SEGMENTATION OF MEDICAL DATA USING EVOLVING PLANE CURVES

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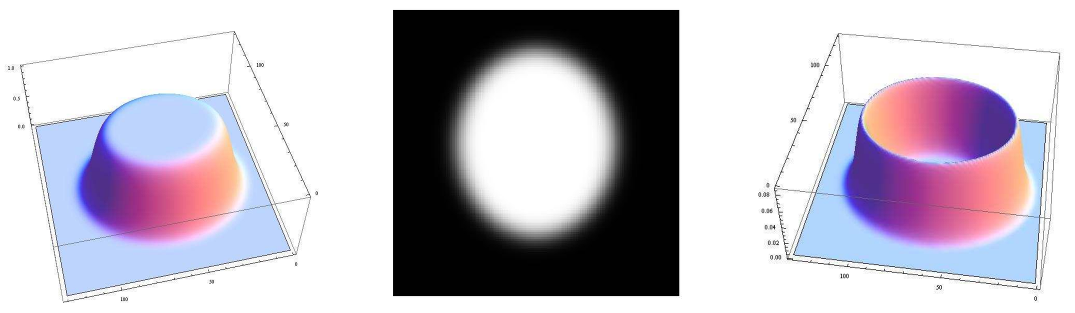
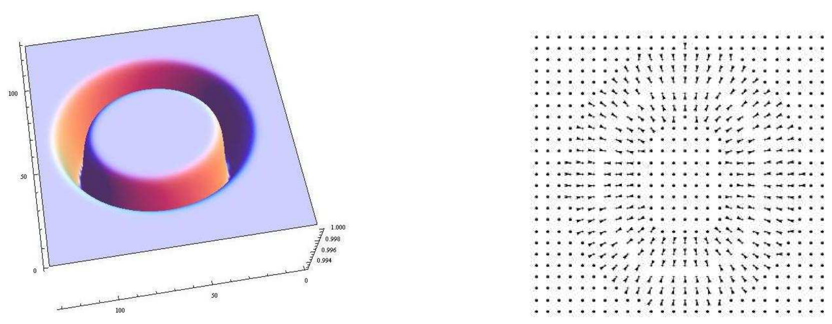
We present mathematical models, computational methods and software solution [6] for computing the plane curve evolution with applications to biological and medical image segmentation. Our solution is based on Lagrangian approach [3, 4, 5]. The plane curve Γ_t, $t \geq 0$ is driven by a suitable velocity vector field [1, 2] projected to the normal of the curve. Furthermore, the motion is regularized by the local curvature of the evolving curve and optimized with respect to discrete curve representation by the asymptotically uniform tangential redistribution [4, 5].

Velocity vector field



$$g(s) = \frac{1}{1+K\,s^2} \qquad \mathbf{v}(x,y) = -\nabla g(|\nabla u(x,y)|) \qquad (1)$$



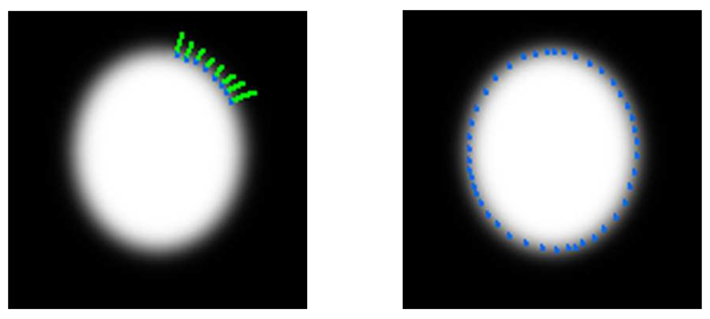
Curve in velocity vector field

The position vector $\mathbf{r}$ of evolving curve Γ_t is driven by differential equation

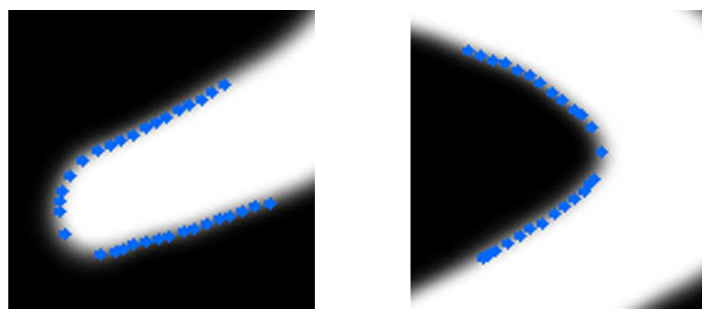
$$\partial_t \mathbf{r} = \mathbf{v} \qquad \mathbf{r}_i^{m+1} = \mathbf{r}_i^m + \tau \mathbf{v} \qquad (2)$$

which is discretized in time and space using gridpoints $\mathbf{r}_i^m$, $i = 1, ..., N$, m denotes time step number and τ time step size.

