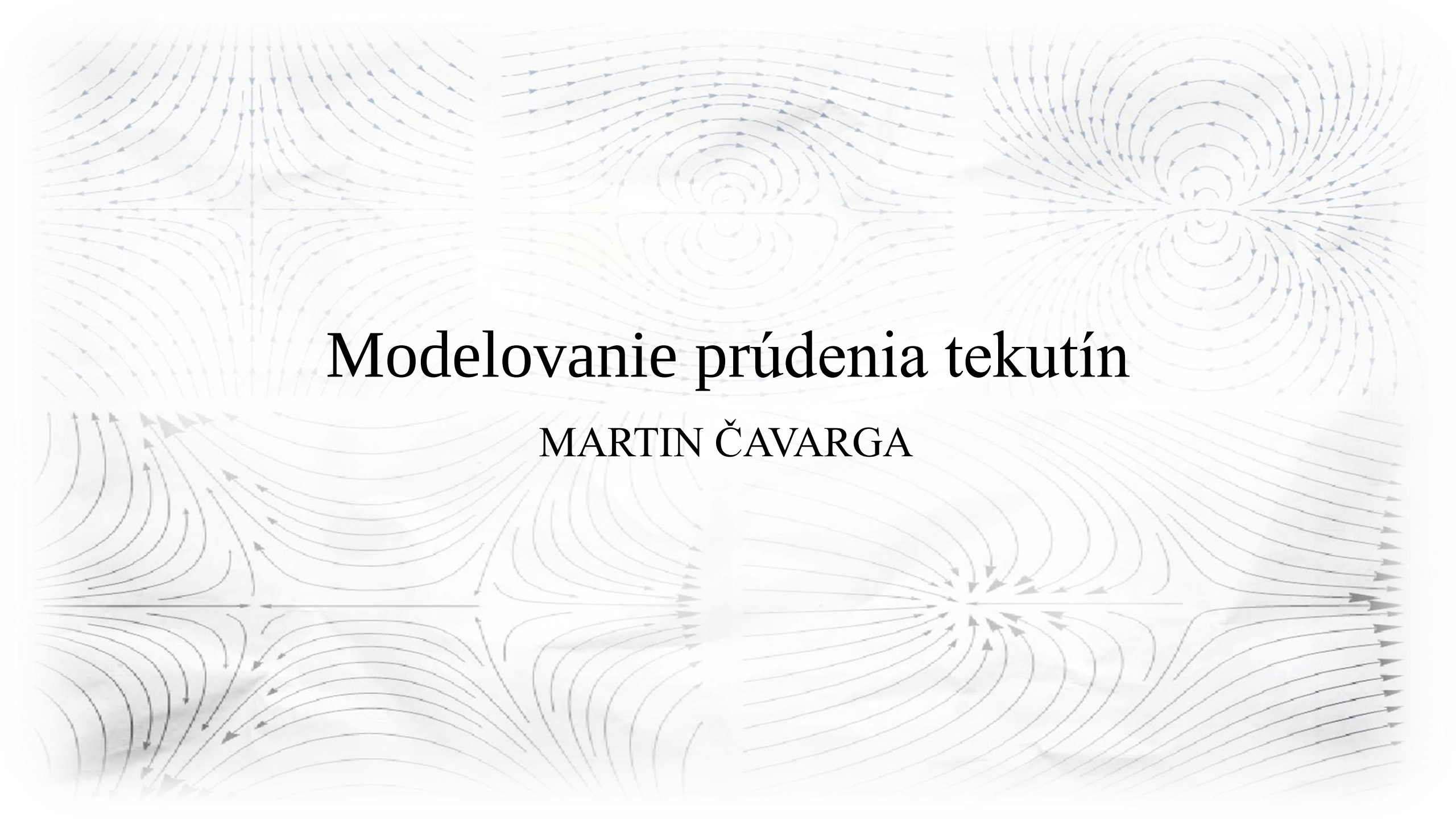


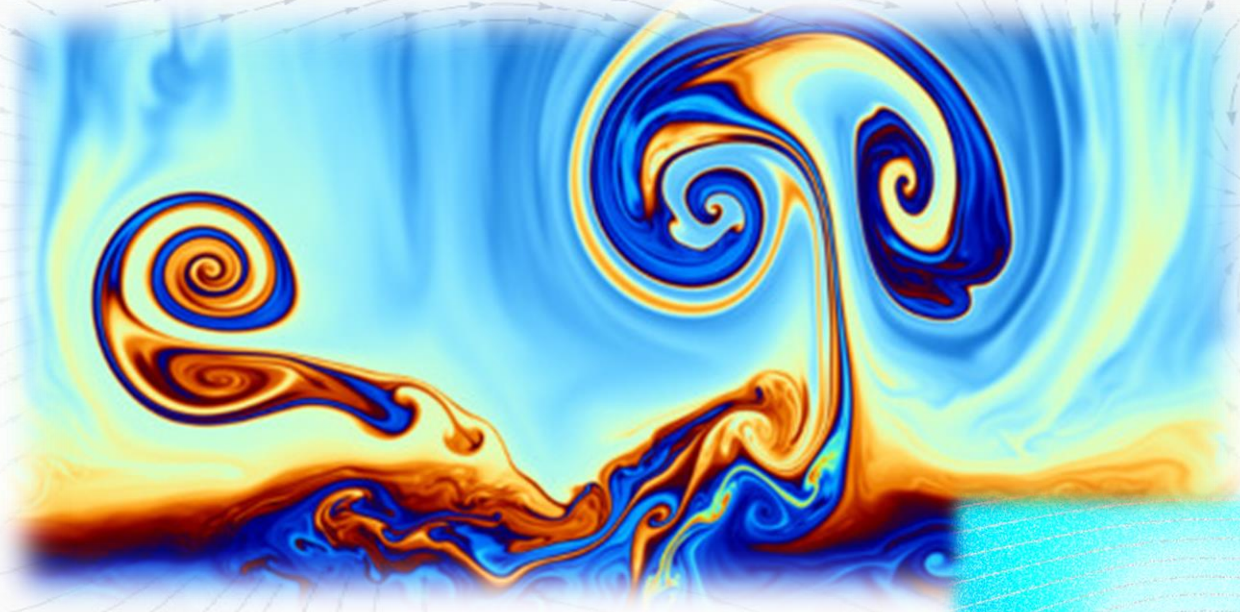
The background of the slide is a light gray field filled with numerous thin, dark gray streamlines. These lines represent the flow of a fluid, with arrows indicating the direction of flow. The streamlines are arranged in various patterns: some are straight and parallel, others are curved and concentric, and some form more complex, swirling patterns. The overall effect is a dense, textured background that visually represents the concept of fluid dynamics.

Modelovanie prúdenia tekutín

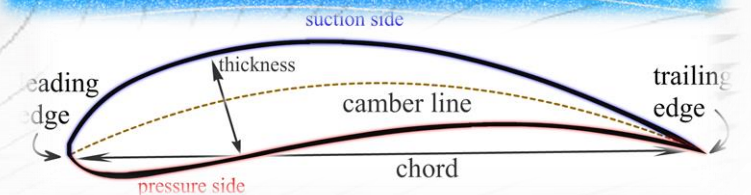
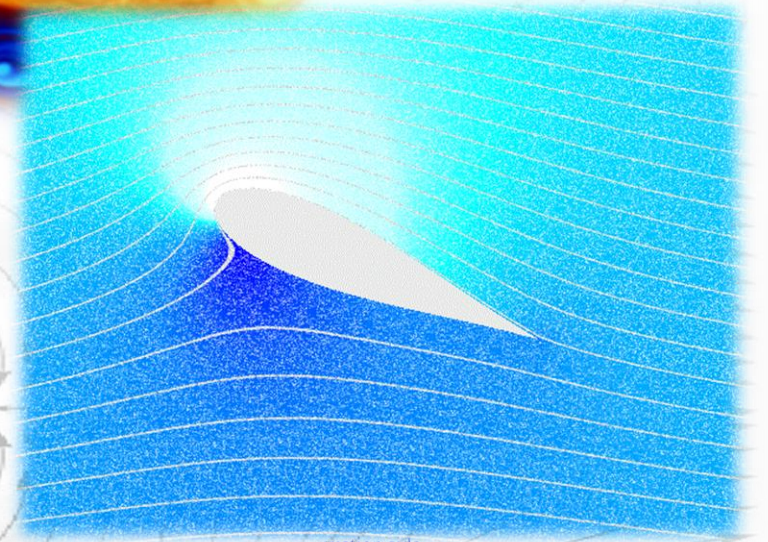
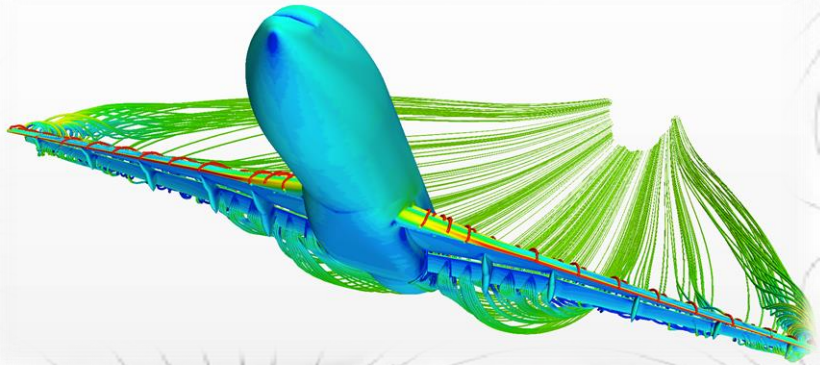
MARTIN ČAVARGA

----- MODELOVANIE PRÚDENIA -----

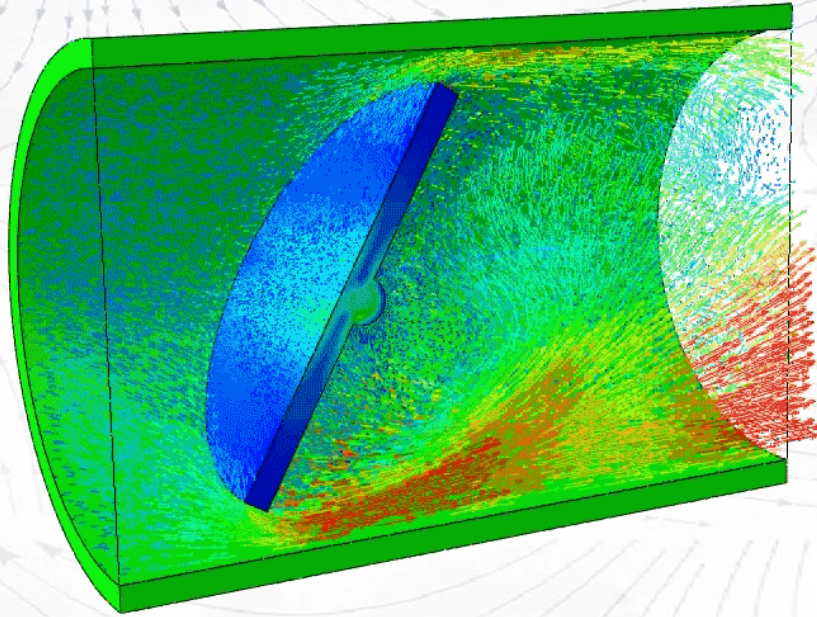
- Čo sú to tekutiny a aké majú vlastnosti?
- Čo znamená modelovať prúdenie?



