

Zlatý rez v matematice a umění

Mgr. art., Mgr. Ladislav Šipeky

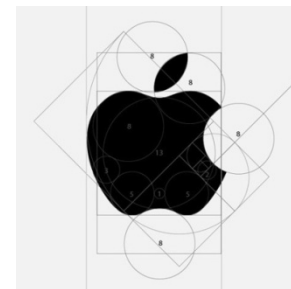
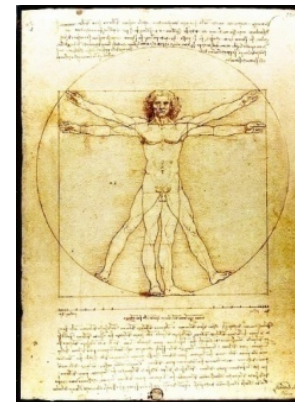
14.4.2013

Pomer prírody a elegancie

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Čo ich spája?



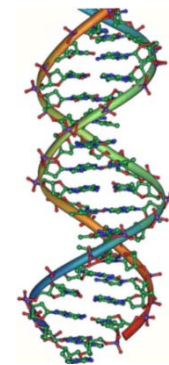
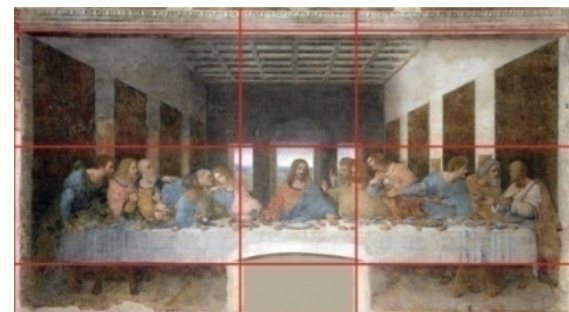
Theme from the Fifth Symphony
L. van Beethoven

motto (5 measures)

$233/377 = 0.618$

377 measures		233 measures	
S	372 measures	S	228 measures

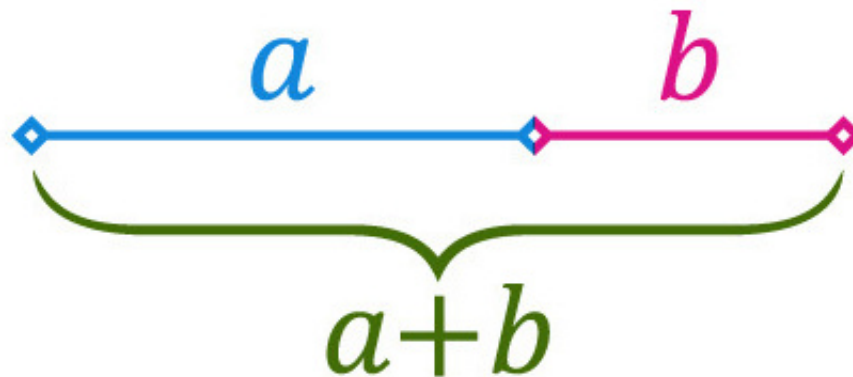
motto (5 measures)



Zlatý rez!

Jednoduchá úloha

Ako rozdeliť úsečku tak, aby sa pomer dĺžky väčšej časti k dĺžke menšej časti úsečky rovnal pomeru dĺžky celej úsečky ku dĺžke jej väčšej časti?



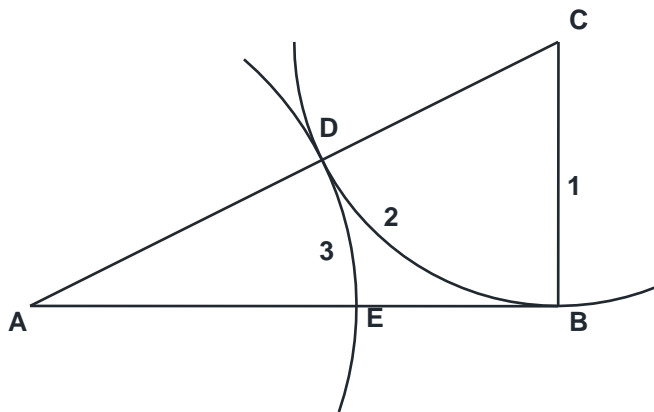
Staroveké Grécko a zlatý pomer

....

