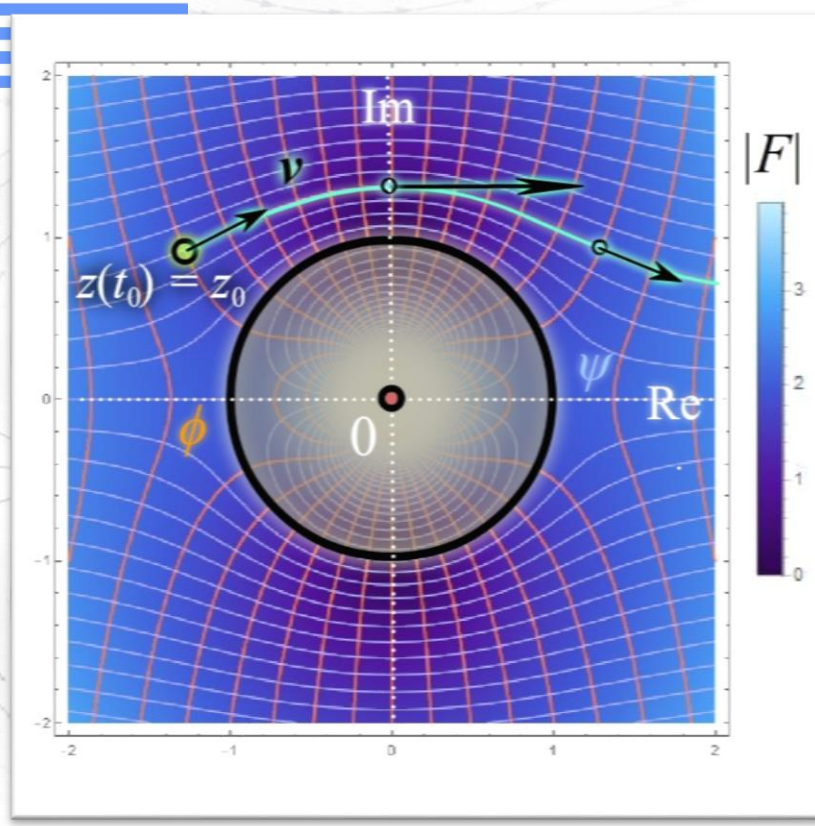
---- PRÚDENIE OKOLO NEKONEČNE DLHÉHO VALCA----