- Aplikácie
- Zjednodušený model prúdenia a dizajn profilov krídiel



----- ČO ZNAMENÁ MODELOVAŤ PRÚDENIE? -----



- Veličiny: $p, \rho, \mathbf{v} = (v_1, v_2, v_3), T, u$
- Zachovanie hmotnosti
- Zachovanie hybnosti (Newtonov 2. p. zákon)
- Vnútorné trenie a vedenie tepla

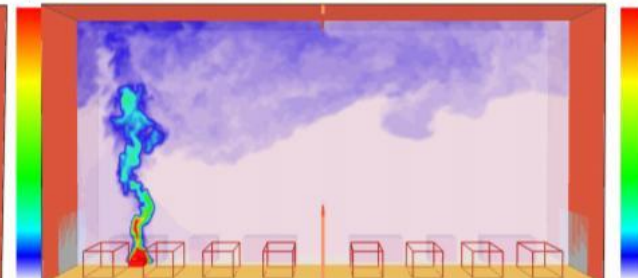
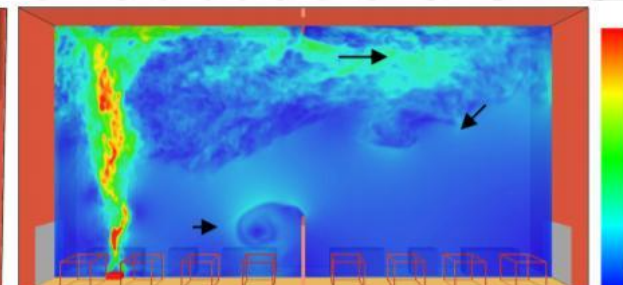
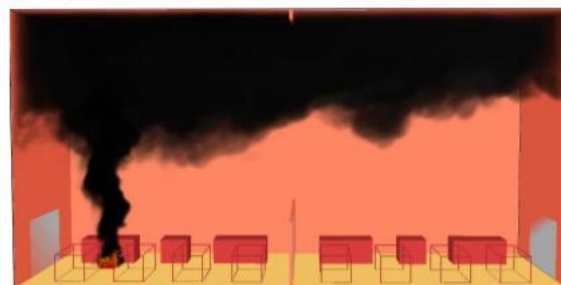
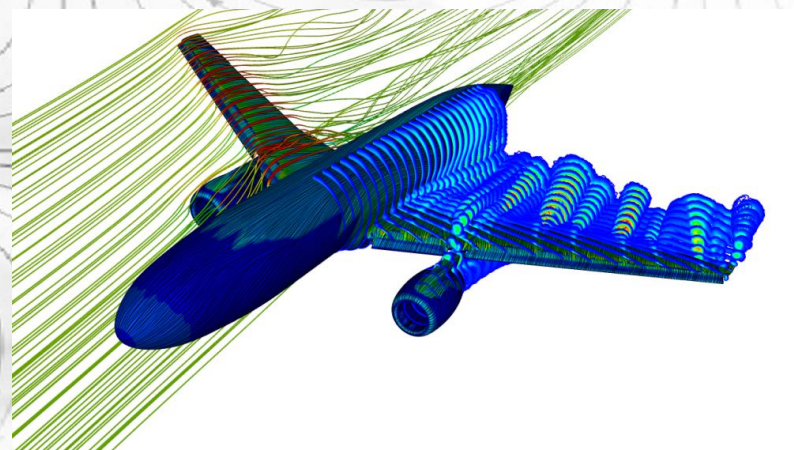
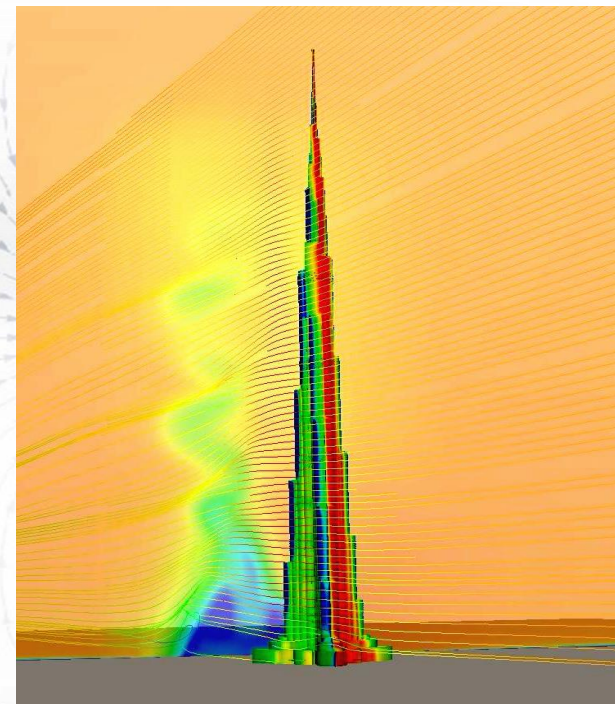
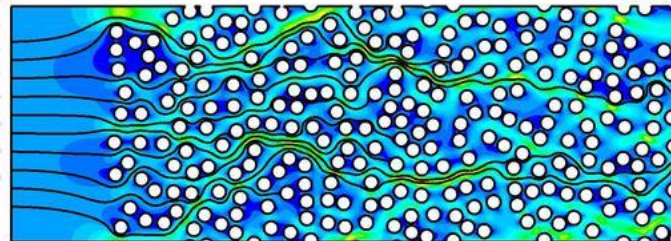
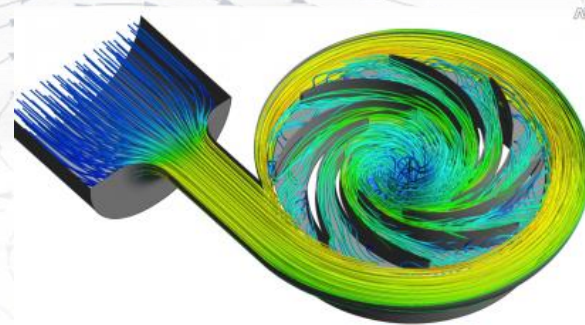
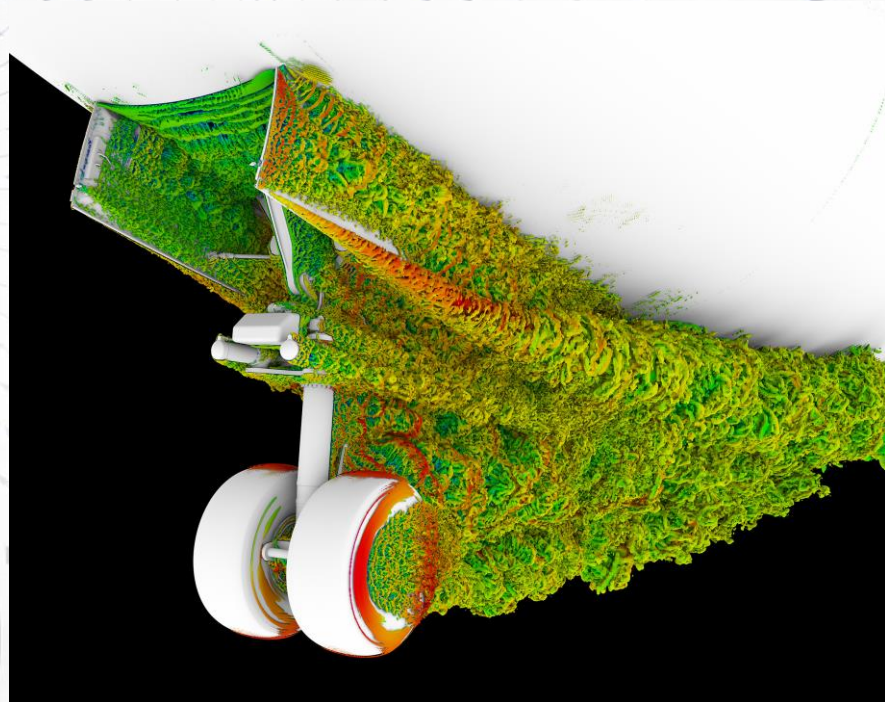
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$-\frac{1}{\rho}p + \mathbf{f} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}$$

$$\begin{aligned} -\nabla p + \rho \mathbf{f} + \nabla \cdot (\lambda \nabla \cdot \mathbf{v} \mathbf{I}) + \nabla \cdot (\mu (\nabla \mathbf{v} + \nabla \mathbf{v}^\top)) &= \\ &= \frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) \end{aligned}$$

$$\begin{aligned} \rho \mathbf{f} \cdot \mathbf{v} + \nabla \cdot \left(\frac{1}{2} (\nabla \mathbf{v} + \nabla \mathbf{v}^\top) \otimes \mathbf{v} \right) + \rho (-k \nabla T) - \nabla \cdot (-k \nabla T) &= \\ &= \frac{\partial}{\partial t} (\rho E) + \nabla \cdot (\rho E \mathbf{v}) \end{aligned}$$

