3D curve evolution algorithm with tangential redistribution for a fully automatic finding of an ideal camera path in virtual colonoscopy

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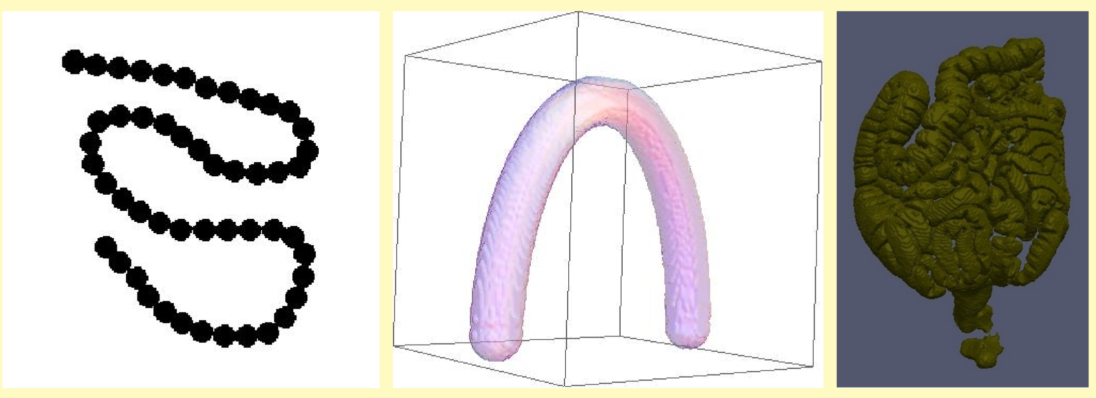
Abstract

We develop new stable method, based on 3D evolving curves, for finding the optimal trajectory of the camera in the virtual colonoscopy. The initial curve is driven by the velocity field in the plane normal to the evolving curve, the evolution is regularized by curvature and accompanied by the suitable choice of tangential velocity. Thanks to the asymtotically uniform tangential redistribution of grid points, originally introduced in this work for 3D evolving curves, and to the fast and stable semi-implicit scheme for solving our proposed intrinsic advection-diffusion PDE, we end up in fast and robust way with the smooth uniformly discretized 3D curve representing the ideal path of the camera in virtual colonoscopy.

The colon segmentation

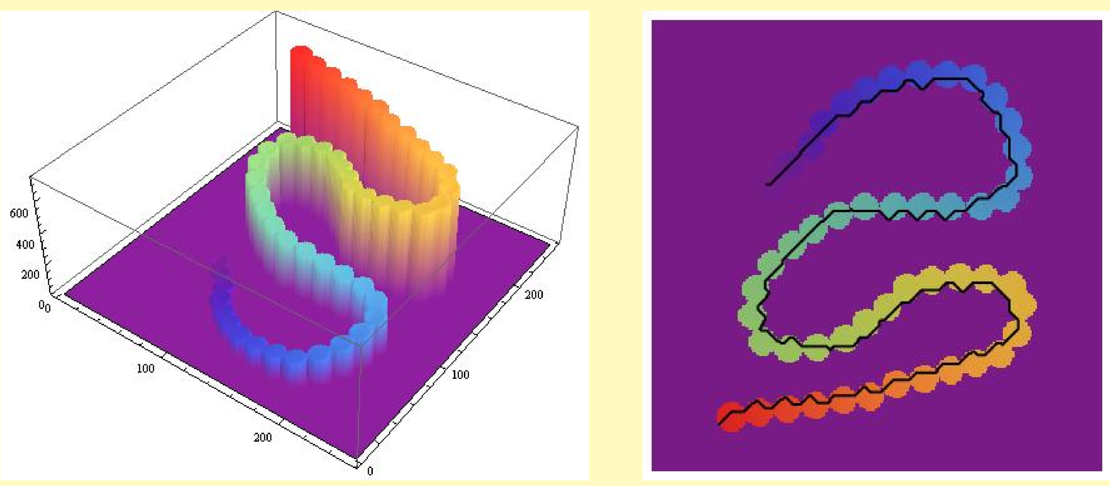
Thresholding - detect all subvolumes filled by the gas
Region growing - find all simply connected parts filled with gas
Removing small subvolumes - subvolume size is counted during region growing
Removing thin subvolumes - compute the distance function of all inner voxels to the border of the segmented subvolume[3] , and check its global maximum