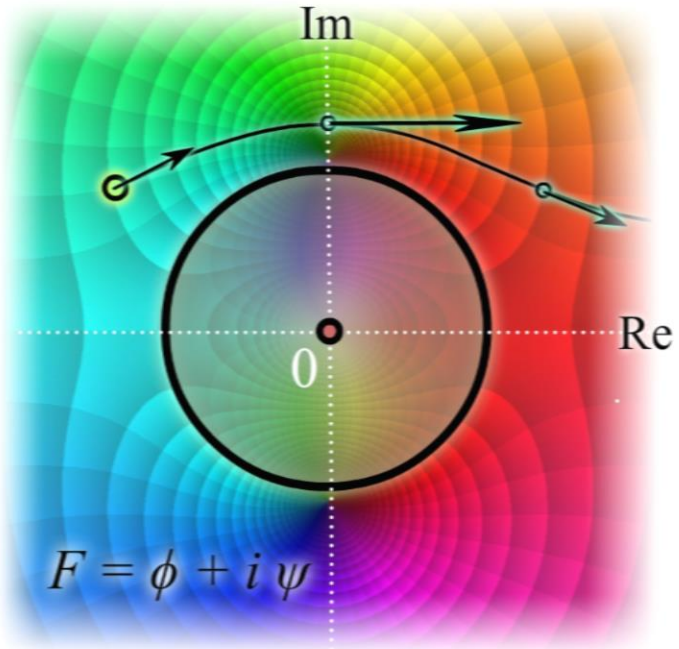


$$F(z) = \left(z + \frac{1}{z}\right)$$

$$F^{-1}(z) = \frac{1}{2}\left(z \pm \sqrt{z^2 - 4}\right)$$

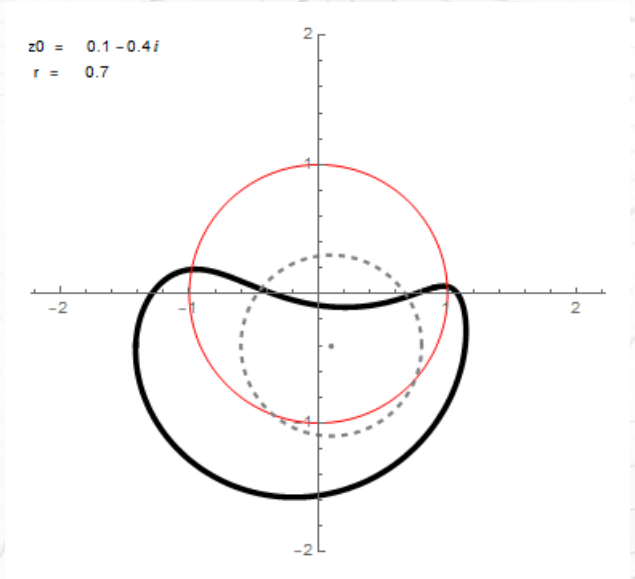


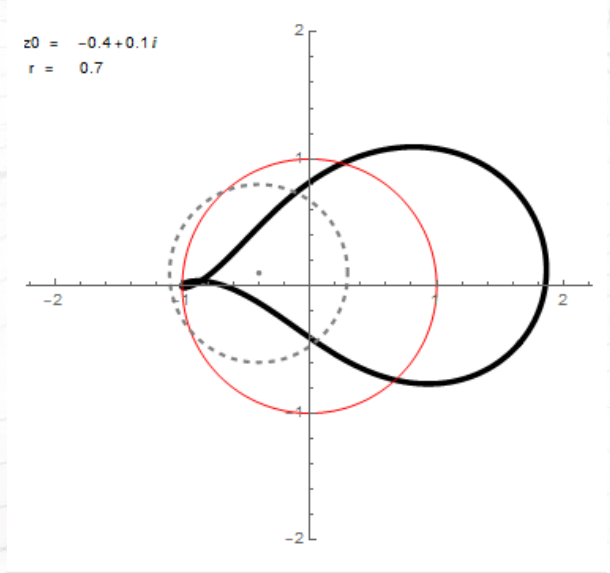


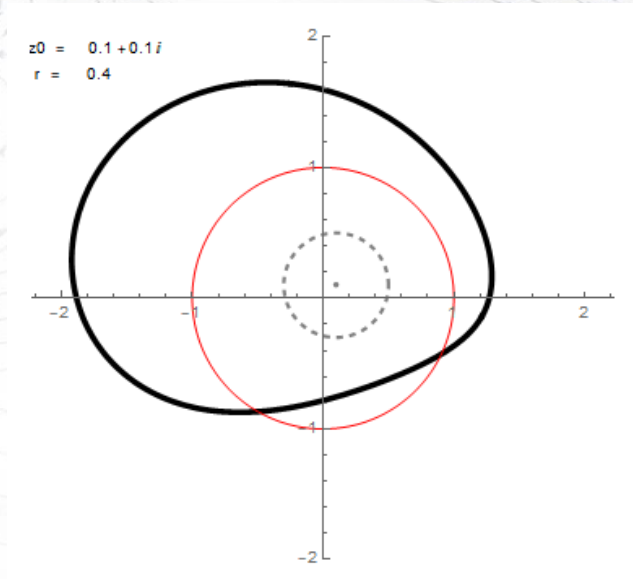


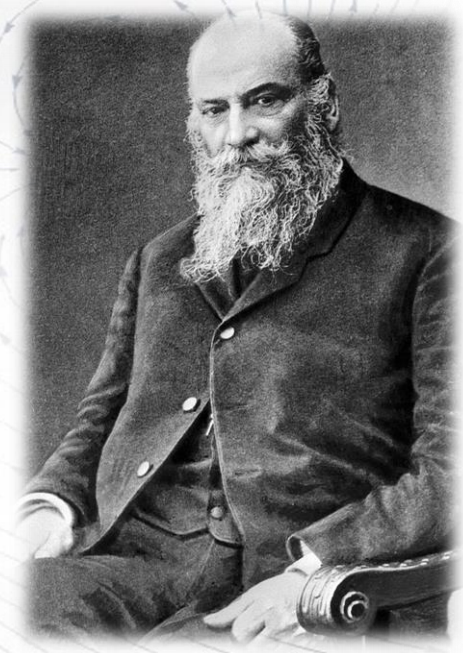
------- ŽUKOVSKÉHO PROFILY -------

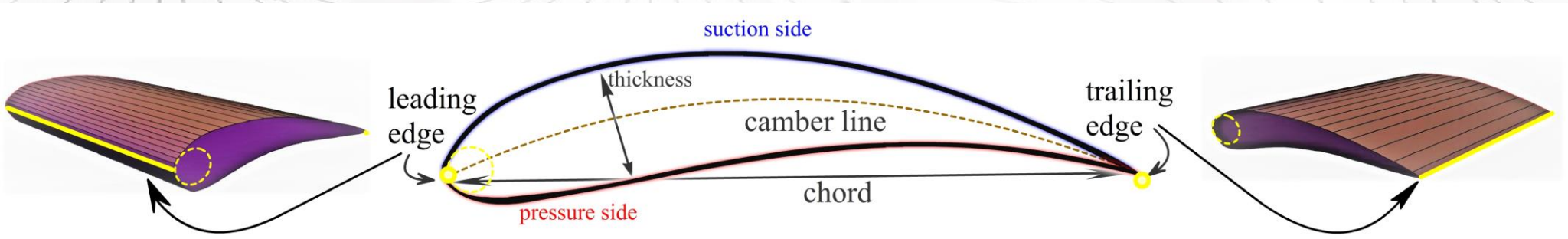
Zobrazujeme kružnice $\gamma(t) = z_0 + re^{it} \quad t \in [0, 2\pi]$ na krivky $\mathcal{A} = F \circ \gamma$