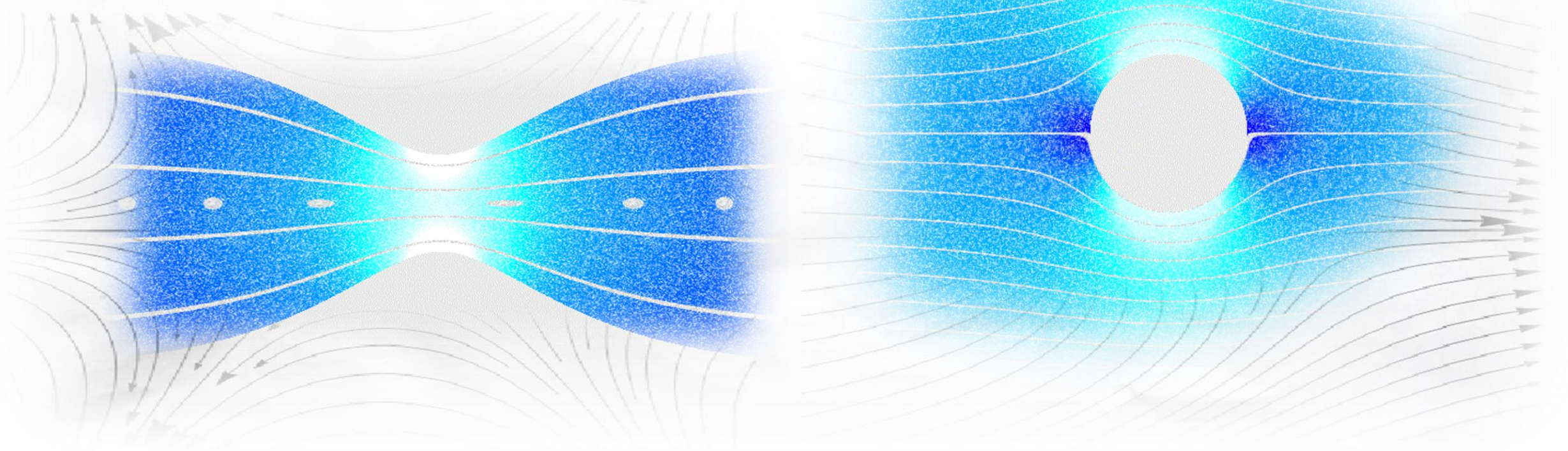
----- APLIKÁCIE -----



----- IDEÁLNE PRÚDENIE -----

Budeme pracovať s prúdením, ktoré spĺňa nasledujúce podmienky:

1. nulová viskozita (vnútorné trenie)
2. nestlačiteľnosť
3. nevírovosť
4. ustálený stav



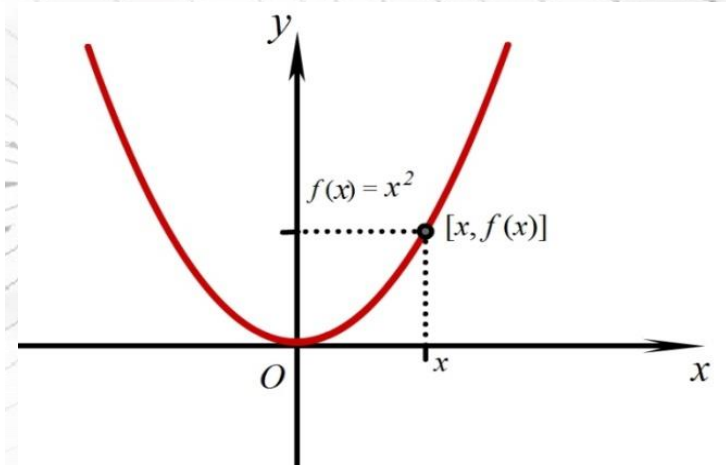
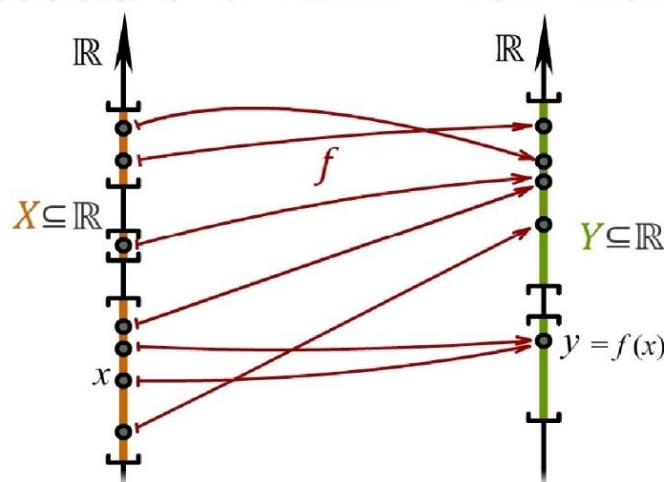
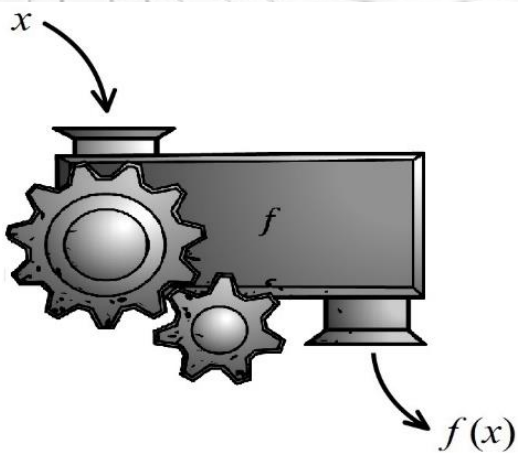
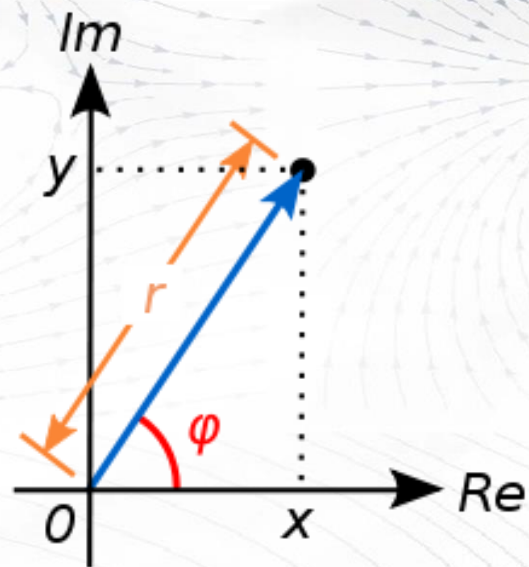
----- K O M P L E X N É Č Í S L A -----

- Niektoré rovnice nemajú riešenie medzi reálnymi číslami $x^2 + 1 = 0$

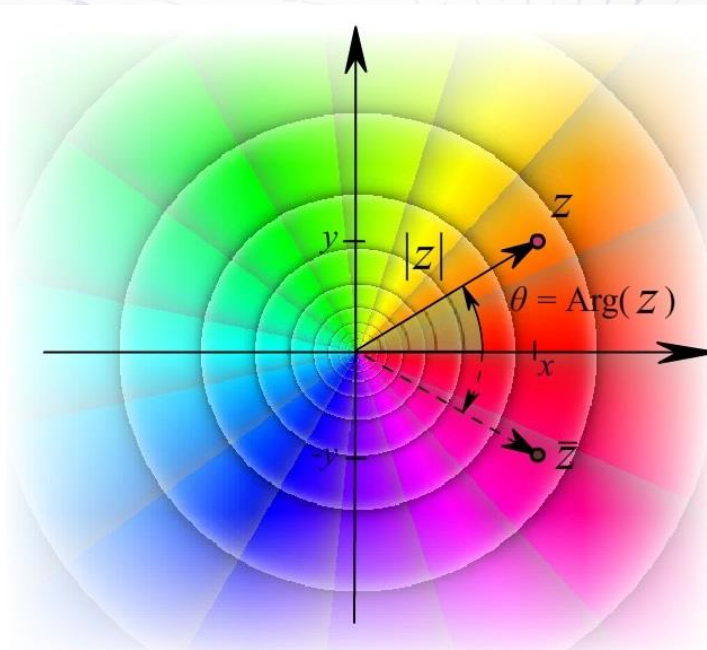
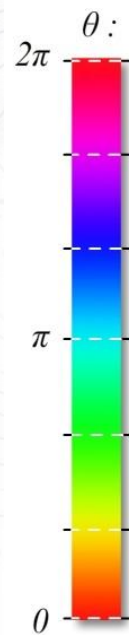
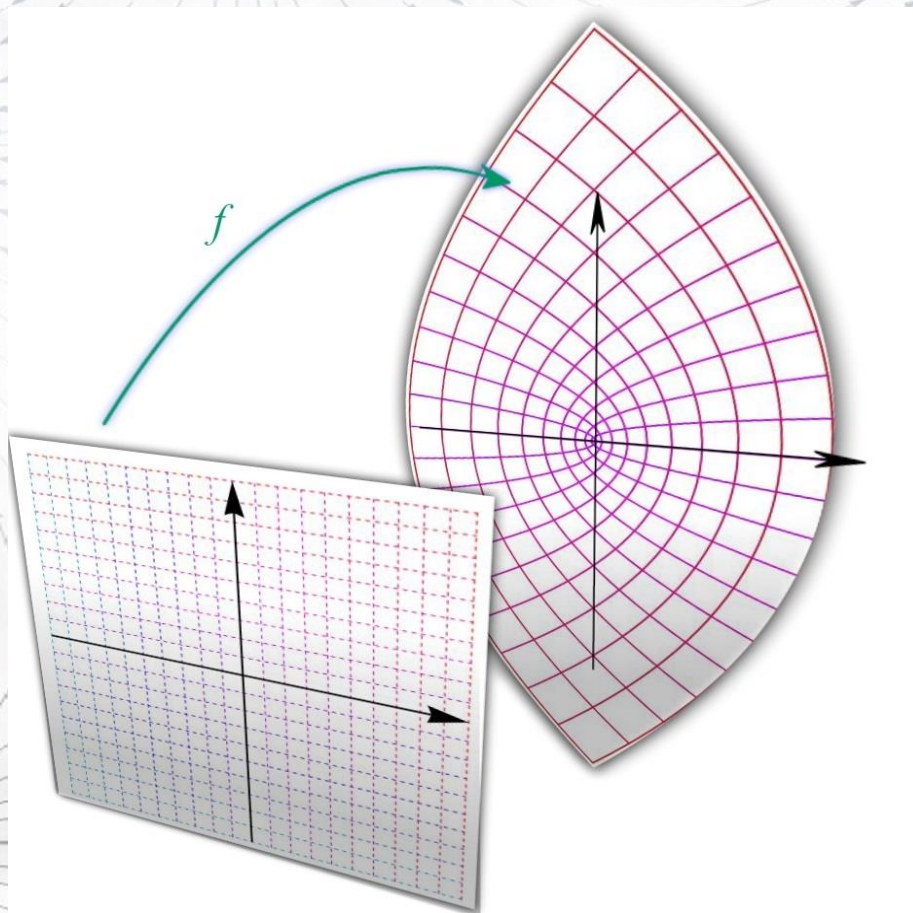
$$x^3 + ax^2 + bx + c = 0$$