The motion starts interactively by user defined uniformly divided line segment which is moved automatically towards the edge.



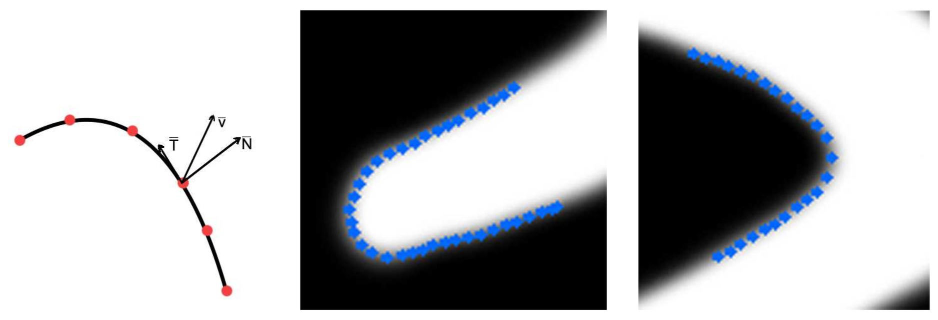
There can be a problem with distribution of curve representing gridpoints due to a discrete character of the vector field.



Modifications of vector field

$$\partial_t \mathbf{r} = v_{\mathbf{N}}\, \mathbf{N} \qquad v_{\mathbf{N}} = \mathbf{v} \cdot \mathbf{N} \qquad (3)$$

The nonlinear term $v_{\mathbf{N}}$ represents projection of $\mathbf{v}$ to the normal $\mathbf{N}$ of the moving curve. Removing non-controlled tangential part of velocity is reasonable for uniformly discretized initial curve [6].



Regularization by curvature

$$\partial_t \mathbf{r} = \mu\, v_{\mathbf{N}}\, \mathbf{N} + \epsilon k \mathbf{N}\,. \qquad (4)$$

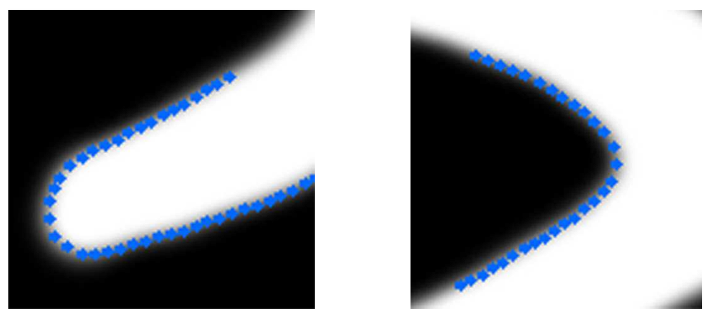
By the influence of local curvature the motion of grid points is more regular, they are tied together in the numerical scheme, μ and ϵ are parameters weighting advection by vector field and regularization by curvature. Due to Frenet's formula

$$k\mathbf{N} = \partial_s \mathbf{T} = \partial_{ss}\mathbf{r} \qquad \partial_t \mathbf{r} = \mu\, v_{\mathbf{N}}\, \mathbf{N} + \epsilon \partial_{ss}\mathbf{r}\,. \qquad (5)$$

where ∂_s denotes derivative with respect to arclength parametrisation s, we obtain intrinsic diffusion equation (5), which is discretized by the explicit flowing finite volume method [3, 5]

$$\mathbf{r}_i^{m+1} = \mathbf{r}_i^m + \tau\mu\, v_{\mathbf{N}}\, \mathbf{N}_i^m + \tau\epsilon \frac{2}{h_{i+1}^m + h_i^m}\left(\frac{\mathbf{r}_{i+1}^m - \mathbf{r}_i^m}{h_{i+1}^m} - \frac{\mathbf{r}_i^m - \mathbf{r}_{i-1}^m}{h_i^m}\right) \qquad (6)$$

$$h_i^m = \sqrt{(x_i^m - x_{i-1}^m)^2 + (y_i^m - y_{i-1}^m)^2}, \qquad \mathbf{r}_i^m = (x_i^m, y_i^m). \qquad (7)$$