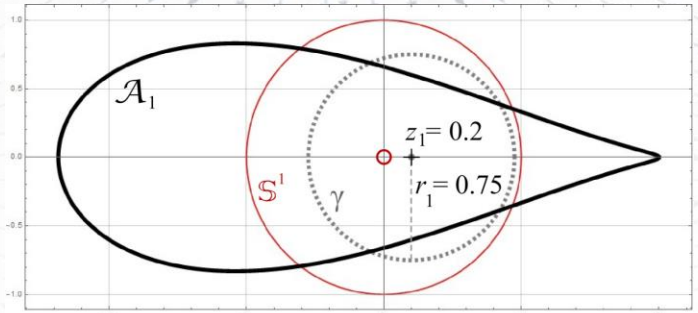


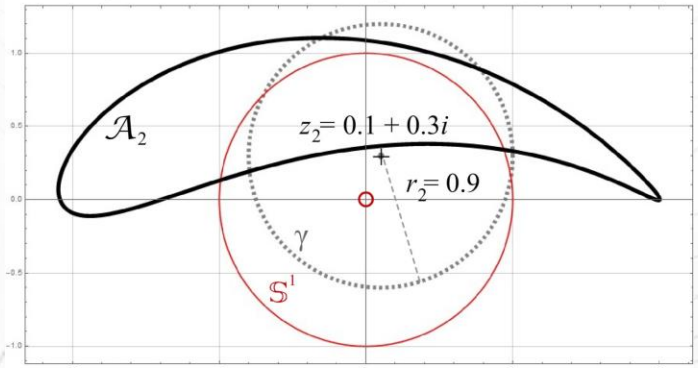


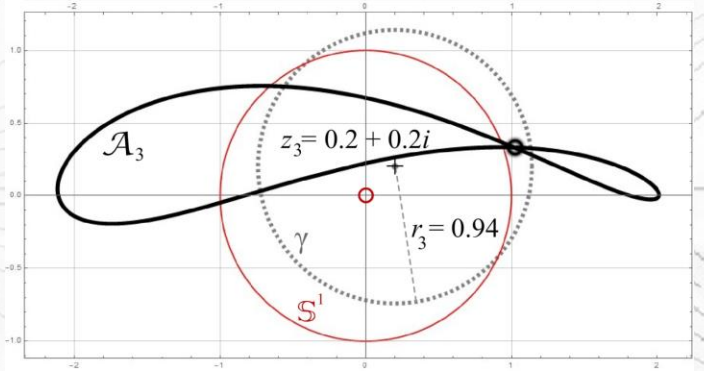


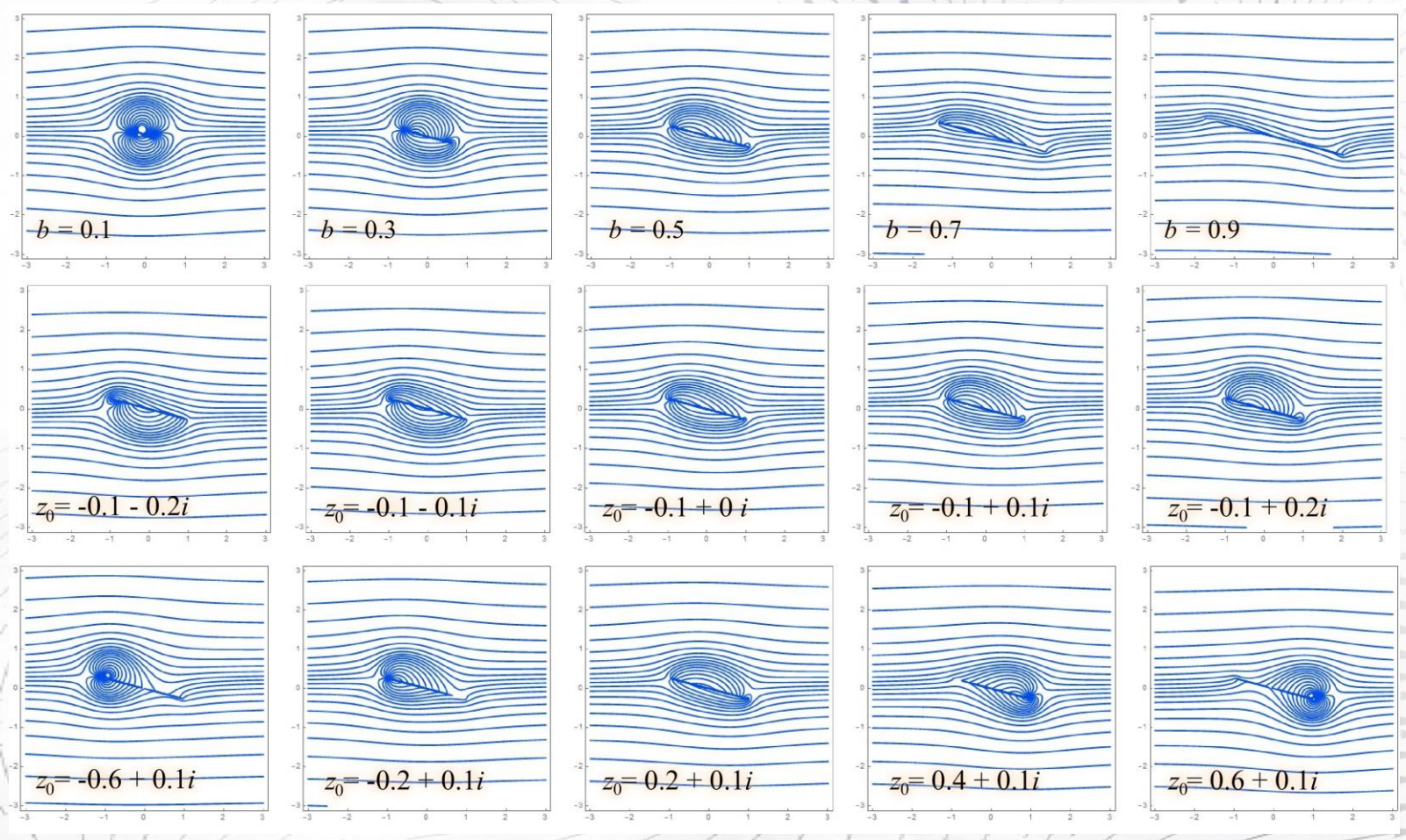


------- ŽUKOVSKÉHO PROFILY -------

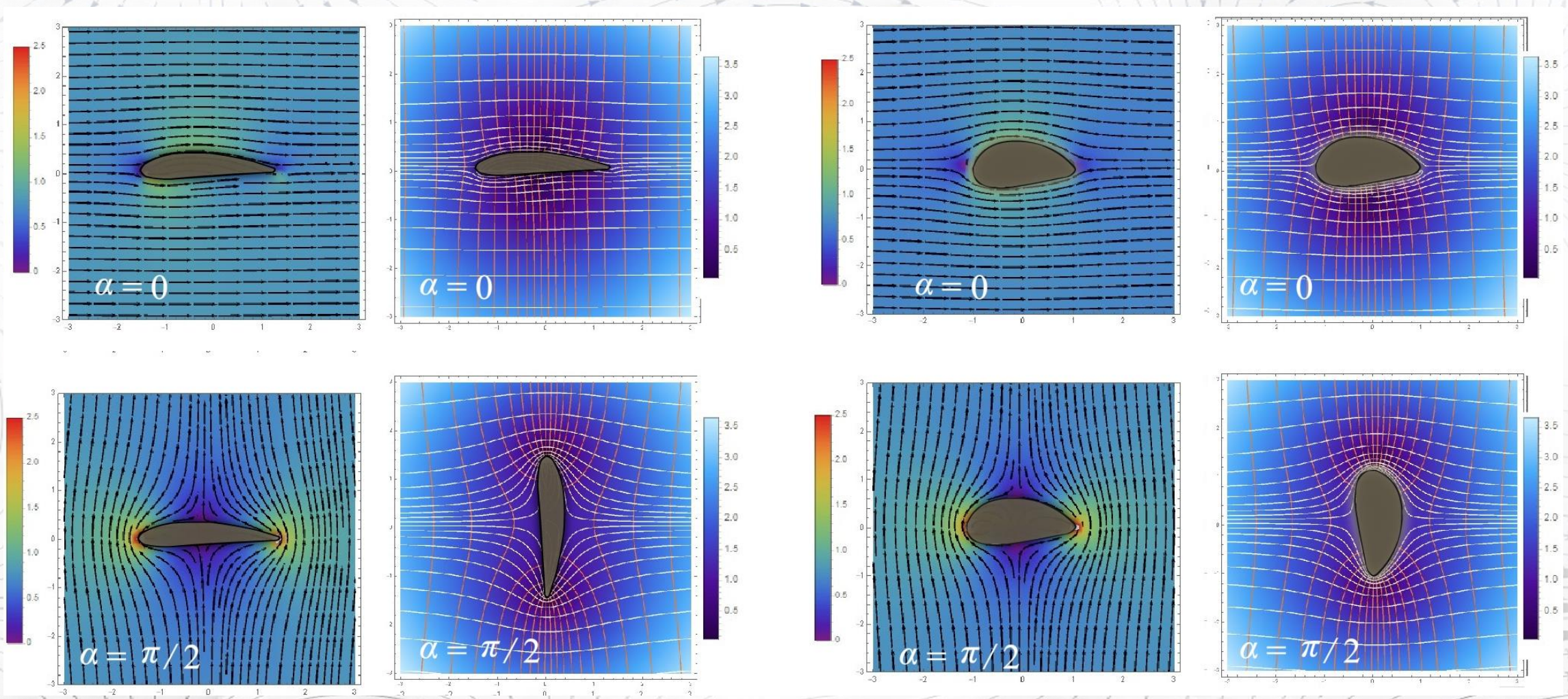