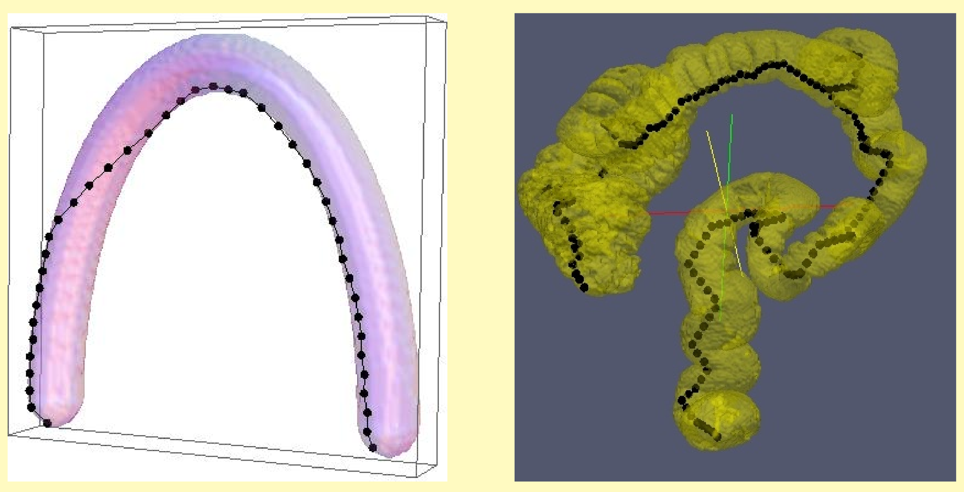
Segmentated test data, large and small intestine

The initial curve

The initial trajectory guess in any colon subvolume is constructed by computing a distance from a point source by the Dijkstra algorithm (in which the graph edges connecting neighbouring voxels have value 1) followed by the backtracking. The voxel coordinates of such path represent the parametric 3D curve, the initial guess of the trajectory inside the subvolume.



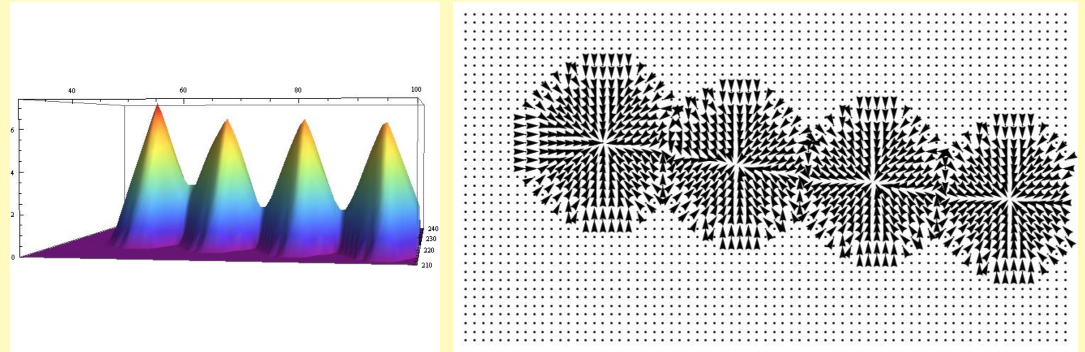
The graph of the distances and the initial trajectory guess.



Initial 3D curve in segmented test and colon data.

The curve in vector field

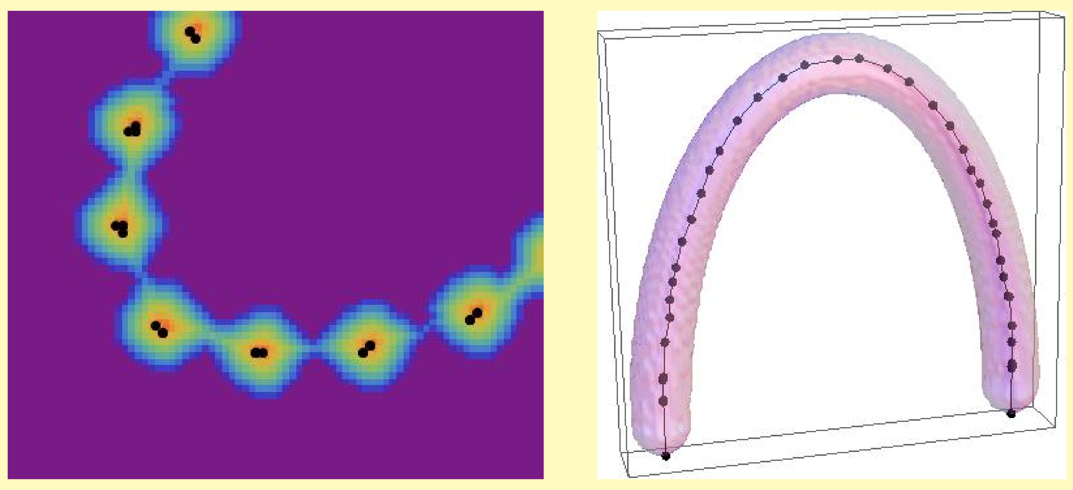
The vector field we compute as $\mathbf{v} = \nabla d$ (d - **distance from subvolume boundary**). We obtain distance function as numerical solution of the time relaxed eikonal equation $d_t + |\nabla d| = 1$.



