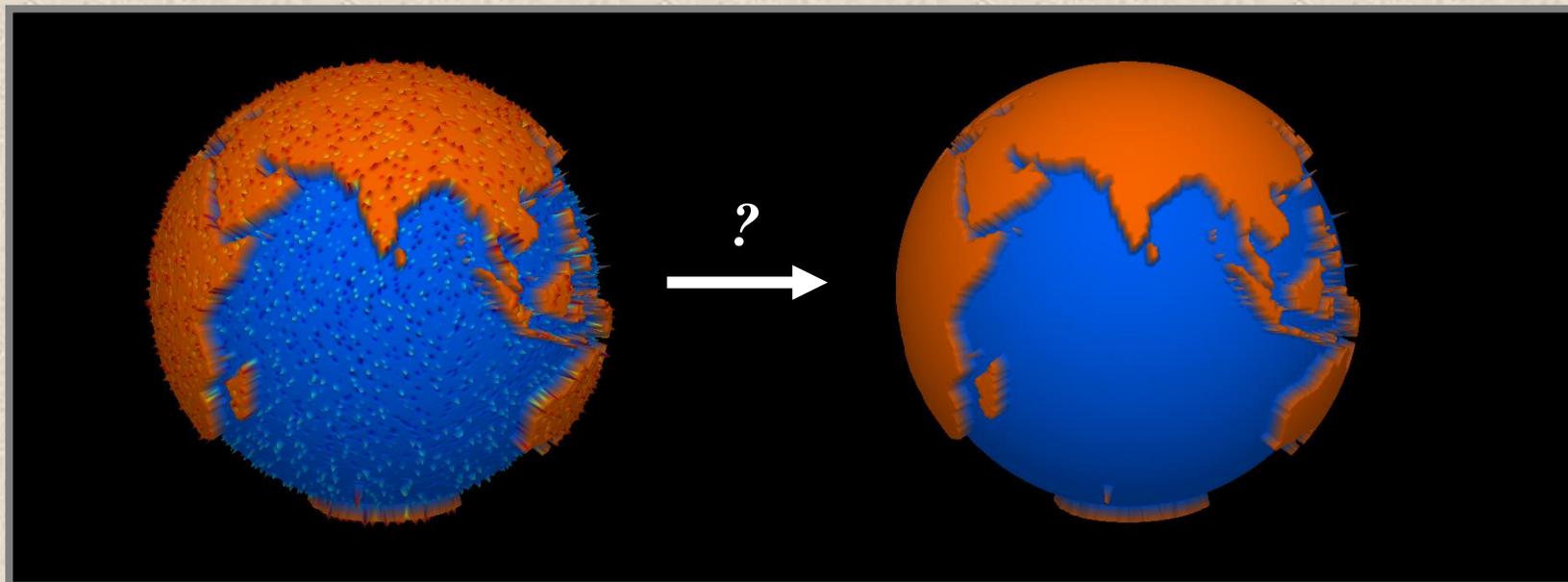


# *Filtrácia dát na povrchu Zeme a na orbitách družicových misií*

**Róbert Čunderlík, Karol Mikula**

[cunderli@svf.stuba.sk](mailto:cunderli@svf.stuba.sk)

[mikula@math.sk](mailto:mikula@math.sk)



# *Obsah*

- **Motivácia**
- **Difúzna filtrácia na uzavretých plochách**  
*(napr. na guli, elipsoide alebo na zemskom povrchu)*
- **Testovací príklad**
- **Filtrovanie priamych meraní družicovej misie GOCE**
- **Filtrovanie dynamickej topografie oceánov**



# Motivácia

- množstvo družicových misií monitorujúcich našu planétu  
⇒ merania sú ovplyvnené šumom