Phidias (cca 480 – 430 p.n.l.) – Cielené (?) využitie

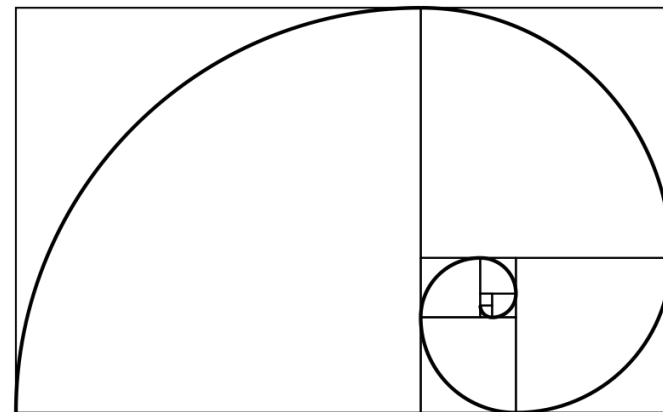
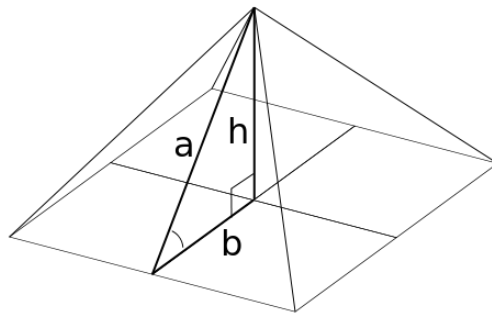
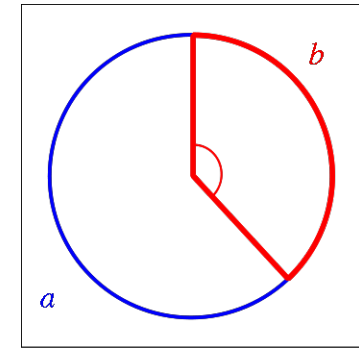
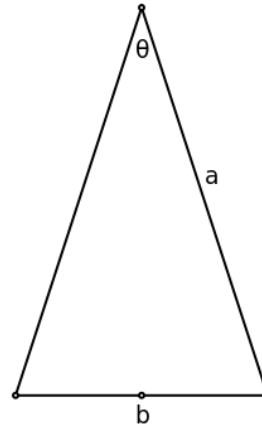
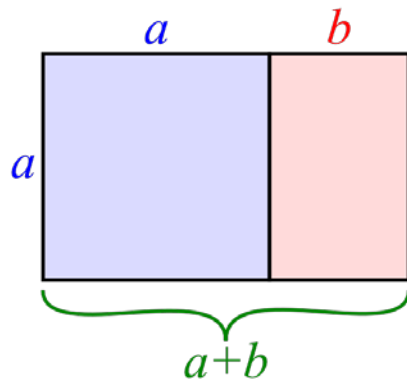
Euklides (cca 325 - 265 p.n.l.) – prvá zmienka

Heron (cca 10-70 n.l.) – „najkrajšia“ konštrukcia:

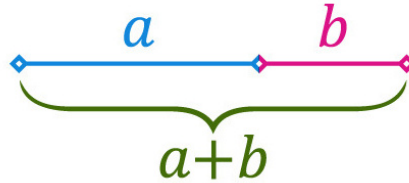


1. narysujeme pravouhlý trojuholník ABC tak, že pravý uhol bude pri vrchole B a dĺžka BC je polovica AB.
2. kružnica so stredom v bode C, s polomerom BC a dostaneme bod D na úsečke AC
3. kružnica so stredom v bode A, s polomerom AD a dostaneme bod E na úsečke AB