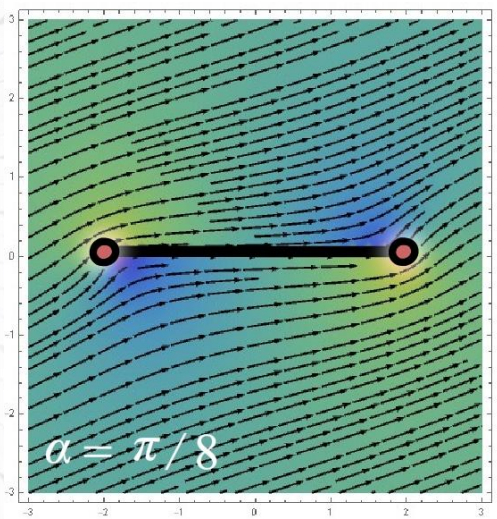


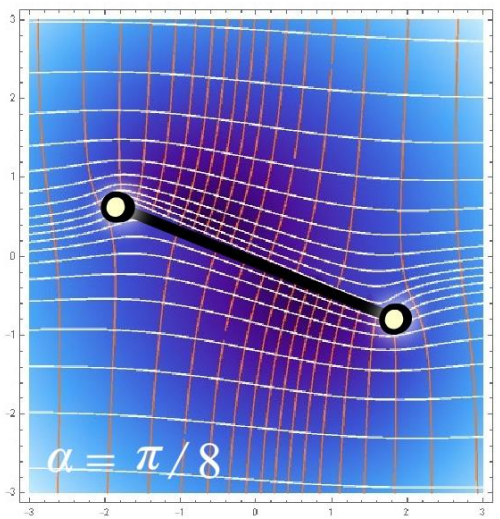


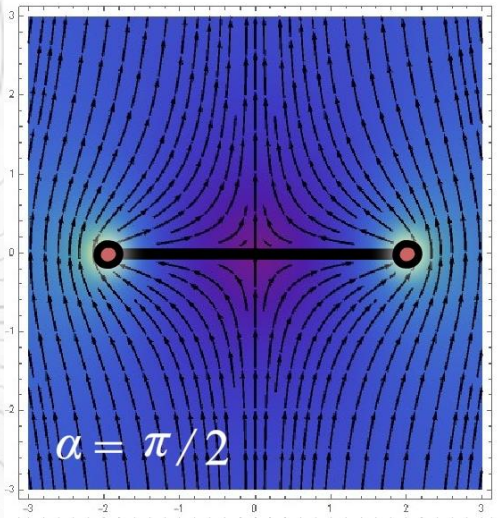
------- ŽUKOVSKÉHO PROFILY -------

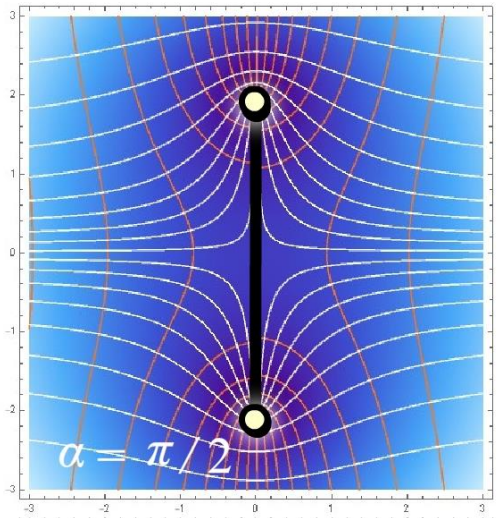


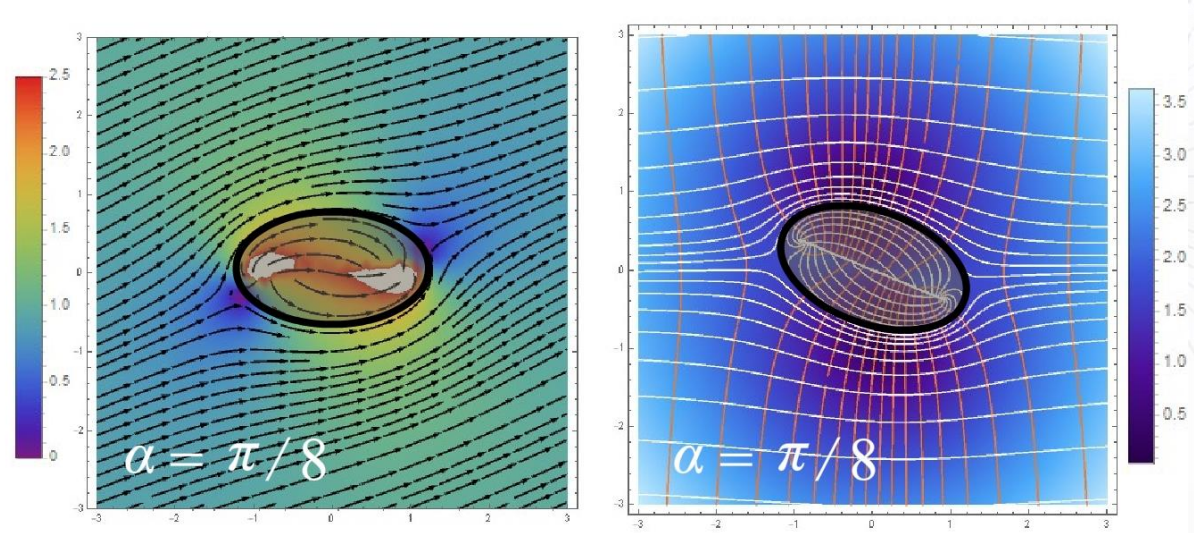