Controlled nontrivial tangential velocity

If we consider general curve evolution in normal and tangential directions

$$\partial_t \mathbf{r} = \beta \mathbf{N} + \alpha \mathbf{T}\,. \qquad (8)$$

with

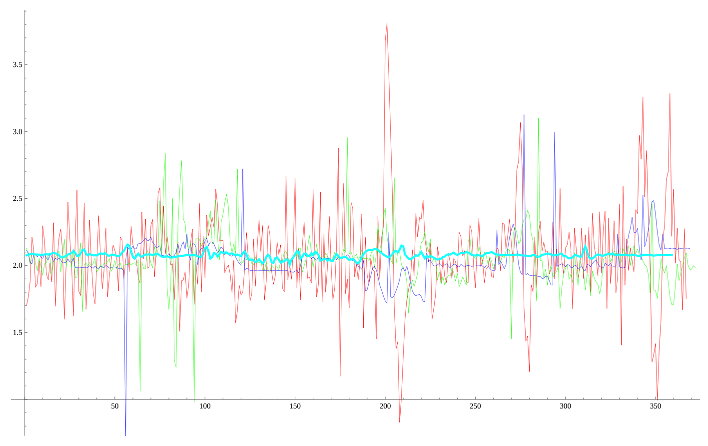
$$\beta = \mu\, v_{\mathbf{N}} + \epsilon k \qquad \partial_s \alpha = k\beta - < k\beta >_\Gamma + \left(\frac{L}{g} - 1\right)\omega\,, \qquad (9)$$

where L is global and g local length of the moving curve, $< k\beta >_\Gamma$ is an average of $k\beta$ along the curve and ω is a relaxation parameter, the gridpoints representing numerically the curve are distributed asymtotically uniformly as time evolves [4]. For this model we get intrinsic advection-diffusion equation

$$\partial_t \mathbf{r} = \mu\, v_{\mathbf{N}} \mathbf{N} + \epsilon\, \partial_{ss}\mathbf{r} + \alpha \partial_s \mathbf{r}. \qquad (10)$$

which is discretized either by explicit (in curve building steps) or semi-implicit (in curve post-processing step) flowing finite volume method [3, 5].

Distribution of grid point distances in the final segmentation curve by the various models. The starting uniform discretization of line segment is given by 2 pixels.



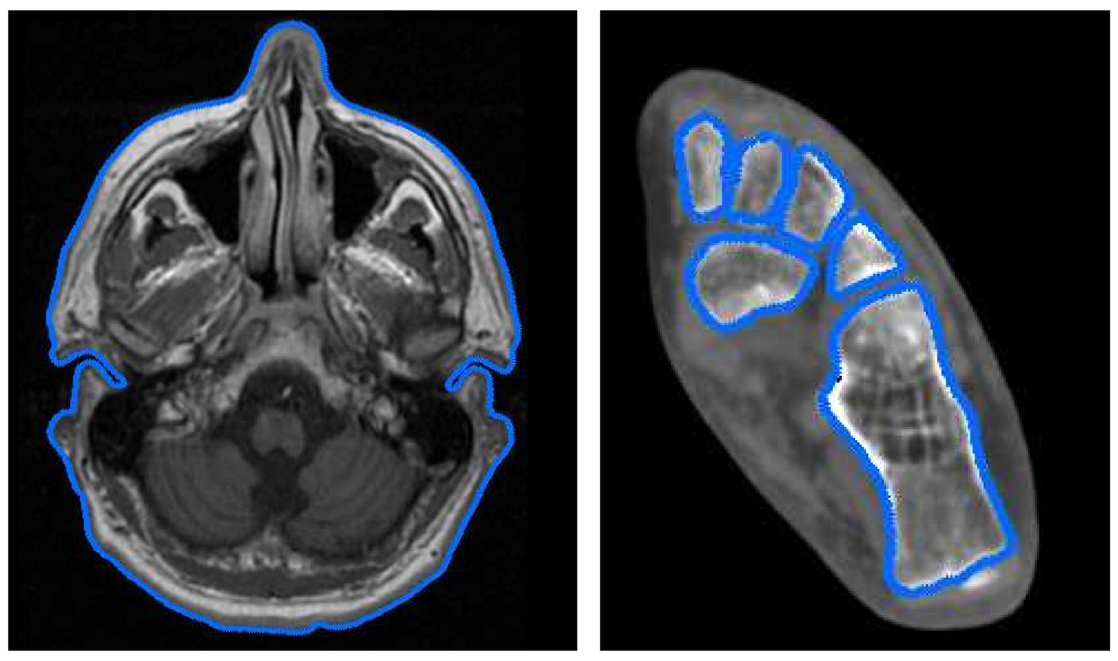
	Min	Max	Variance
model (2)	0.61681	3.59267	0.12216
model (3)	0.89949	3.17837	0.04209
model (4)	1.32320	3.21941	0.03333
model (8)	1.83412	2.19442	0.00092