„Najkrajší obdĺžnik“ a iné zlaté útvary



Φ, φ [Phi]



$$\frac{a}{b} = \frac{a+b}{a}$$

Substitúcia: $\frac{a}{b} = \varphi$

$$\varphi = 1 + \frac{1}{\varphi}$$

$$\varphi^2 - \varphi - 1 = 0 \rightarrow \begin{aligned} \varphi_1 &= \frac{1 + \sqrt{5}}{2} = 1,618\,033\,988\dots \\ \varphi_2 &= \frac{1 - \sqrt{5}}{2} = -0,618\,033\,988\dots \end{aligned}$$

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1,618\,033\,988\dots$$

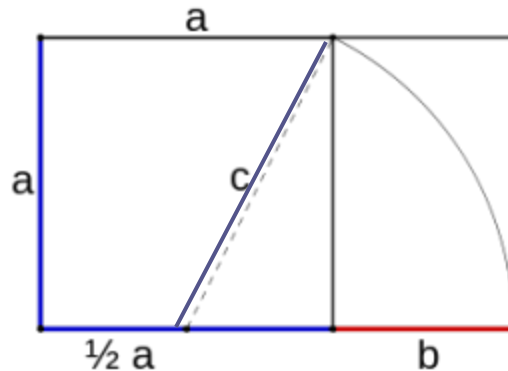
Φ, φ [Phi] pre fajšmekrov

1.618033988749894848204586834365638117720309179805762862135448622705260462818
902449707207204189391137484754088075386891752126633862223536931793180060766
726354433389086595939582905638322661319928290267880675208766892501711696207
032221043216269548626296313614438149758701220340805887954454749246185695364
864449241044320771344947049565846788509874339442212544877066478091588460749
988712400765217057517978834166256249407589069704000281210427621771117778053
153171410117046665991466979873176135600670874807101317952368942752194843530
567830022878569978297783478458782289110976250030269615617002504643382437764
861028383126833037242926752631165339247316711121158818638513316203840052221
657912866752946549068113171599343235973494985090409476213222981017261070596
116456299098162905552085247903524060201727997471753427775927786256194320827
505131218156285512224809394712341451702237358057727861600868838295230459264
787801788992199027077690389532196819861514378031499741106926088674296226757
560523172777520353613936210767389376455606060592165894667595519004005559089
502295309423124823552122124154440064703405657347976639723949499465845788730
396230903750339938562102423690251386804145779956981224457471780341731264532
204163972321340444494873023154176768937521030687378803441700939544096279558
986787232095124268935573097045095956844017555198819218020640529055189349475
926007348522821010881946445442223188913192946896220023014437702699230078030
852611807545192887705021096842493627135925187607778846658361502389134933331
223105339232136243192637289106705033992822652635562090297986424727597725655
086154875435748264718141451270006023890162077732244994353088999095016803281
121943204819643876758633147985719113978153978074761507722117508269458639320
456520989698555678141069683728840587461033781054443909436835835813811311689
938555769754841491445341509129540700501947754861630754226417293946803673198
058618339183285991303960720144559504497792120761247856459161608370594987860
069701894098864007644361709334172709191433650137157660114803814306262380514

Φ, φ [Phi]

$$\varphi = 1 + \frac{1}{\varphi} \Rightarrow \varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

$$\varphi = \frac{1 + \sqrt{5}}{2} \Rightarrow$$



Ako na kalkulačke:

$$\varphi = \dots + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}}$$

Fibonacci Leonardo Pisano Bigollo (cca 1170 n.l. - 1250 n.l.) a nesmrteľné králiky

Akou rýchlosťou sa môžu králiky rozmnožovať za ideálnych podmienok?