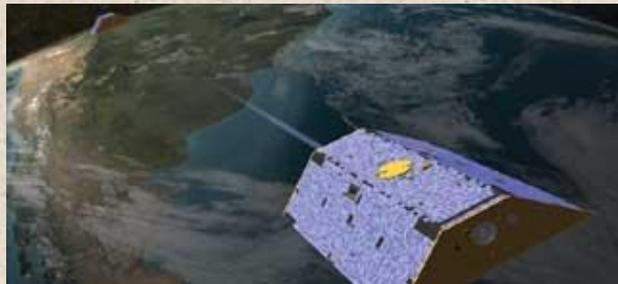
**Dôvody:** - prechod signálu atmosférou (troposférou a ionosférou)  
- nepresnosť polohy a orientácie družicovej misie  
- chyby samotného meracieho zariadenia

## Tiažové pole zeme a jeho časové variácie

**CHAMP** (2000-2010)



**GRACE** (2002-now)



**GOCE** (2009-2013)



## Družicová altimetria – variácia hladiny oceánov

- **Seasat** (1978), **Geosat** (1985), **TOPEX-Poseidon** (1992-2006)
- **ERS-2**, **Jason-1**, **ENVISAT**, **Jason-2**, ...



# Motivácia

## Vplyv oceánov na klímu

### Vplyv filtrácie dát na interpretáciu výsledkov

**The Oceans' Role in Climate**

**Jason-1**  
Measuring Ocean Surface Topography From Space

The Jason-1 satellite, an international mission with France, extends ocean topography measurements into the 21st century. Jason-1 continues the quest to better understand our planet through long-term monitoring of Earth's oceans.

CLIMATE RESEARCH

OFFSHORE INDUSTRIES

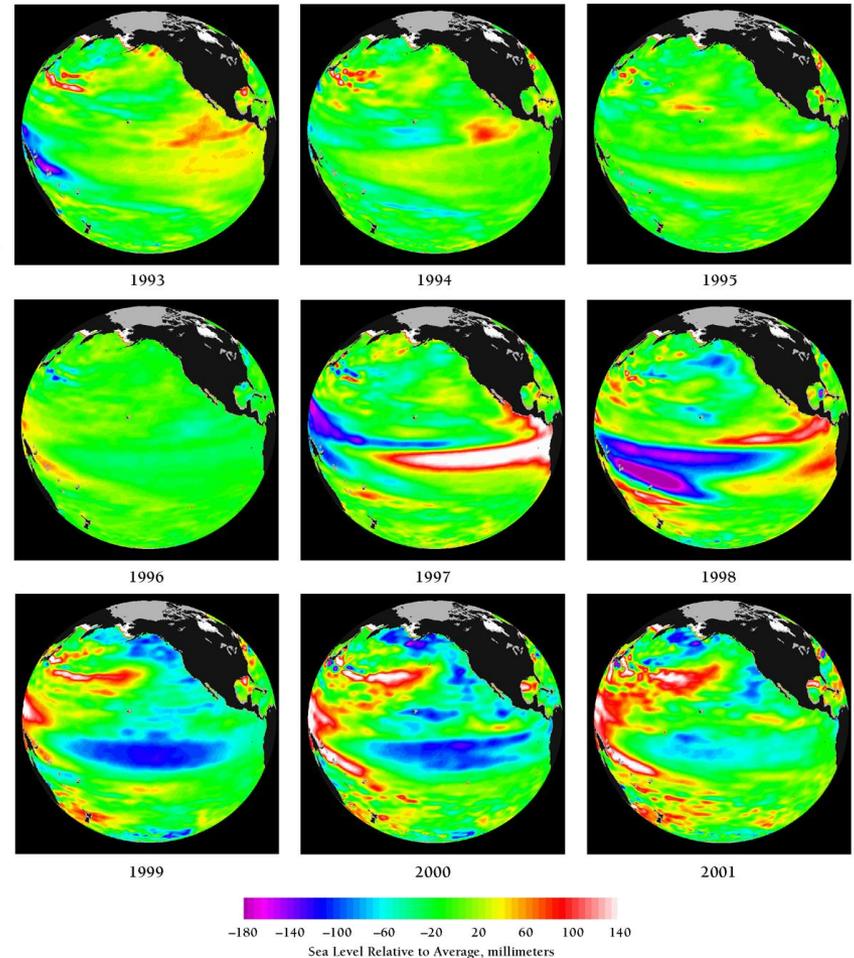
MARINE MAMMAL RESEARCH

SHIP ROUTING

TOPEX/Poseidon revolutionized our knowledge of ocean circulation and its effects on global climate change. This satellite provided measurements that led to early predictions of the great El Niño 1997-98, which caused extreme disruptions to weather patterns worldwide.