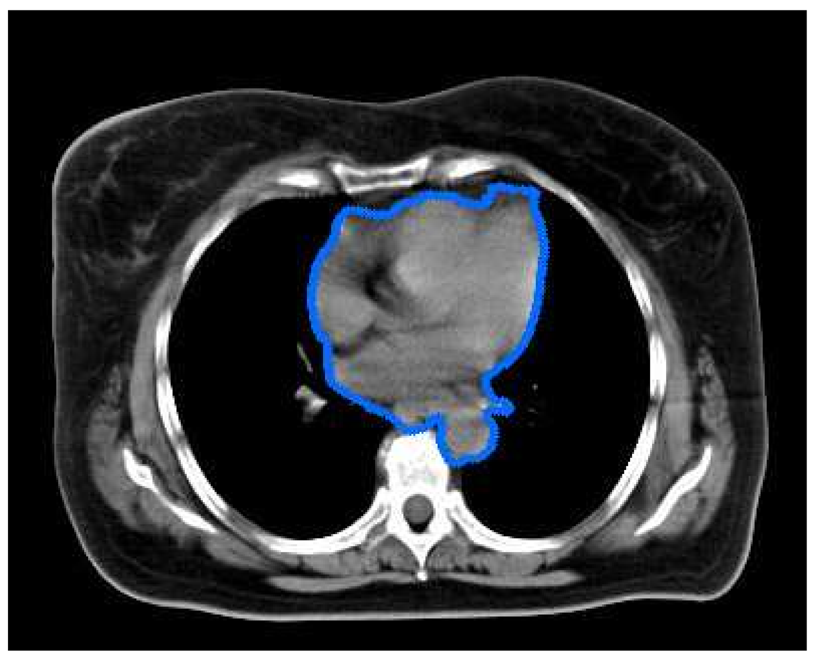
Stopping criterion

The curve is stopped at the m-th time step, if

$$|(v_{\mathbf{N}})_i^m| < 0.01 \quad \text{or} \quad (v_{\mathbf{N}})_i^m\, (v_{\mathbf{N}})_i^{m-1} < 0\,,\ \forall i. \qquad (11)$$