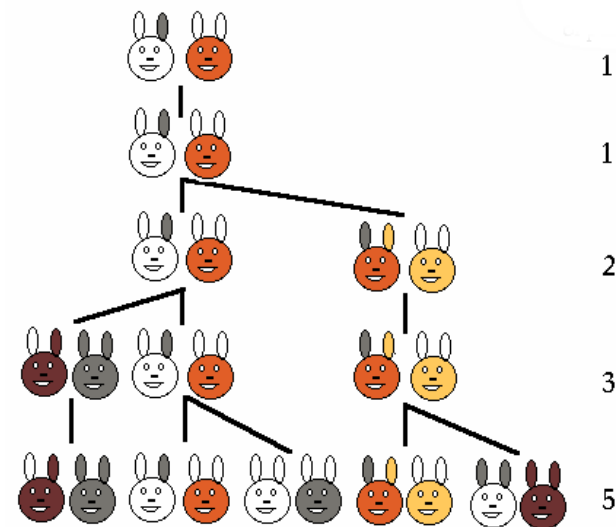
- Králiky sú schopné sa páriť vo veku jedného mesiaca tak, že na konci druhého mesiaca samička privedie na svet ďalší pár králikov.
- Králiky nikdy nezomrú

Fibonacci a nesmrtelné králíky

Fibonacciho postupnost:
1, 1, 2, 3, 5, 8, 13, 21, 34 ...

$$F_n = F_{n-2} + F_{n-1}$$

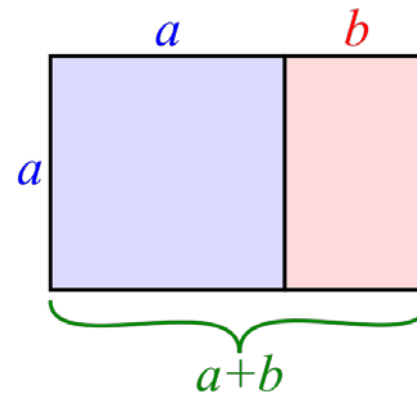
$$\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = \varphi$$



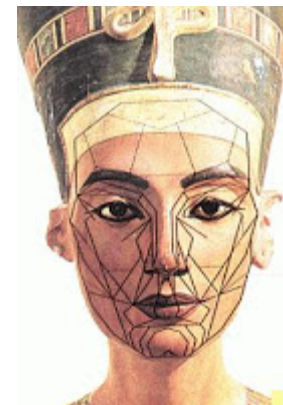
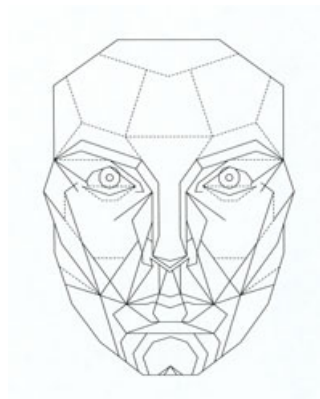
Johannes Kepler (1571–1630)

Psychológia, krása...

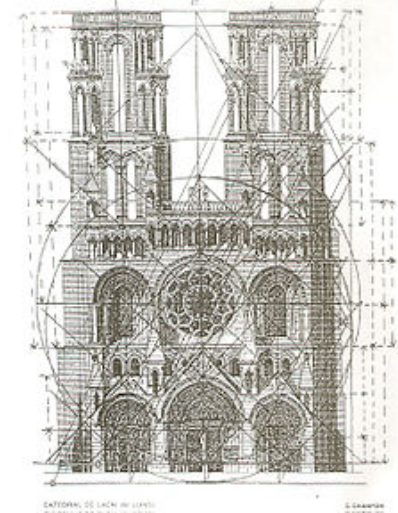
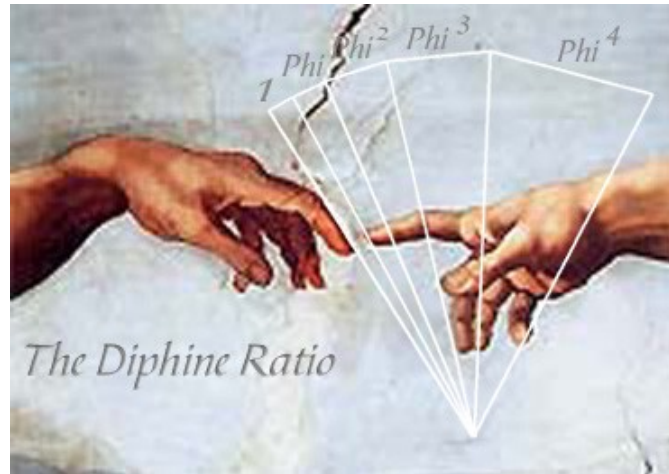
- Najkrajší obdĺžnik?