- uvádzame $i = \sqrt{-1}$

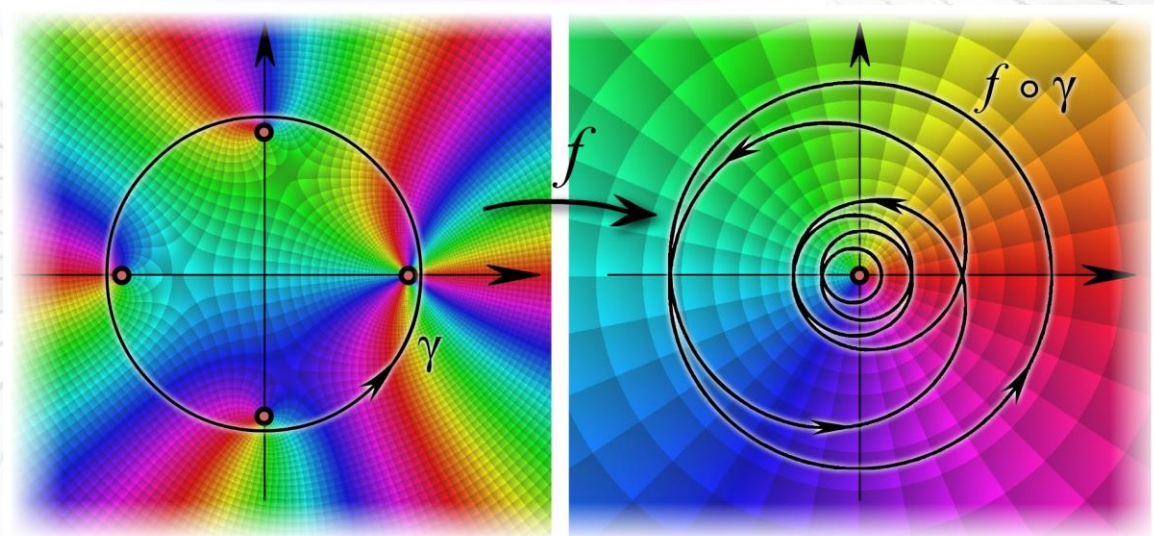
- všeobecne $z = x + iy$



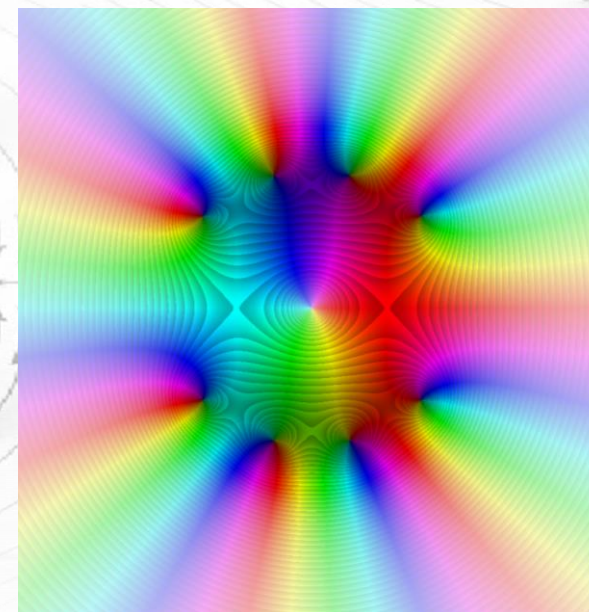
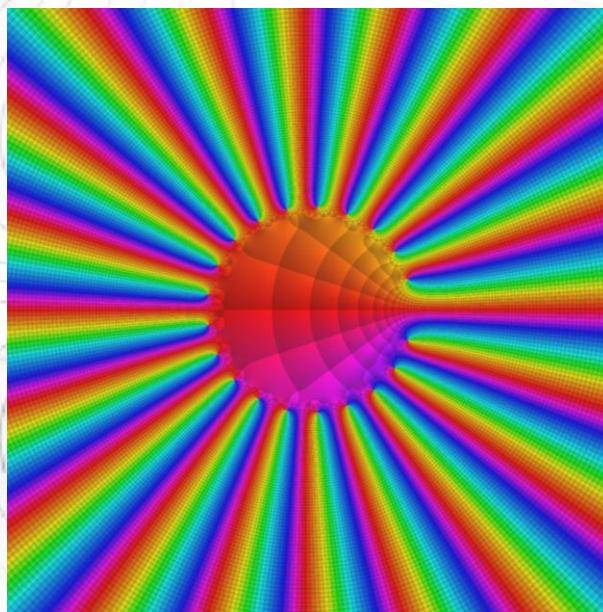
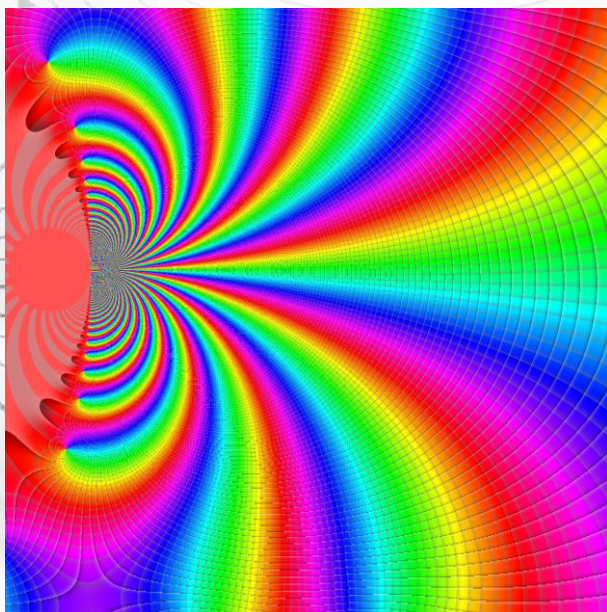
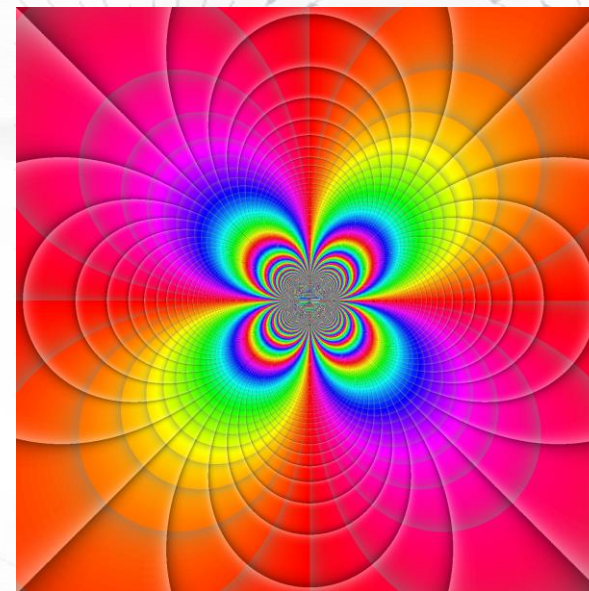
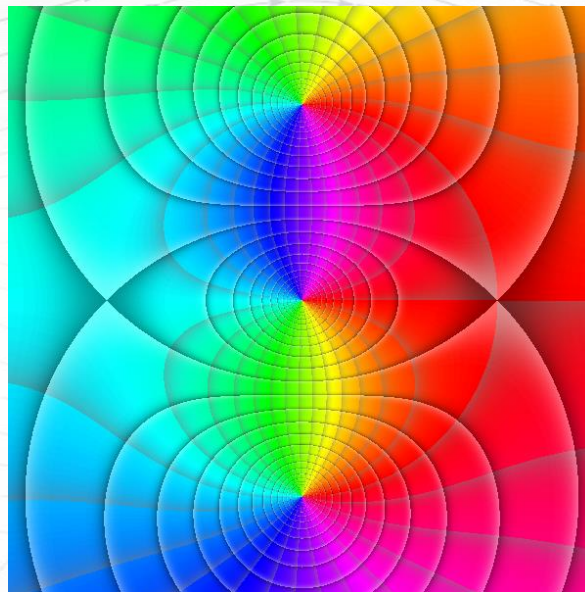
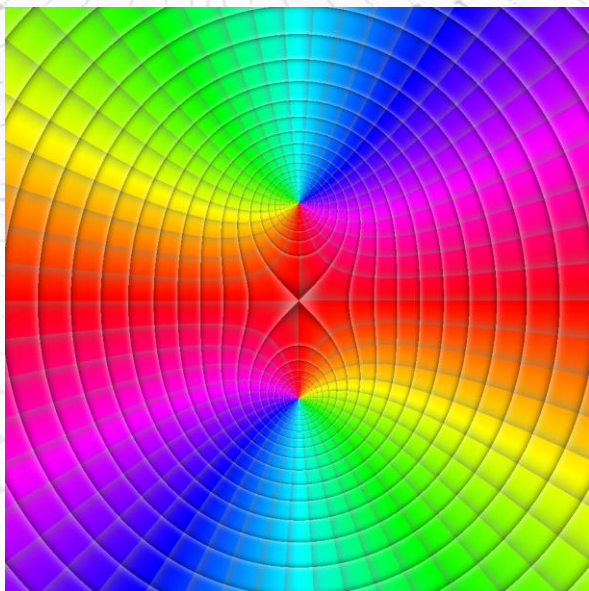
----- K O M P L E X N É F U N K C I E -----



