Geodetic data filtering by nonlinear diffusion equations on the Earth surface

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Introduction

We define a surface finite volume method for the numerical solution of parabolic partial differential equations on closed manifold which in our case represents the Earth surface. The main idea is based on approximation of the surface by finite number of triangles and using the Green`s theorem for Laplace-Beltrami operator to define a weak formulation of diffusion equation on the manifold. By the finite volume approximation of the weak formulation we obtain a system of linear equations which can be efficiently solved in each discrete time step by an iterative solver. Using a *semi-implicit* time discretization we extend the method to solve the nonlinear Perona-Malik diffusion equation which at the same time reduces a noise and keeps the edges and other details important for correct interpretation of the real data. In our application the initial condition is given by the satellite model of disturbing potential ITG-Grace03s and we apply both linear Laplace-Beltrami and nonlinear Perona-Malik type diffusion on closed manifold to obtain filtered geodetic data on the Earth surface.

The Perona-Malik equation

The regularized Perona-Malik model of form

$$\frac{\partial u}{\partial t} - \nabla_S u \cdot (g(v) \nabla_S u) = 0$$

on surface ω represents a generalization of and the classical Perona -Malik model which is widely used in image processing.

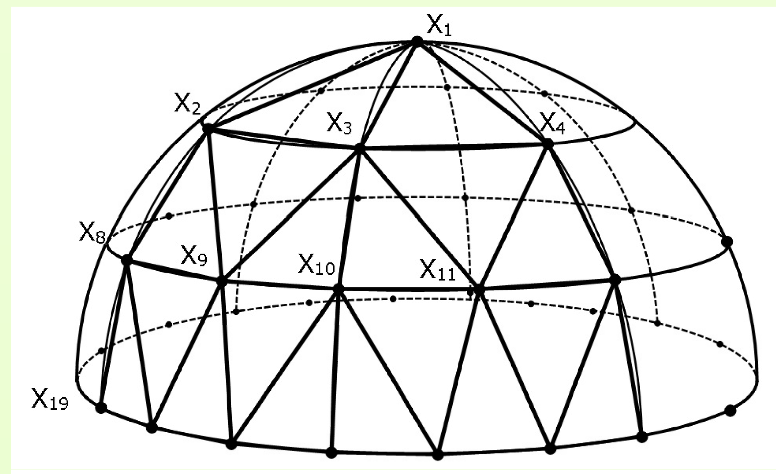
- u - **scalar function**
- $g(v) = \frac{1}{1 + Hv}$ - **edge indicator**
- $H \geq 0$ - **controls sensitivity of method**
- $v = \left| \nabla_S u^{\sigma} \right|$ - u^{σ} **solution of linear difusion** $(g(v) \equiv 1)$ **after small time step** σ
- **Weak formulation** $\int_{V_i} \frac{\partial u}{\partial t} dx - \sum_{q=1}^{Q_i} \int_{\partial V_{iq}} g(v) \nabla_S u \cdot \vec{\eta}_{iq} ds = 0$

•Numerical approximation (Used: semi-implicit time discretization, Green`s theorem and space discretization)

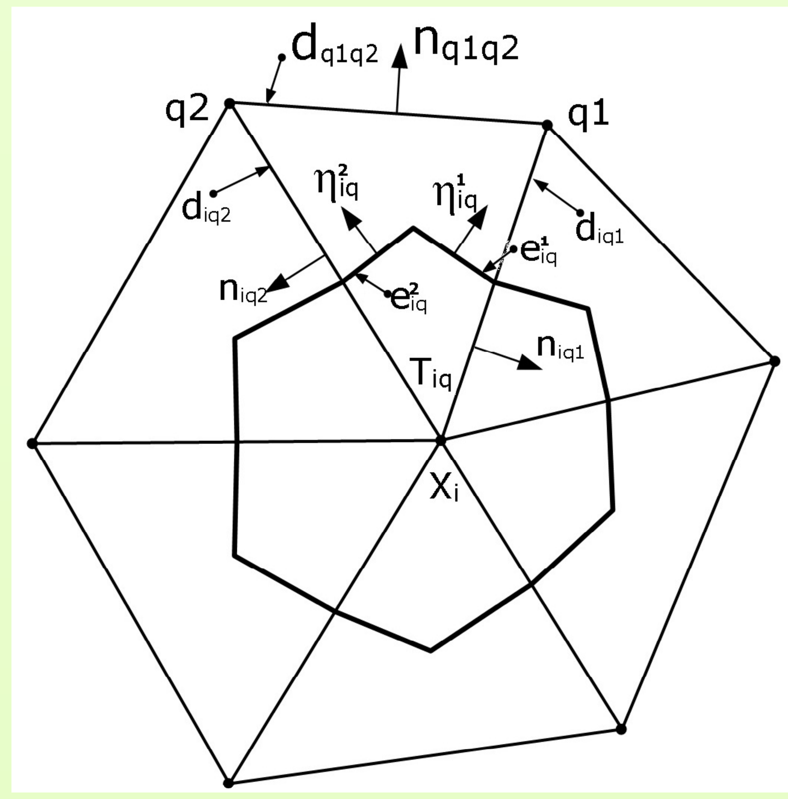
$$m(V_i)\frac{u_i^k - u_i^{k-1}}{\tau} - \sum_{q=1}^{Q_i} \left(m(e_{iq}^1) \vec{\eta}_{iq}^1 \cdot P_{T_{iq}}^k g\left(\left| P_{T_{iq}}^{\sigma,k-1} \right| \right) + m(e_{iq}^2) \vec{\eta}_{iq}^2 \cdot P_{T_{iq}}^k g\left(\left| P_{T_{iq}}^{\sigma,k-1} \right| \right) \right) = 0$$