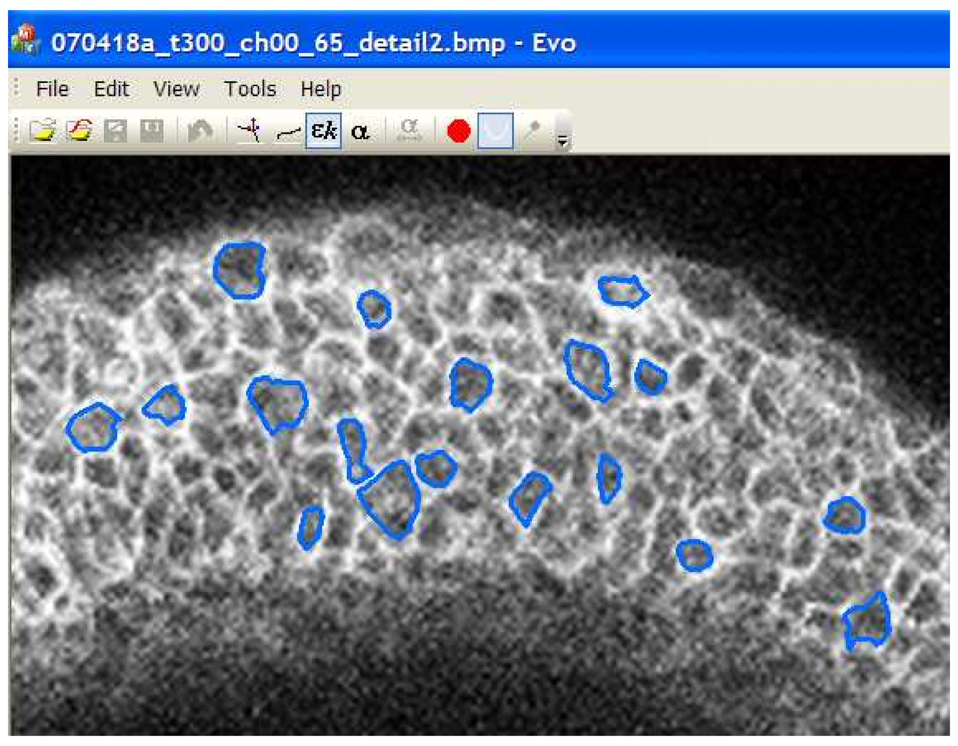
Medical image segmentation



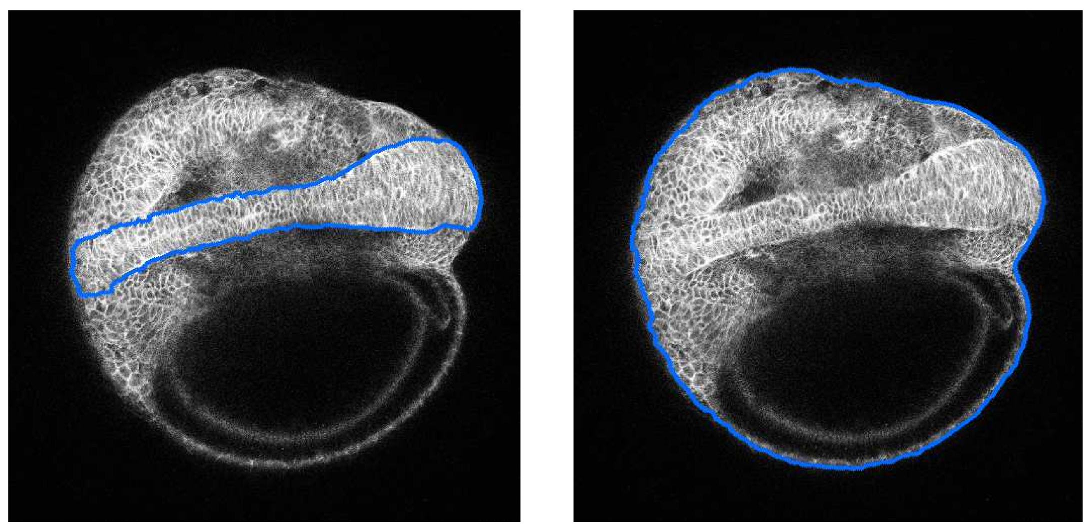


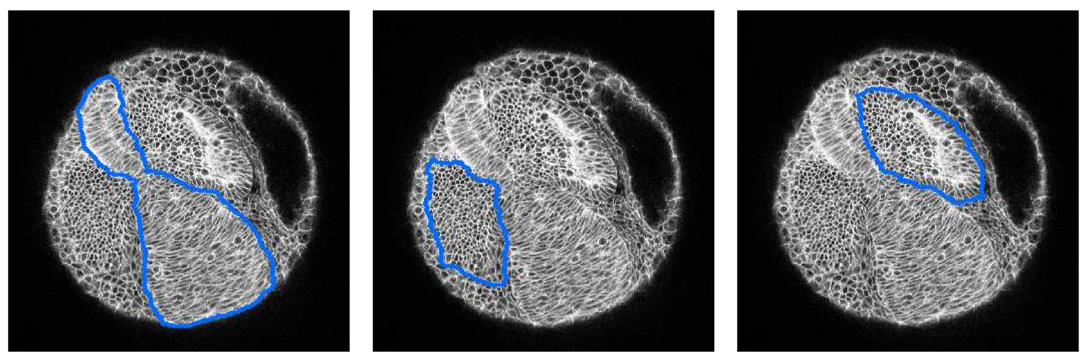
Zebrafish embryogenesis image segmentation