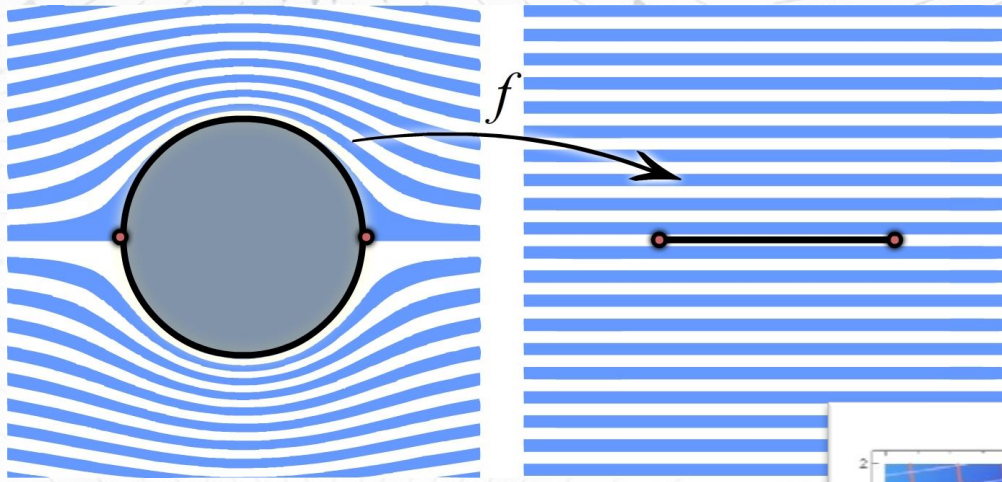
$$f(z) = z^2$$



----- KOMPLEXNÉ FUNKCIE -----

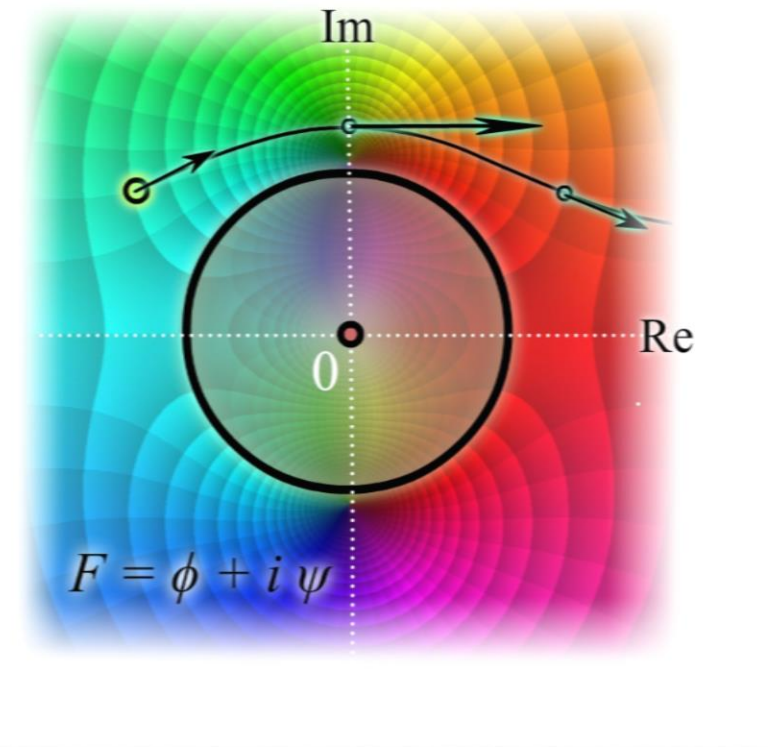
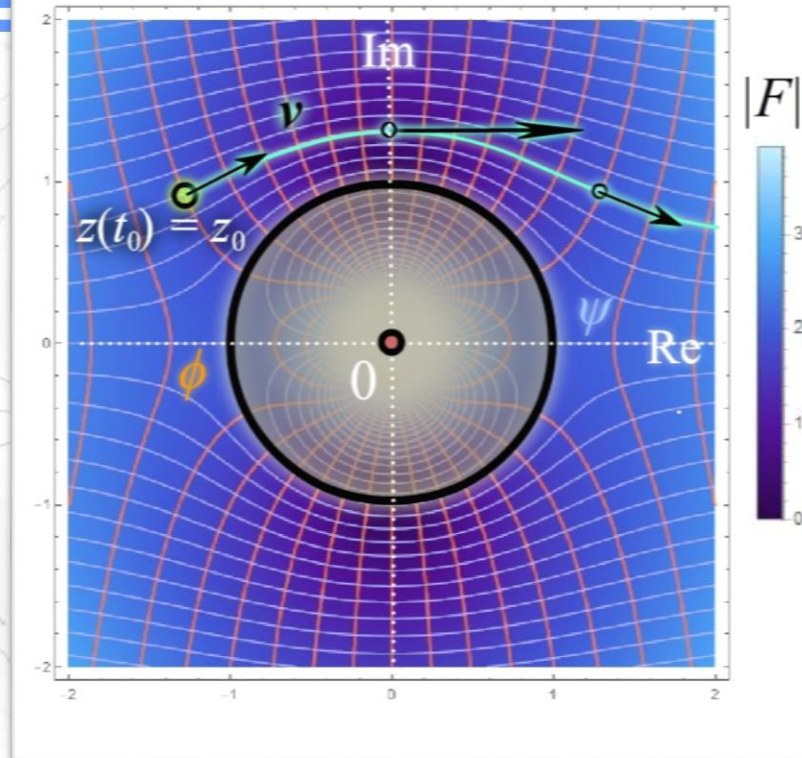
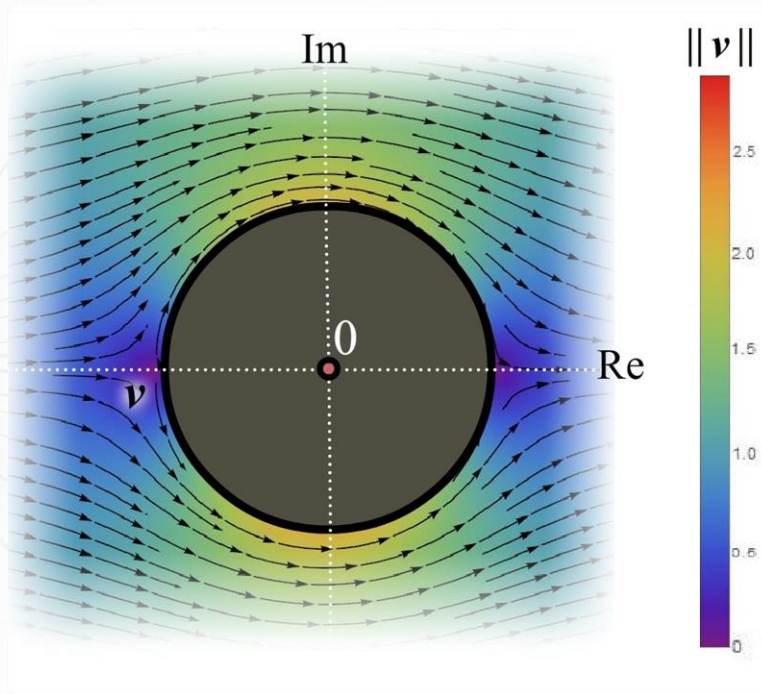


---- PRÚDENIE OKOLO NEKONEČNE DLHÉHO VALCA ----



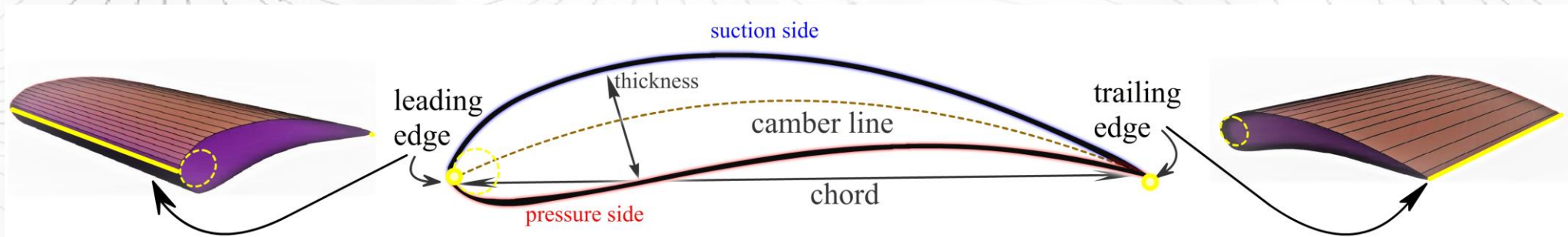
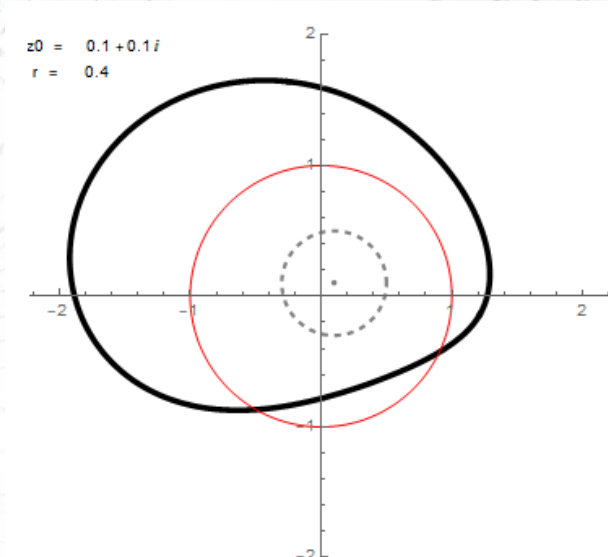
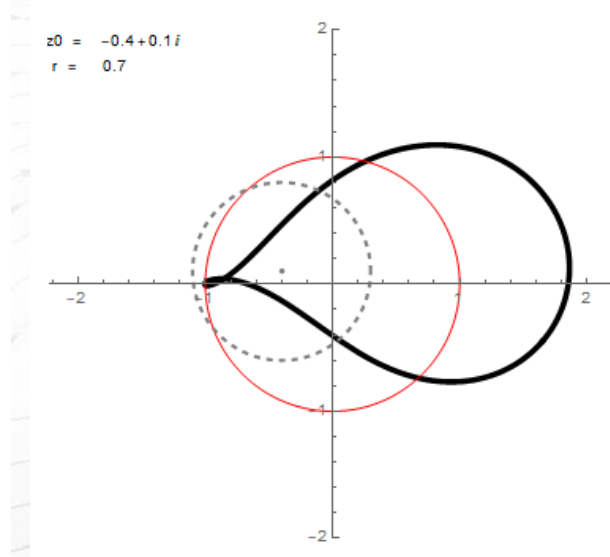
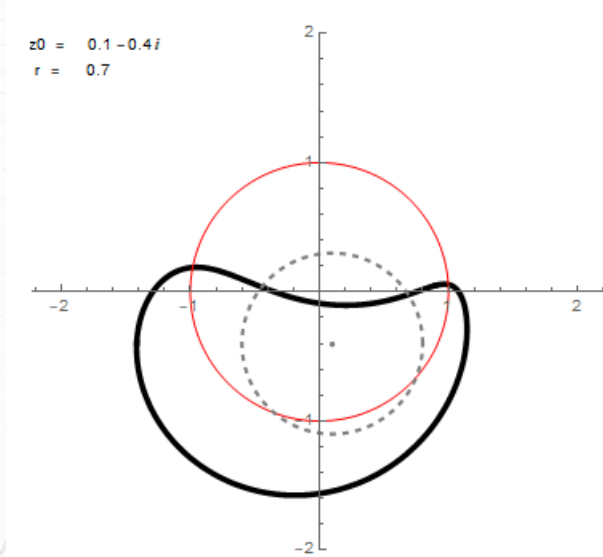
$$F(z) = \left(z + \frac{1}{z} \right)$$