Detailed graph of distance function to the boundary of 2D testing shape and computed vector field

The simplest model for the motion of the curve in the vector field

$$\partial_t \mathbf{r} = \mathbf{v}(\mathbf{r}).$$

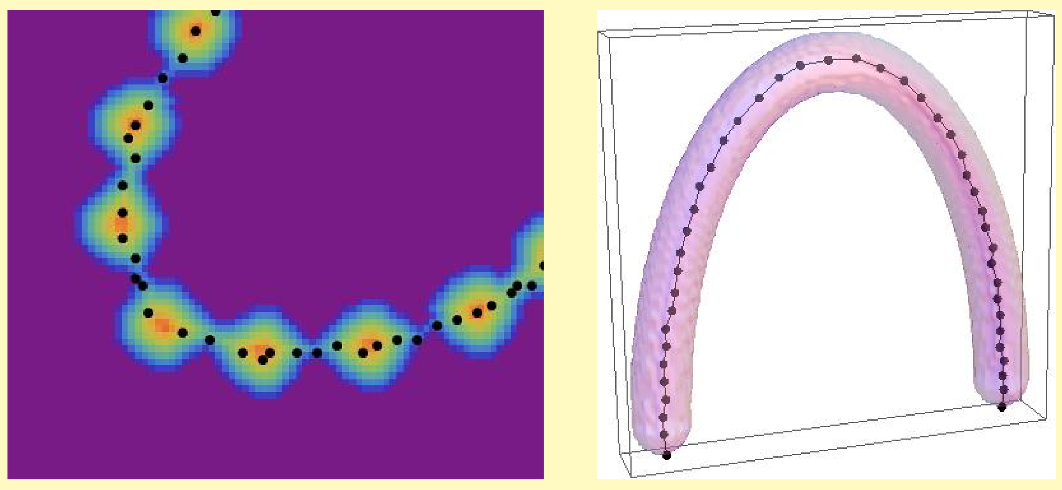


The results obtained using the velocity field given as $\mathbf{v} = \nabla d$ on segmented object in 2D and 3D test data.

Motion in the normal plane

The motion of the curve can be decomposed $\partial_t \mathbf{r} = \beta \mathbf{N} + \alpha \mathbf{T}$, where $\mathbf{N}$ resp. $\mathbf{T}$ is the unit normal resp. tangent vector to the curve.
Overall shape of the evolving curve is determined only by the normal velocity component (α can be zero).
The projection of vector field v to the curve normal plane is defined by $\mathbf{N_v} = \mathbf{v} - (\mathbf{T}.\mathbf{v})\mathbf{T}$, $k\mathbf{N}$ denotes **curvature vector** and the regularized curve motion in the normal plane is given by

$$\partial_t \mathbf{r} = \mu \mathbf{N_v} \; + \epsilon k \mathbf{N}.$$



The results obtained using the projection of the original vector field into the normal plane to the evolving 3D curve accompanied by the curvature regularization.