- Najkrajšie tváre?



Aj tu ...



Theme from the Fifth Symphony
L. van Beethoven

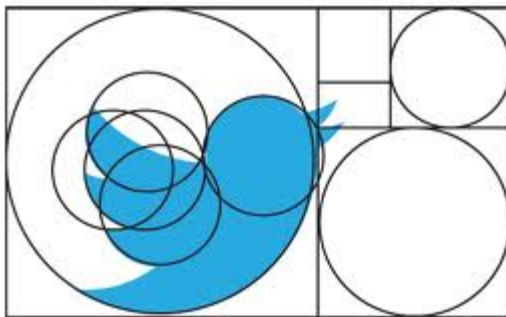
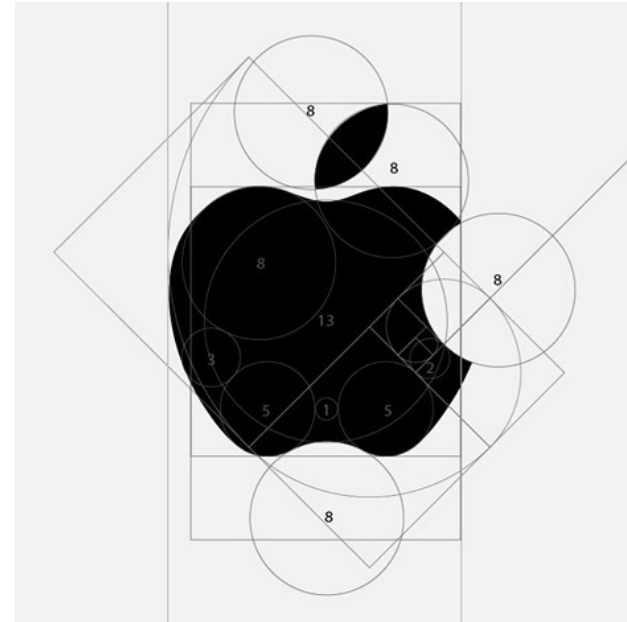
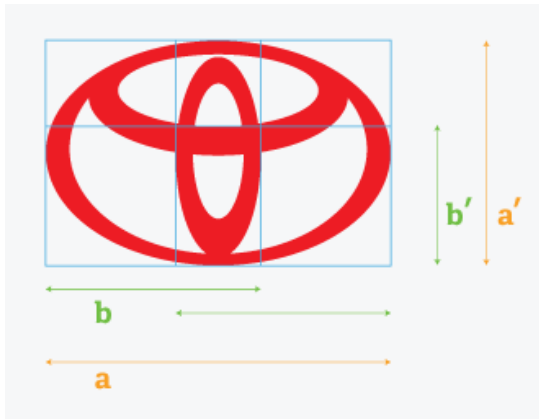
The image shows a musical score for the 'Theme from the Fifth Symphony' by Ludwig van Beethoven. The score is in 2/4 time and features a melody in the treble clef and a bass line in the bass clef. The melody is divided into five measures, each highlighted with a blue background and numbered 1 through 5. The first measure is marked *ff* and the fifth measure is marked *p*. Below the score, a table shows the structure of the piece in measures:

377 measures		233 measures	
5	372 measures	5	228 measures

293/377 = 0.618

motto (5 measures)