$$F^{-1}(z) = \frac{1}{2} (z \pm \sqrt{z^2 - 4})$$

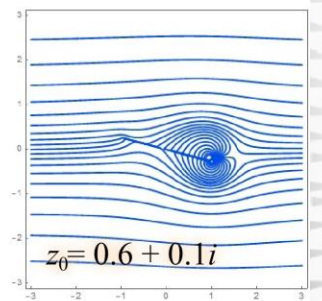
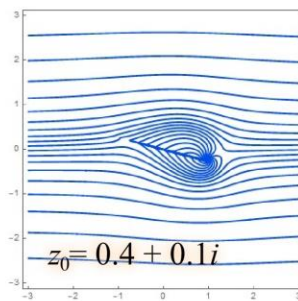
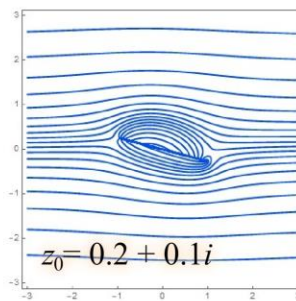
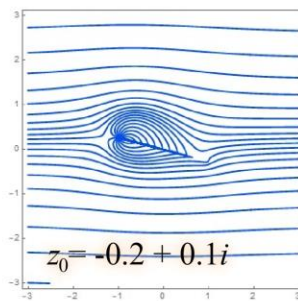
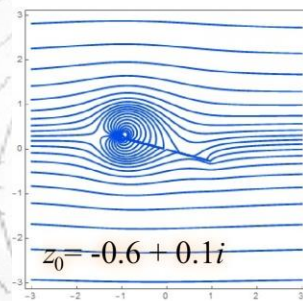
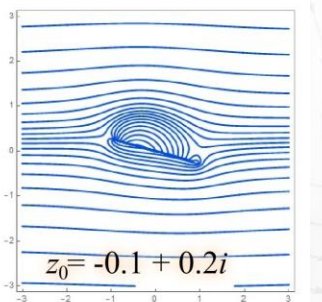
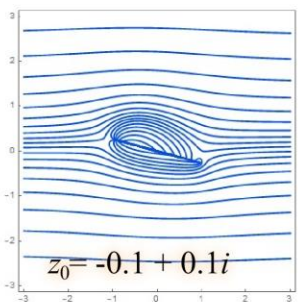
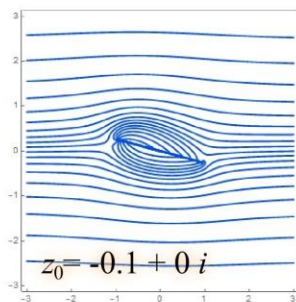
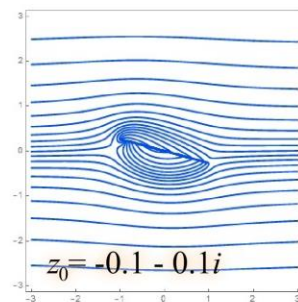
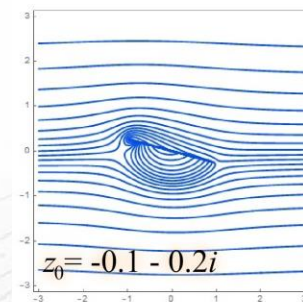
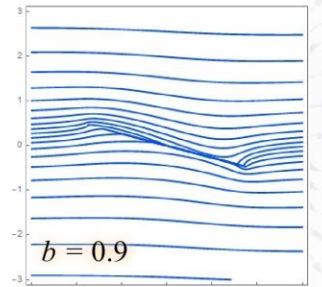
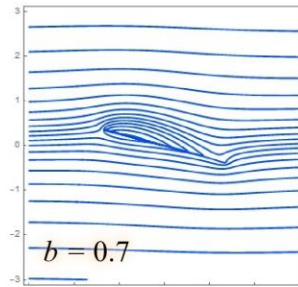
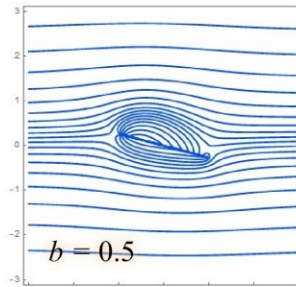
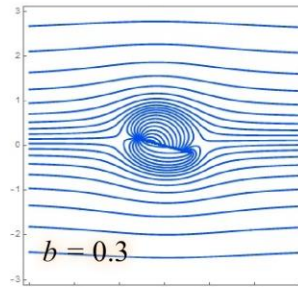
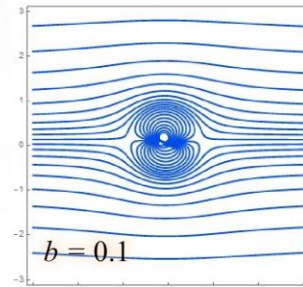
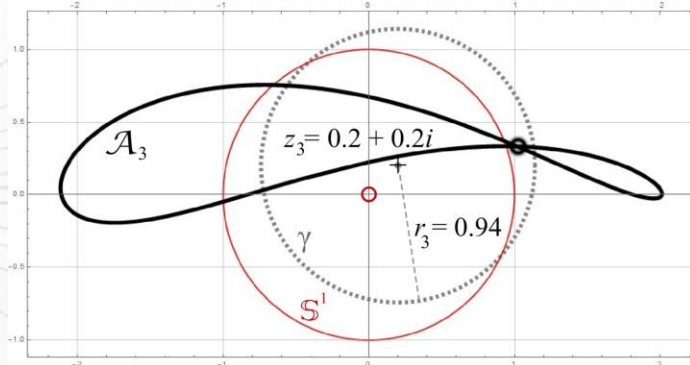
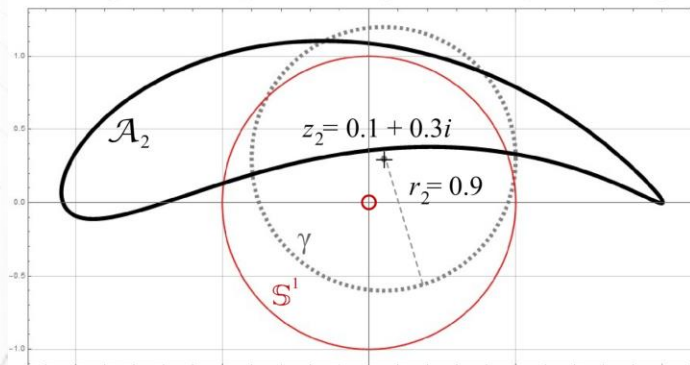
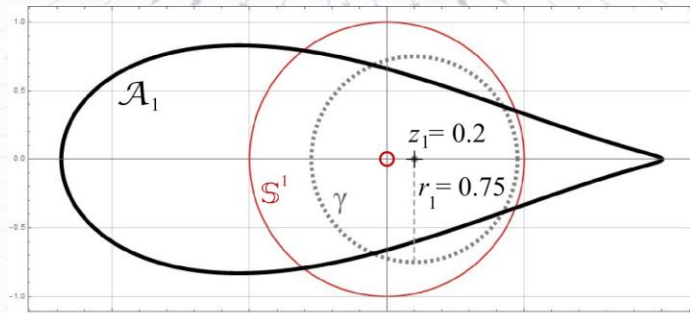


----- ŽUKOVSKÉHO PROFILY -----

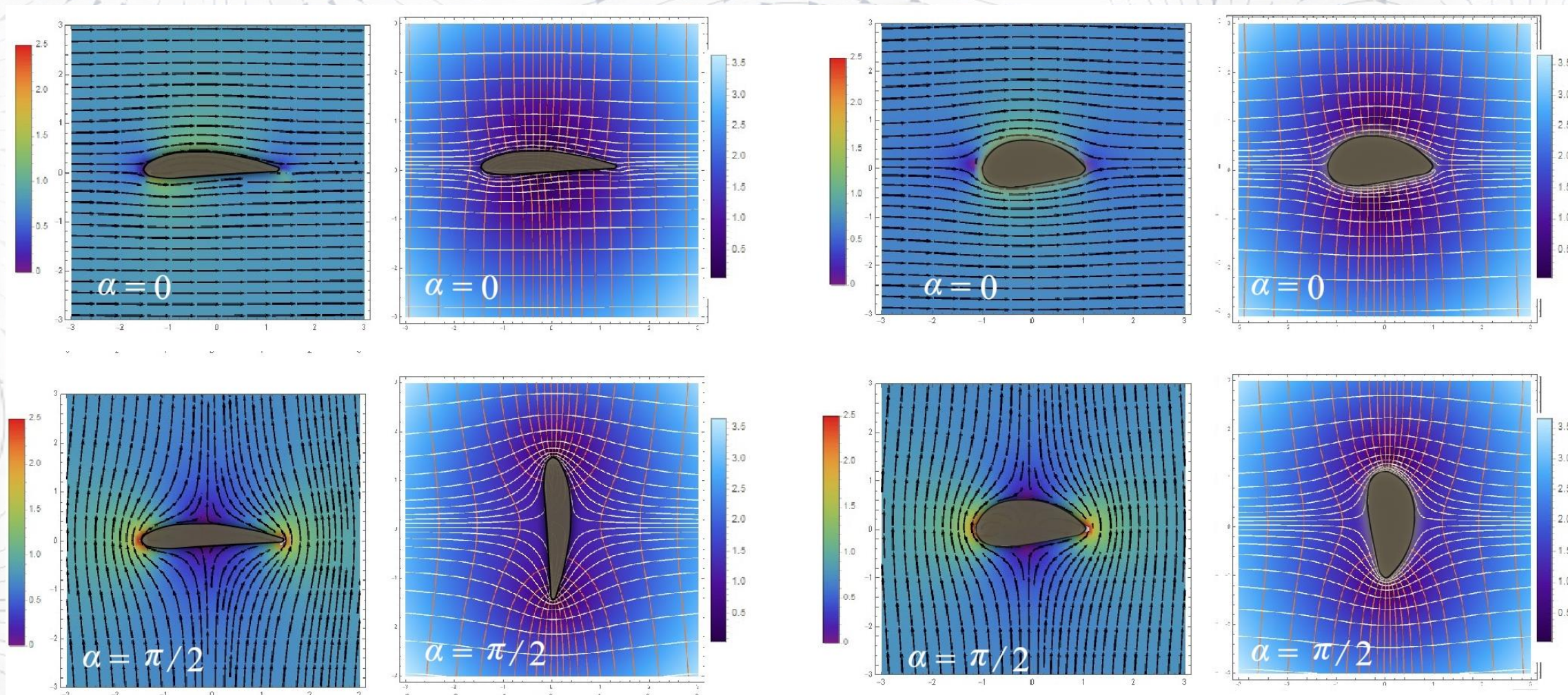
Zobrazujeme kružnice $\gamma(t) = z_0 + re^{it}$ $t \in [0, 2\pi]$ na krivky $\mathcal{A} = F \circ \gamma$