Discretization

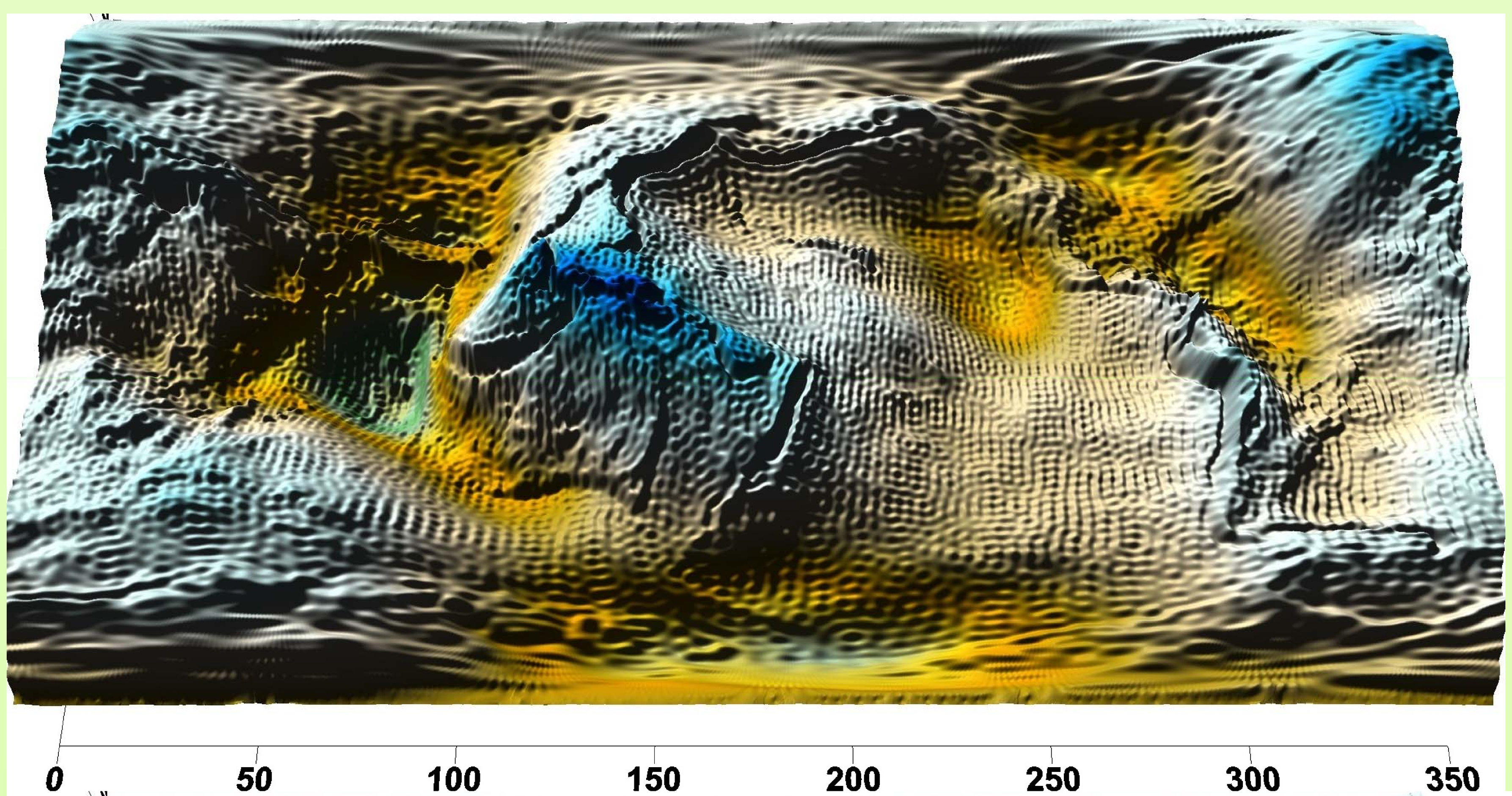
Primary grid:

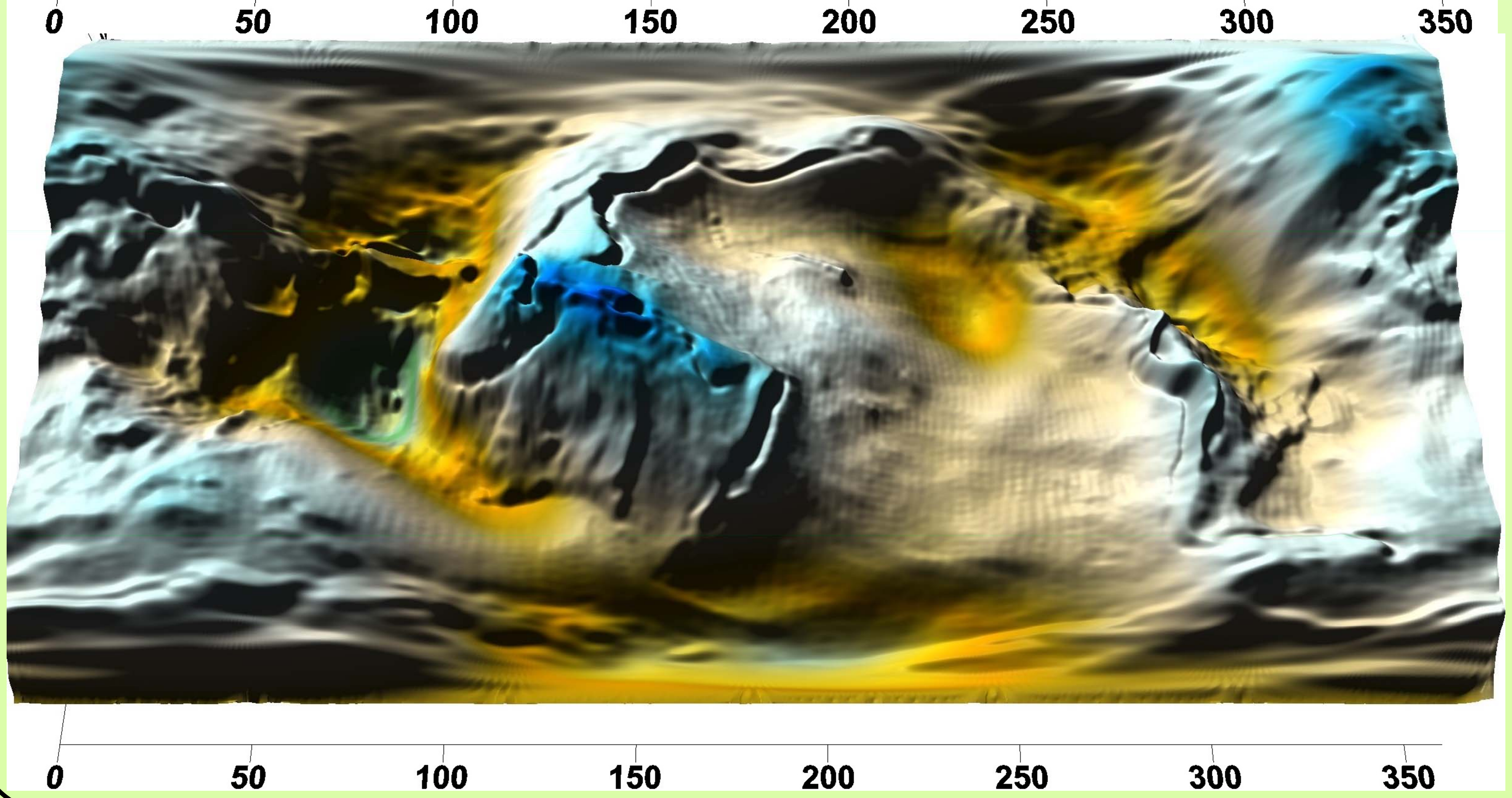


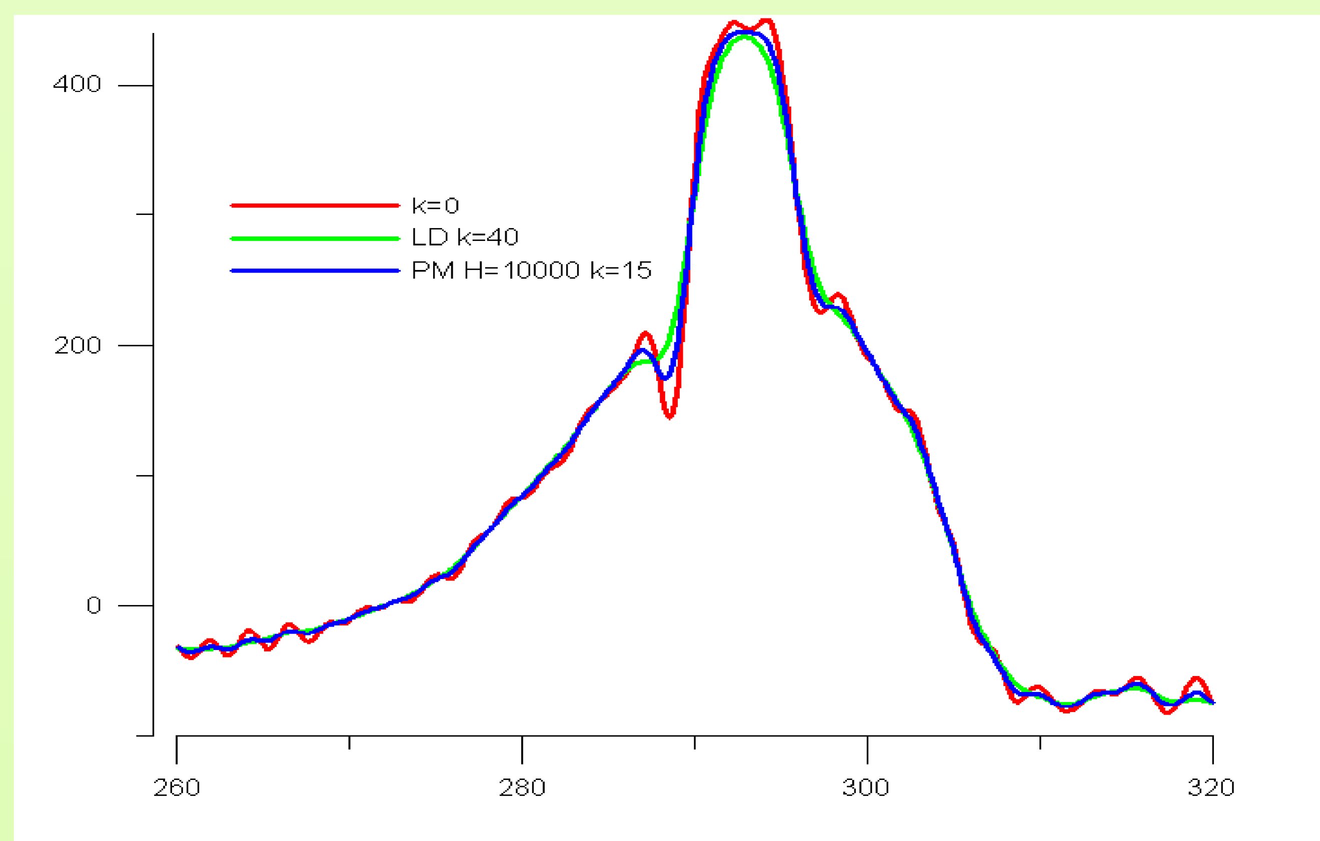
Finite volume:



Experiments







Computational details

- **We use both models (linear and nonlinear)**
- **initial condition will be provided by**
- **ITG Grace03 – satellite geopotential model of disturbing potential**
  - **– modeled by spherical harmonics up to degree 180**
  - **– "*striping*" noise (truncation error)**

-Space discretization: 1215002 points on Earth surface(≈ 0.2 deg)

-Time step: $\tau = \frac{1}{N} \sum_{i=1}^{N} m(V_i)$