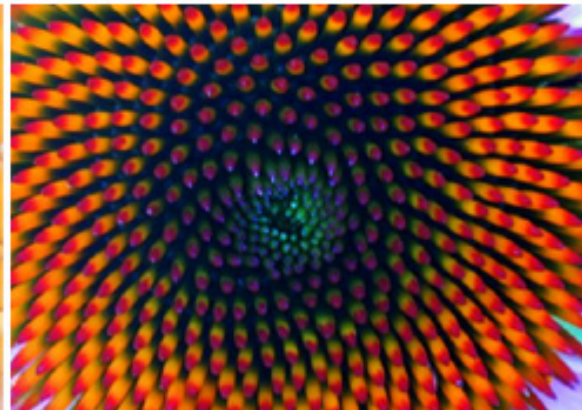
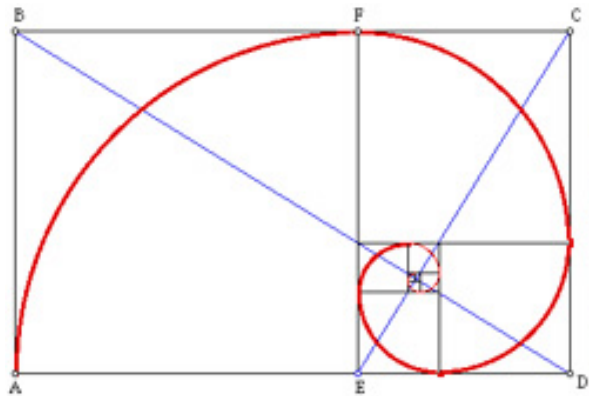
Aj tu, aj tu ...



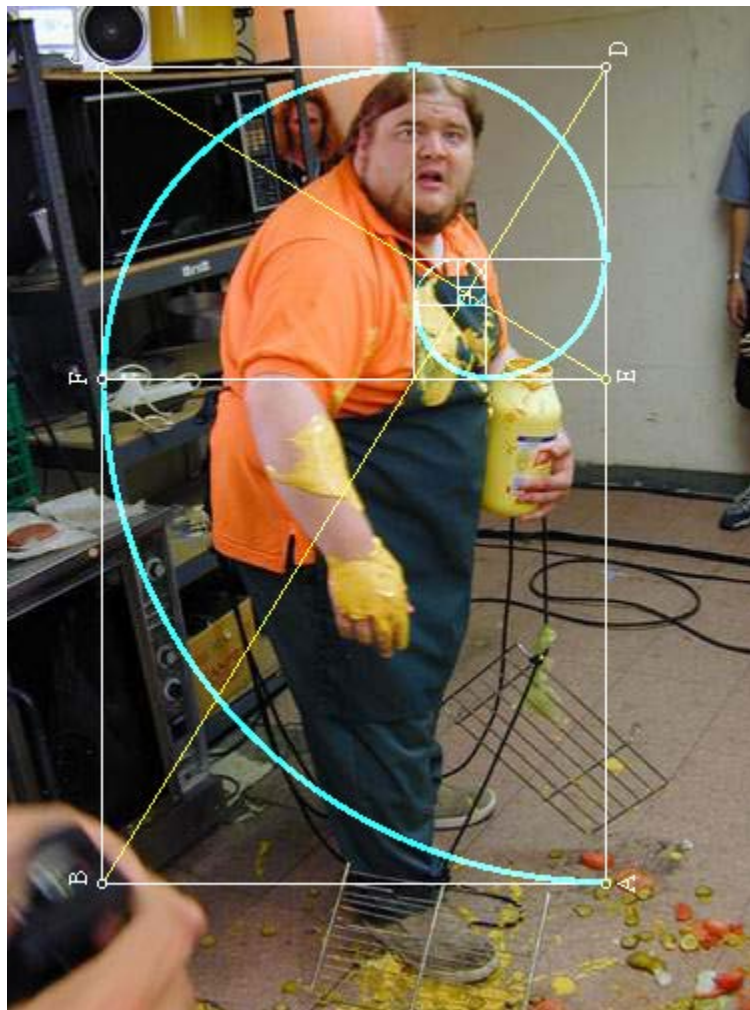
ShamanAKA11



Aj tu, aj tu, aj tu ...



Aj tu ???



DNA (deoxyribonukleová kyselina)

$$\frac{\text{výška twistu}}{\text{šířka twistu}} = \varphi = 1,6180339\dots$$



Ďakujem za pozornosť

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