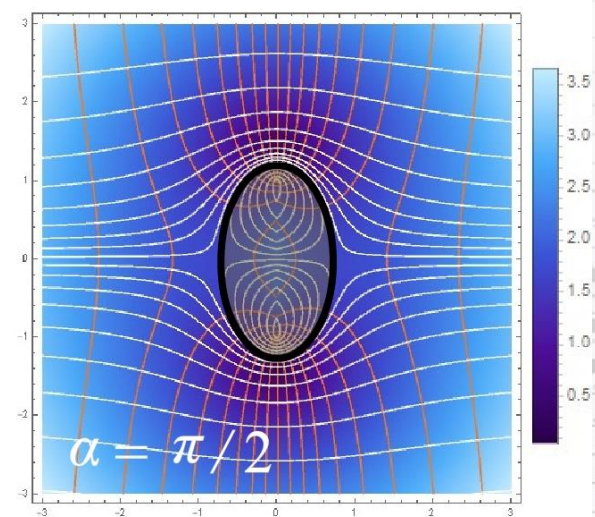
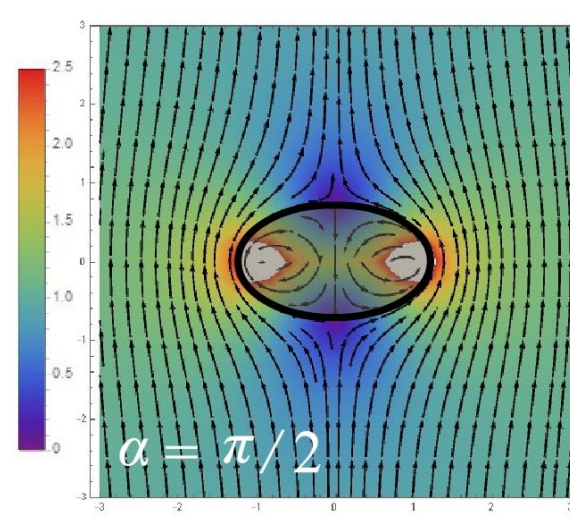
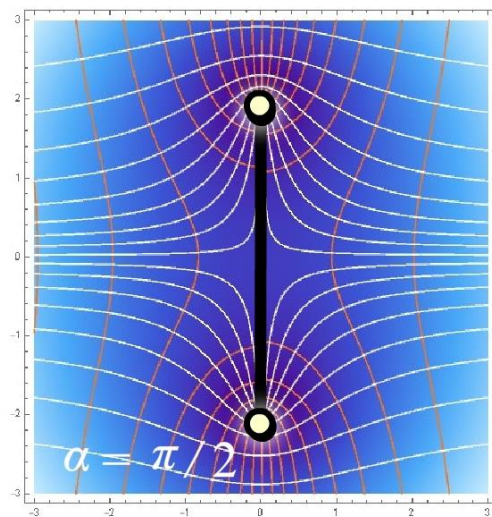
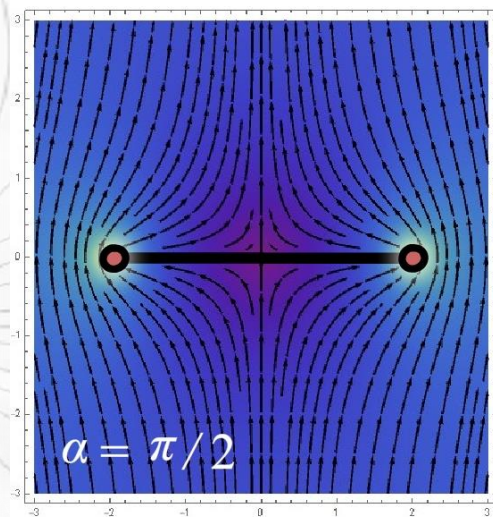
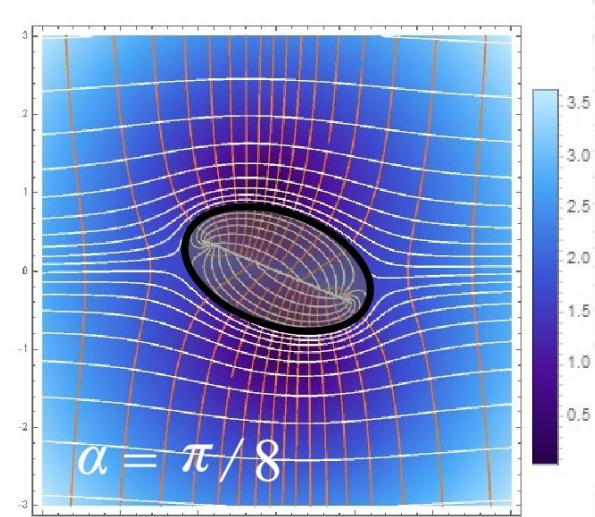
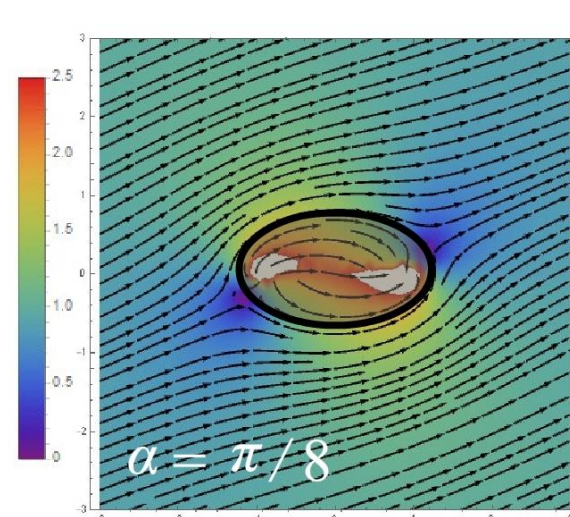
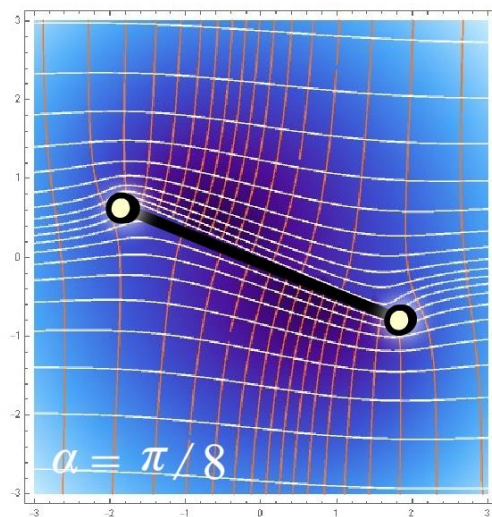
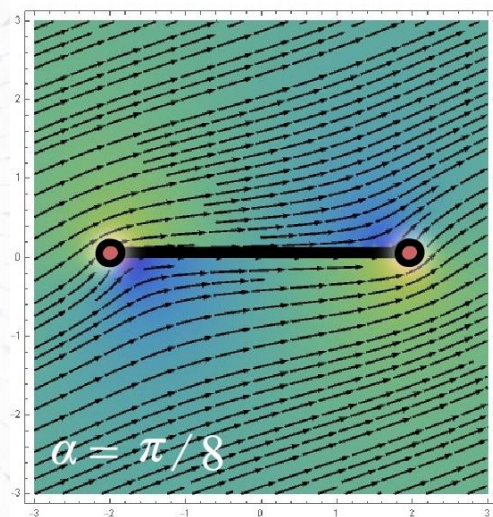
----- ŽUKOVSKÉHO PROFILY -----