The optimal path - determining of suitable tangential velocity

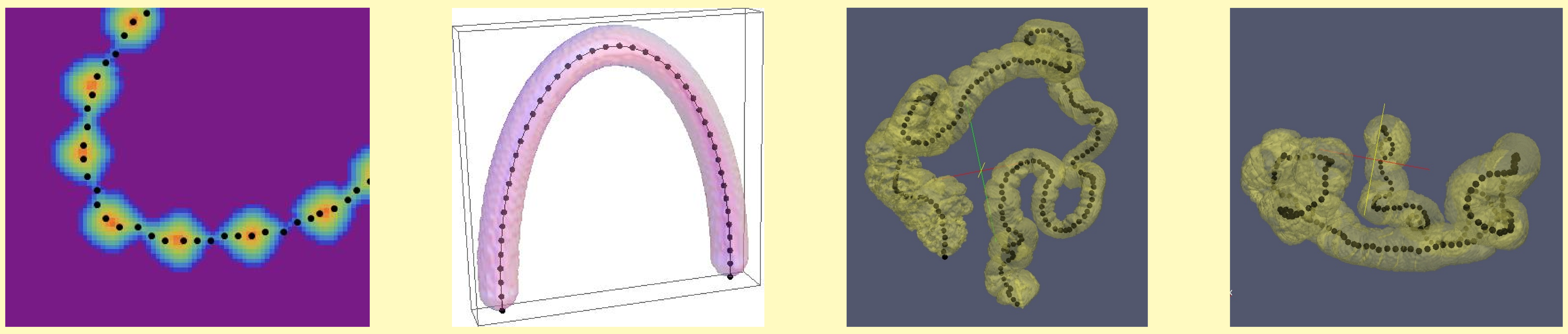
We introduce the local orthogonal basis smoothly varying along the 3D curve, cf. [1]. It will consist of $\mathbf{T}$ and $\mathbf{N_1} = \frac{\mathbf{N_v}}{|\mathbf{N_v}|}$ and $\mathbf{N_2} = \mathbf{N_1} \times \mathbf{T}$ (orthogonal vectors in the normal plane). Let us define $k_1 = k\mathbf{N}.\mathbf{N}_1$ and $k_2 = k\mathbf{N}.\mathbf{N}_2$. The evolution equation can be written as $\partial_t \mathbf{r} = U\mathbf{N_1} + V\mathbf{N_2} + \alpha\mathbf{T}$, with normal components given by $U = k_1 + \mu|\mathbf{N_v}|$ and $V = k_2$. **The tangential velocity α guarateeing the asymptotically uniform redistribution of 3D curve grid points [2]** we obtain as solution of equation

$$\partial_s \alpha = Uk_1 + Vk_2 - < Uk_1 + Vk_2 >_\Gamma + \left(\frac{L}{g} - 1\right)\omega_r, \qquad (1)$$

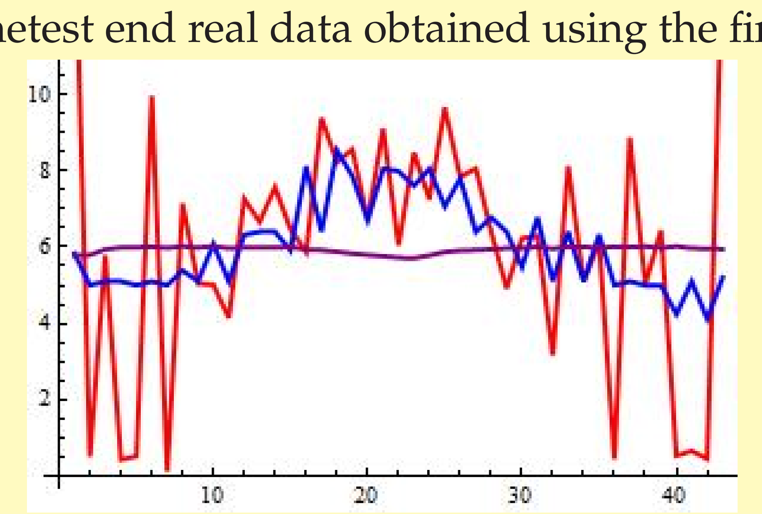
where ω_r is a speed of redistribution process, L resp. g denotes global, resp. local curve length. Since $\mathbf{T} = \partial_s \mathbf{r}$ and $k\mathbf{N} = \partial_{ss}\mathbf{r}$ we get our final 3D curve evolution model in the form of the following intrinsic advection-diffusion PDE with driving force

$$\partial_t \mathbf{r} = \mu\, \mathbf{N_v} + \epsilon\, \partial_{ss}\mathbf{r} + \alpha \partial_s \mathbf{r} \qquad (2)$$

with the Dirichlet boundary conditions.



The results for thetest end real data obtained using the final model (1)-(2).



Comparison of the grid point distances: the first (red), the second (blue)and the third (violet) model.

References

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[2] Mikula, K., Ševčovič, D., Balažovjech, M.: A simple, fast and stabilized flowing finite volume method for solving general curve evolution equations, Comm. Comp. Physics, Vol. 7, No. 1 (2010) pp. 195-211.

[3] Rouy, E., Tourin, A.: Viscosity solutions approach to shape-from-shading,SIAM Journal on Numerical Analysis 29 No. 3 (1992) pp. 867–884.

[4] Deschamps, T., Cohen., L.D.: Fast extraction of minimal paths in 3D images and application to virtual endoscopy, Medical Image Analysis, volume 5, Issue 4, December 2001