------- OVÁL A DOSKA -------

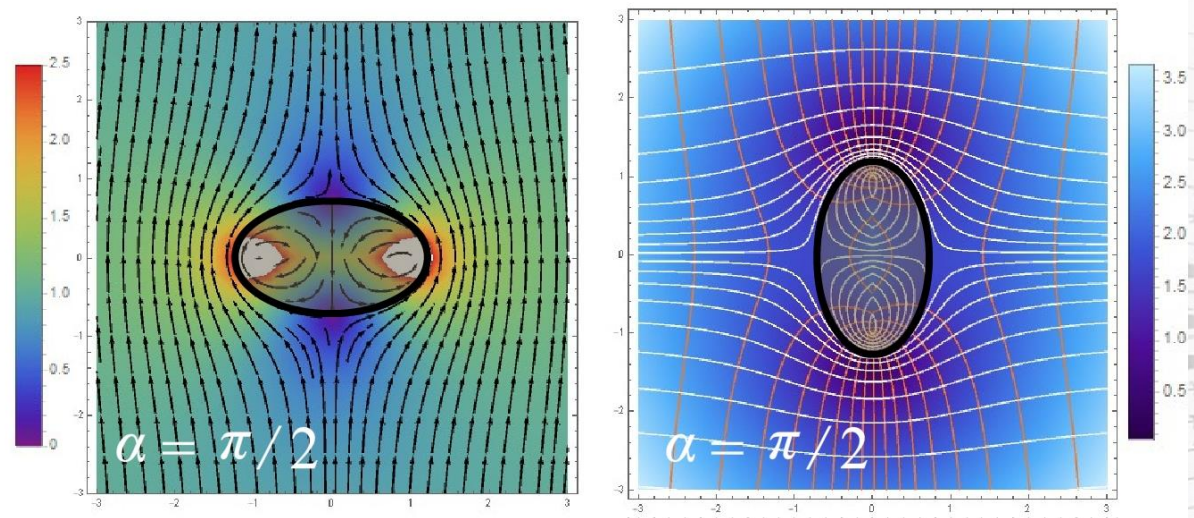




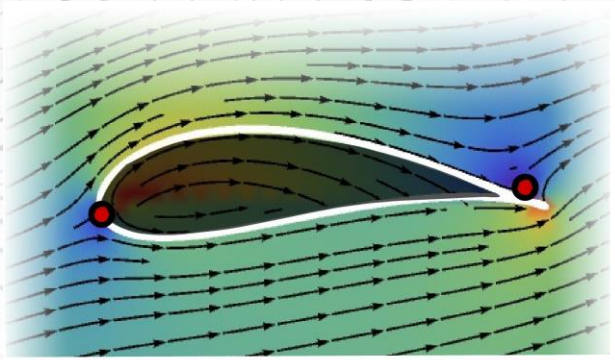


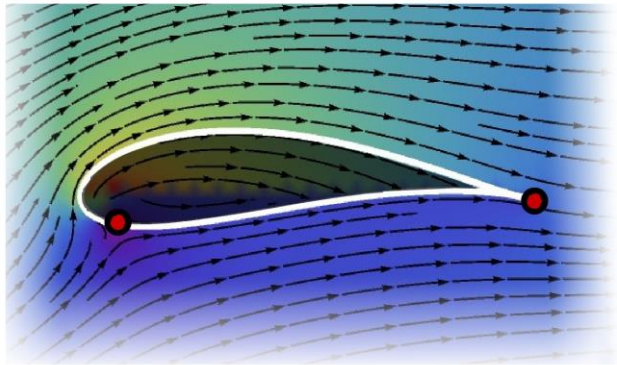


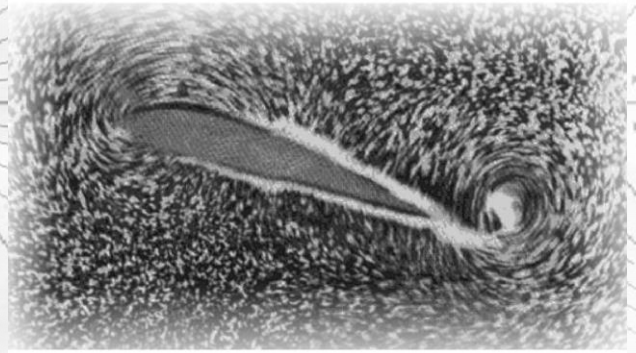