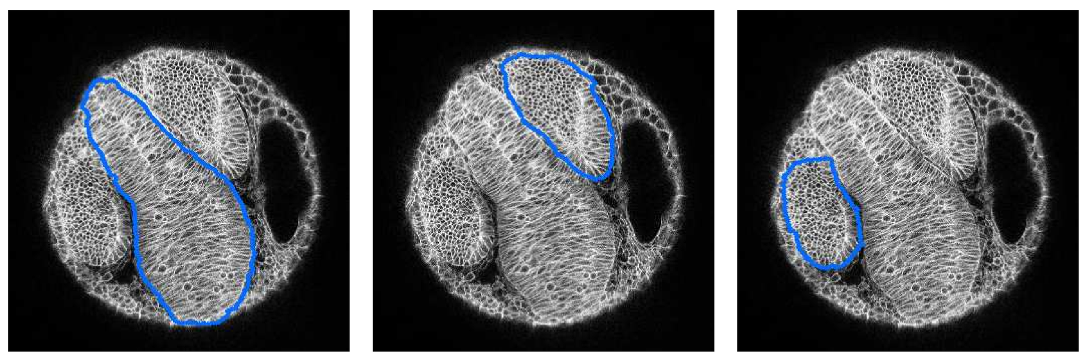
View of **Evo** application with segmented cells