----- ŽUKOVSKÉHO PROFILY -----



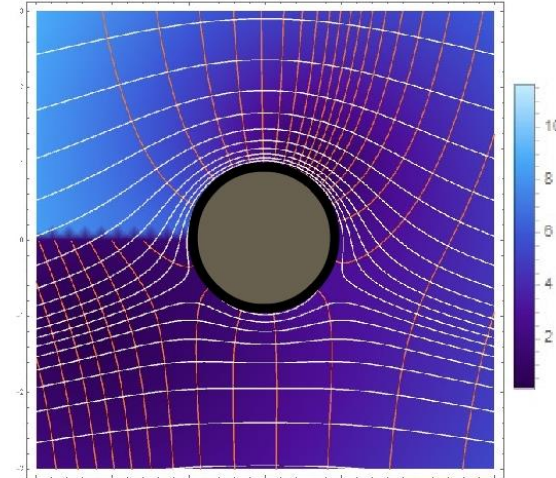
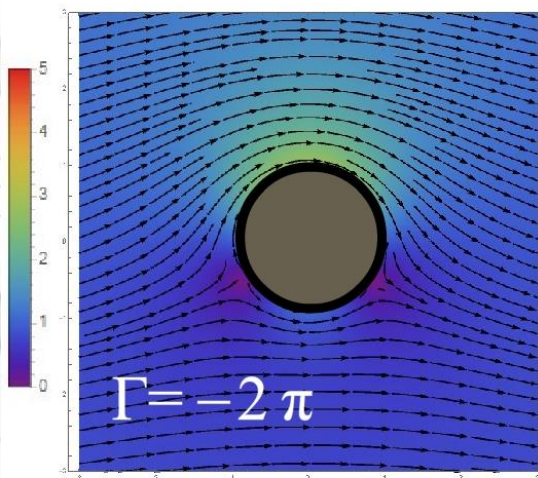
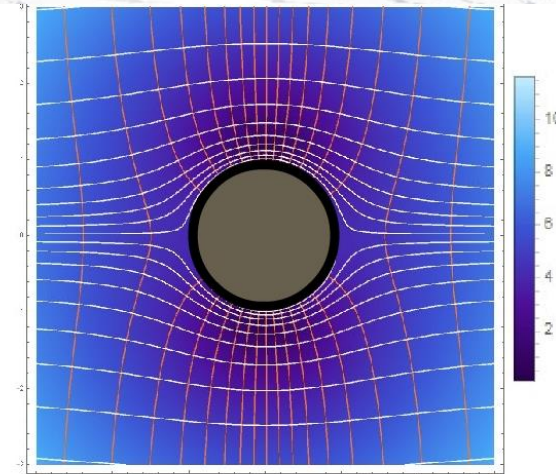
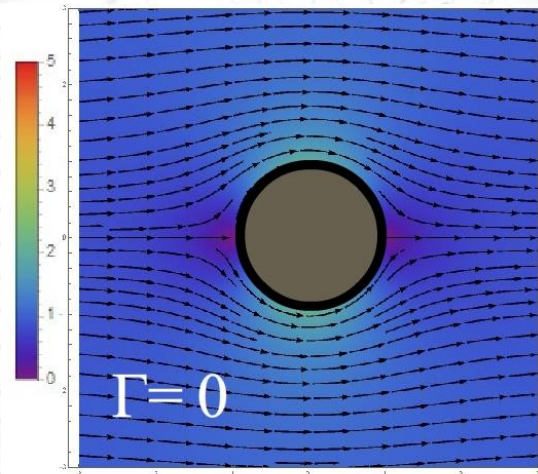
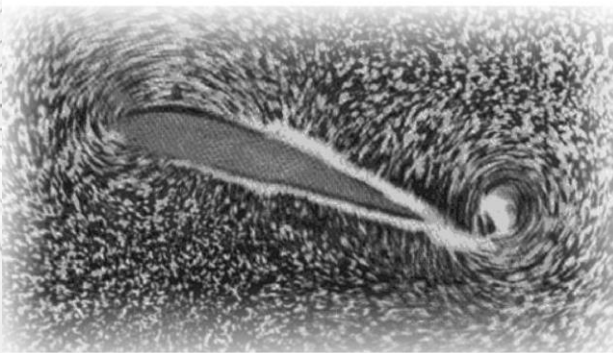
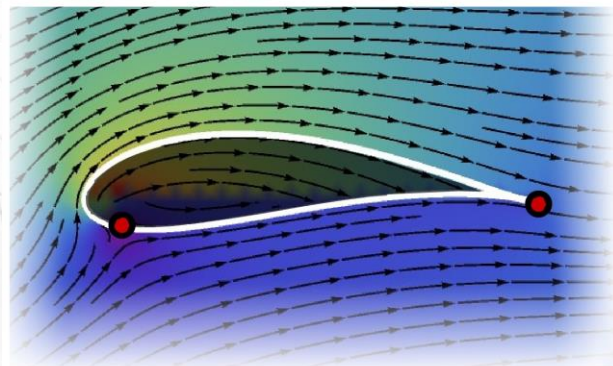
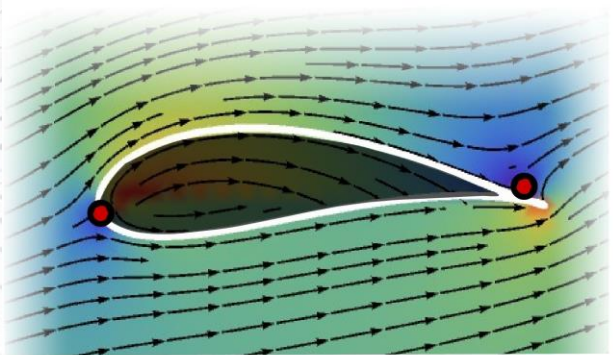
----- OVÁL A DOSKA -----



----- PREČO JE KRÍDLO NADNÁŠANÉ? -----

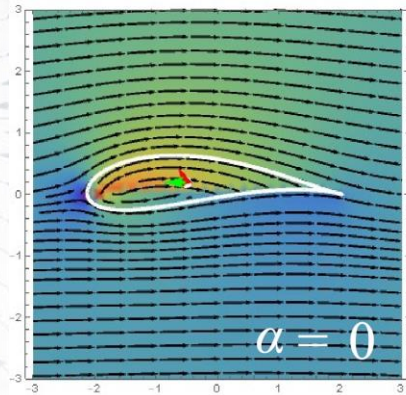
$$F(z) = v_{\infty} \left(z + \frac{a^2}{z} \right) - i \frac{\Gamma}{2\pi} \log(z)$$

Γ - cirkulácia



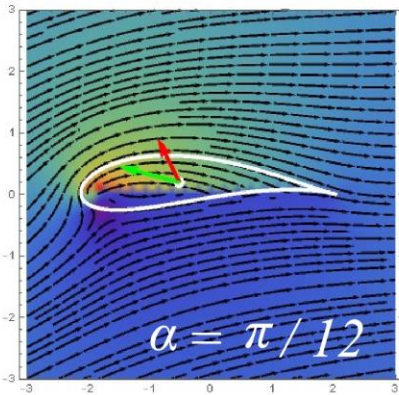
----- PREČO JE KRÍDLO NADNÁŠANÉ? -----

Porovnanie zdvihových síl (*Blasius-Chaplygin* vs. *Kutta-Žukovského* formula):



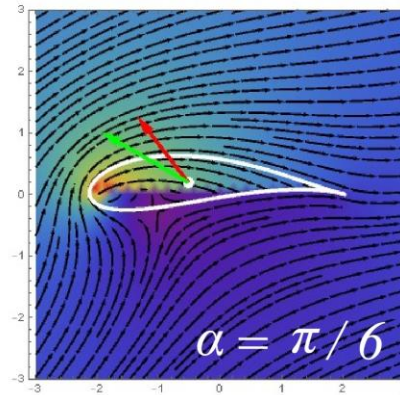
$$F_1 = -1.892$$

$$F_2 = -0.629 + 1.128i$$



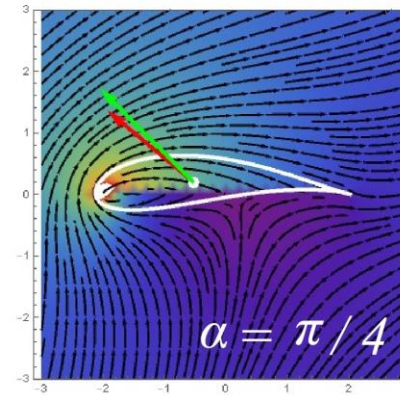
$$F_1 = -5.004 + 1.361i$$

$$F_2 = -1.844 + 3.708i$$



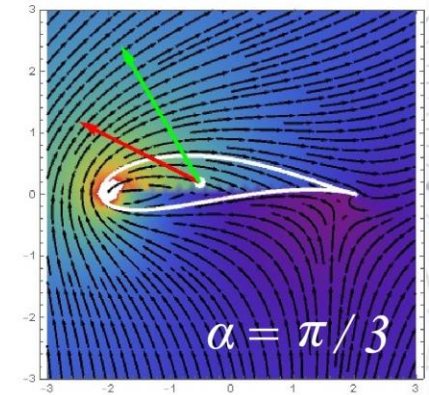
$$F_1 = -7.114 + 4.123i$$

$$F_2 = -4.185 + 5.392i$$



$$F_1 = -7.691 + 7.691i$$

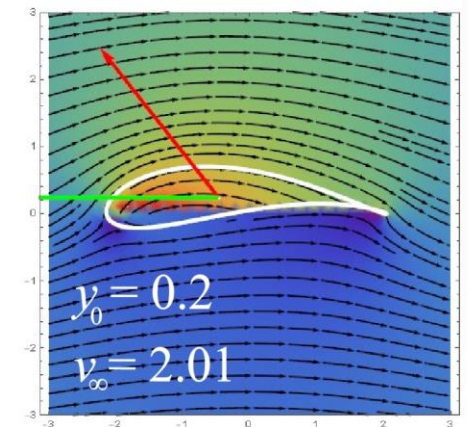
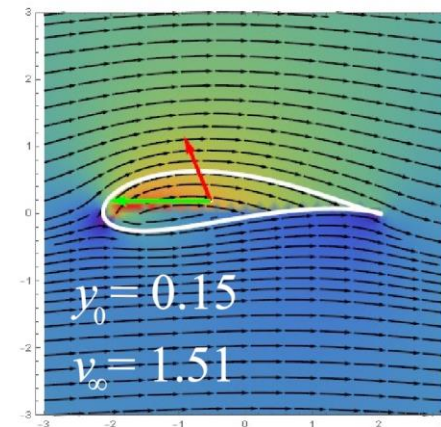
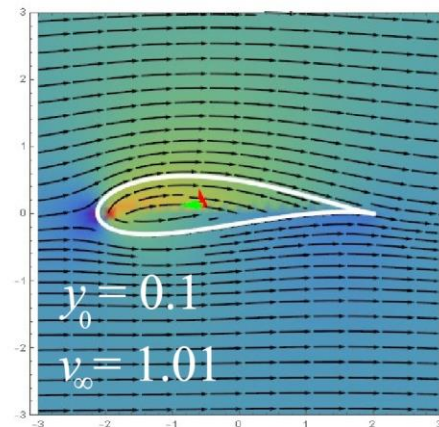
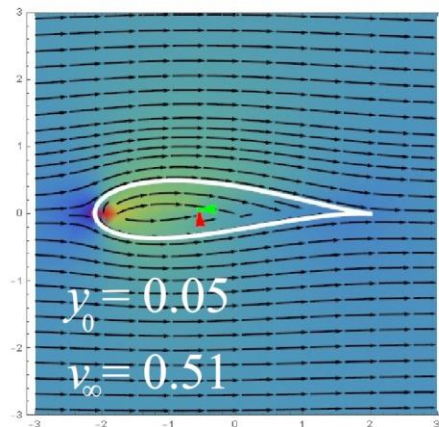
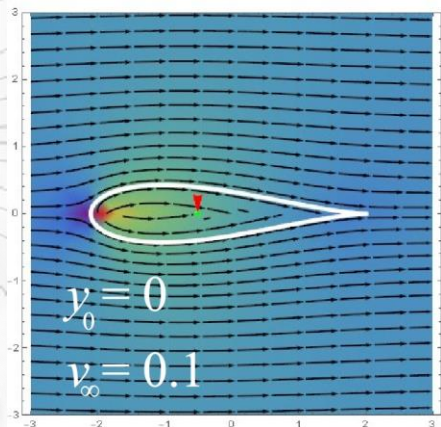
$$F_2 = -7.103 + 5.865i$$



$$F_1 = -6.448 + 11.169i$$

$$F_2 = -9.884 + 4.967i$$

Vzlet lietadla:



Ďakujem za pozornosť