------- PREČO JE KRÍDLO NADNÁŠANÉ? -------

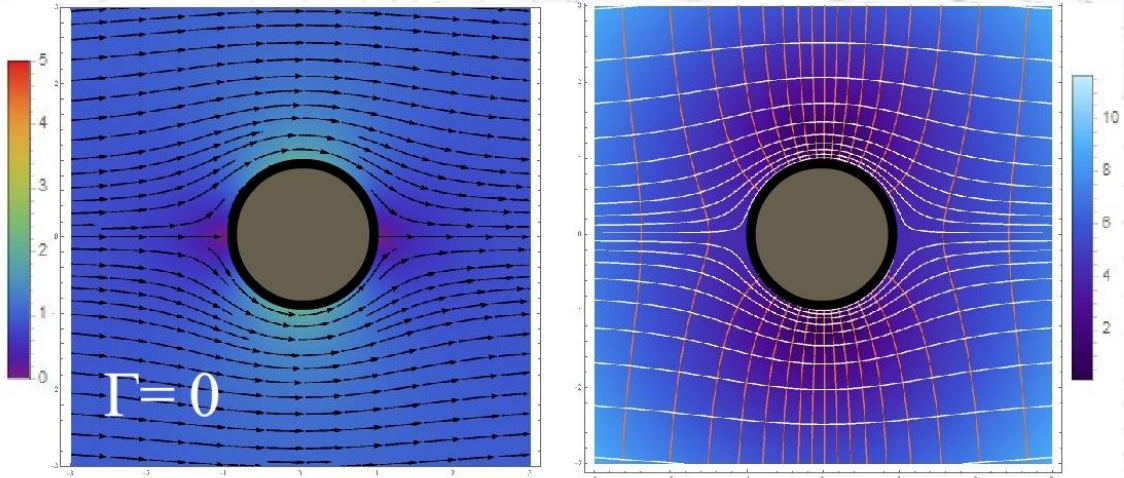


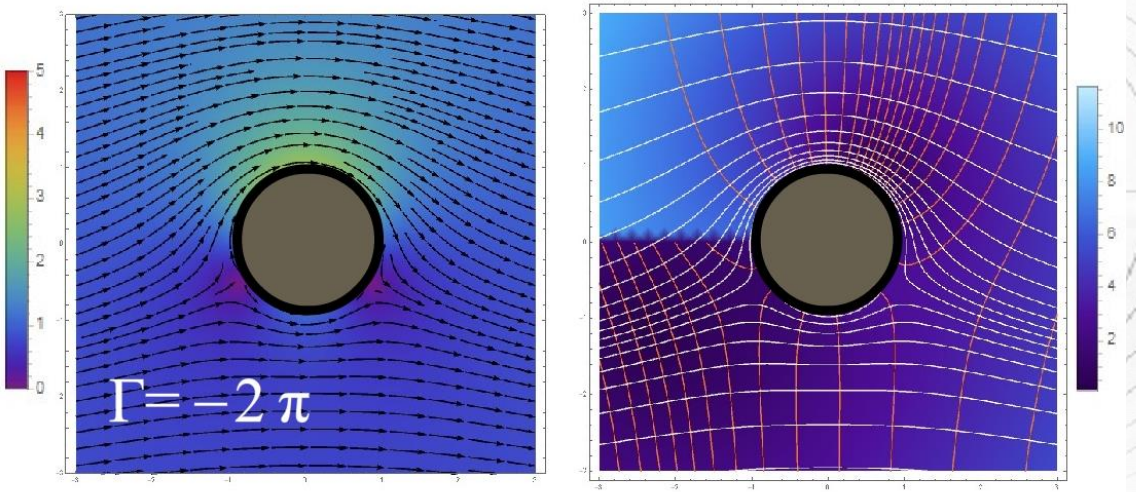




$$F(z) = v_\infty \left(z + \frac{a^2}{z} \right) - i \frac{\Gamma}{2\pi} \log(z)$$

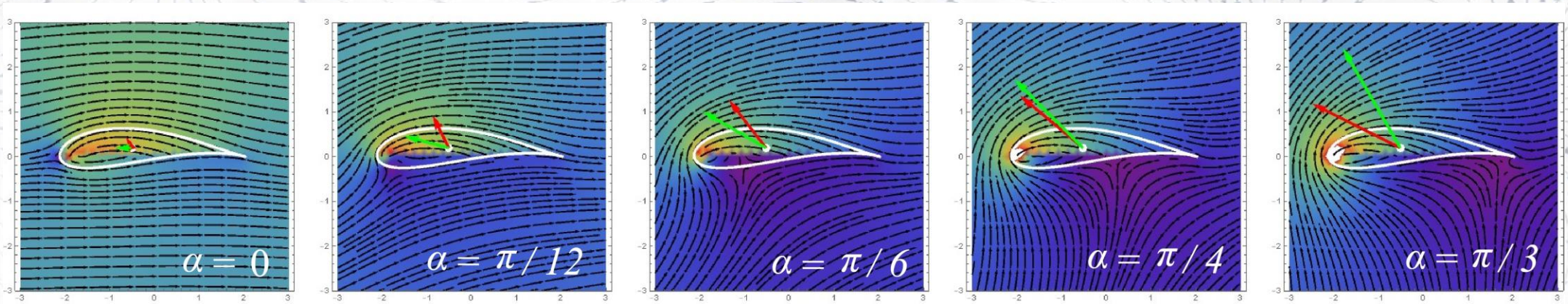