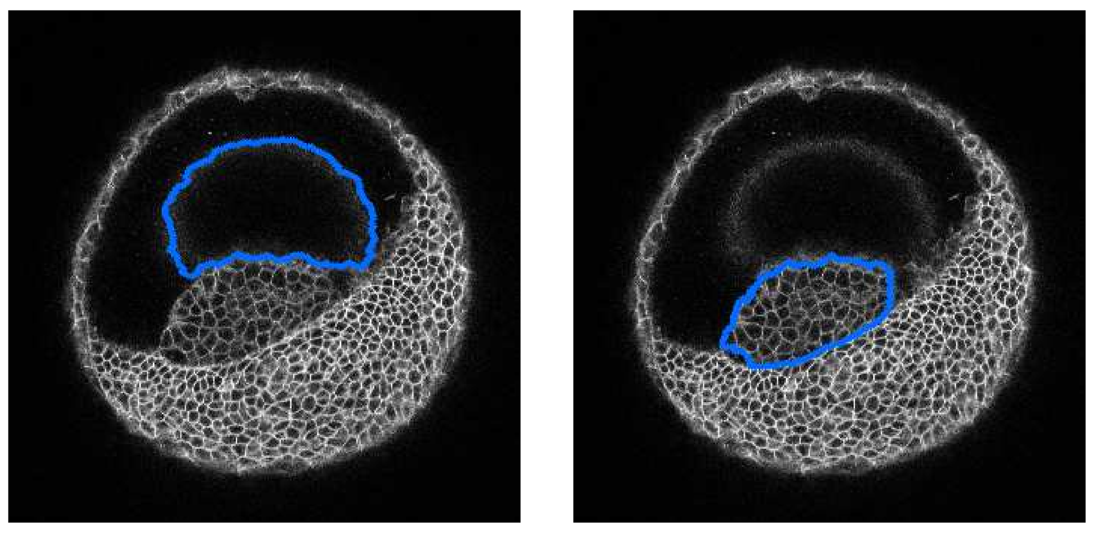


Segmented structures in zebrafish embryogenesis

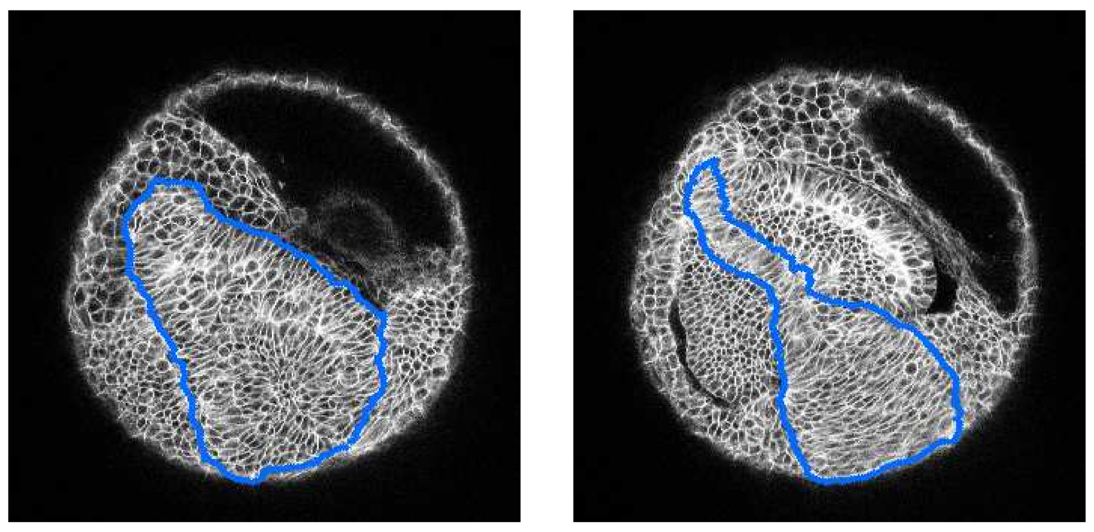


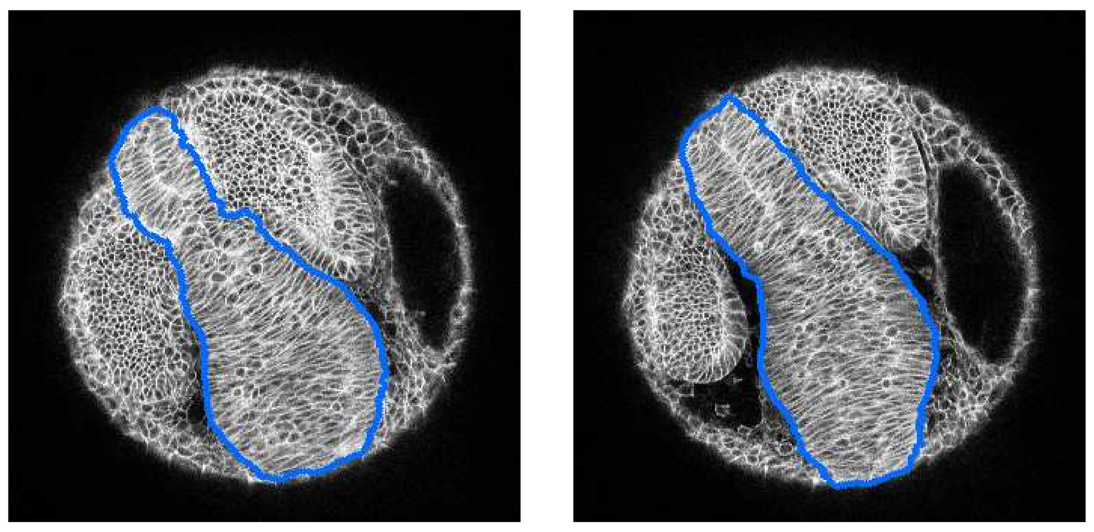






Segmented evolving structure in embryogenesis





References

[1] V. Caselles, R. Kimmel, G. Sapiro, *Geodesic active contours*, International Journal of Computer Vision, 22 (1997), pp. 61-79.

[2] S. Kichenassamy, A. Kumar, P. Olver, A. Tannenbaum, A. Yezzi, *Conformal curvature flows: from phase transitions to active vision*, Arch. Rational Mech. Anal., 134 (1996), pp. 275-301.

[3] K. Mikula, D. Ševčovič, *Evolution of plane curves driven by a nonlinear function of curvature and anisotropy*, SIAM J. Appl. Math., 61 (2001), pp. 1473-1501.

[4] K. Mikula, D. Ševčovič, *A direct method for solving an anisotropic mean curvature flow of planar curve with an external force*, Mathematical Methods in Applied Sciences, 27 (2004) pp. 1545-1565.

[5] K. Mikula, D. Ševčovič, M. Balažovjech *A simple, fast and stable flowing finite volume method for solving general curve evolution equations*,Preprint 2008-10-1, Department of Mathematics and Descriptive Geometry, Slovak University of Technology, Bratislava (2008)

[6] J.Urbán, *Evolution of plane curves in image processing*, Bachelor thesis (in Slovak), Slovak University of Technology in Bratislava, 2009