Γ - cirkulácia





- - - - - - - P R E Č O J E K R Í D L O N A D N Á Š A N É ? - - - - - - -

Porovnanie zdvihových síl (*Blasius-Chaplygin vs. Kutta-Žukovského formula*):



$F_1 = -1.892$
$F_2 = -0.629+1.128i$

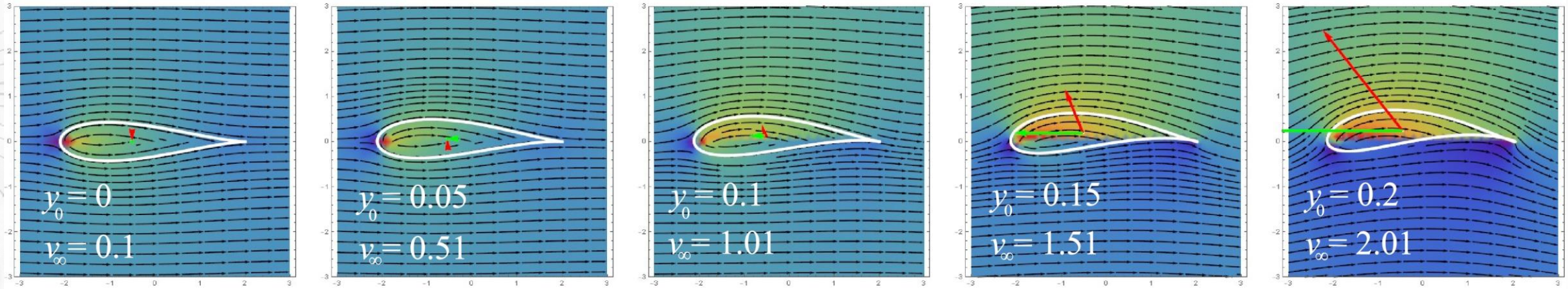
$F_1 = -5.004+1.361i$
$F_2 = -1.844 + 3.708i$

$F_1 = -7.114+4.123i$
$F_2 = -4.185 + 5.392i$

$F_1 = -7.691+7.691i$
$F_2 = -7.103 + 5.865i$

$F_1 = -6.448+11.169i$
$F_2 = -9.884+4.967i$

Vzlet lietadla:



Ďakujem za pozornosť