Jason-1 will build on TOPEX/Poseidon's monitoring of the Earth's oceans, continuing the goal of building long-term data sets of Earth's ocean surface topography.

<http://sealevel.jpl.nasa.gov>



# Difúzna filtrácia dát na uzavretých plochách

- filtrovanie šumu v dátach sa dá realizovať aj pomocou riešenia tzv. rovnice vedenia tepla  
⇒ často sa používa v spracovaní obrazu (image processing)

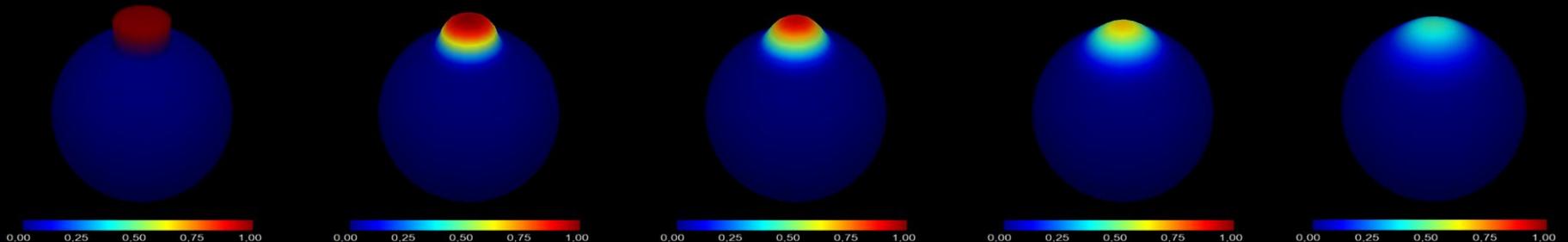
## Rovnica vedenia tepla na ploche

$$\frac{\partial u(x,t)}{\partial t} - \Delta_S u(x,t) = 0$$

$\Delta_S$  - the Laplace–Beltrami operator (druhé derivácie)  
 $u(x,t)$  – skalárna funkcia daná na ploche

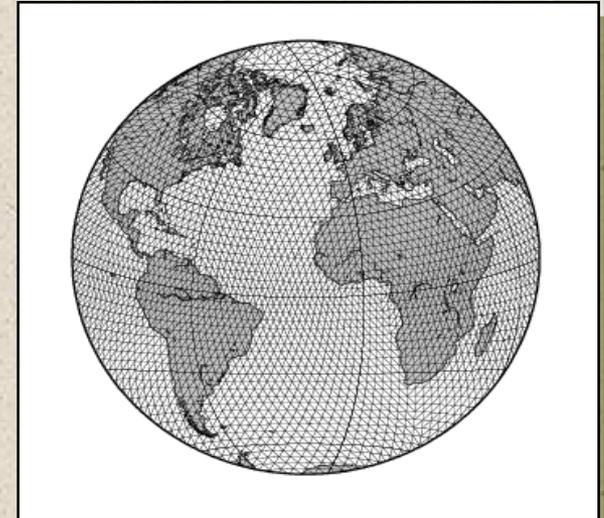
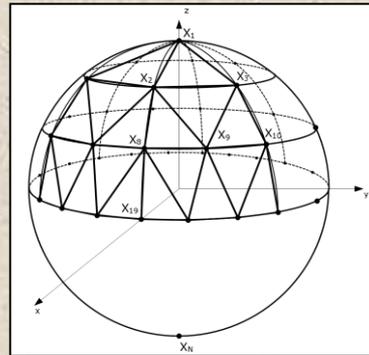
- vstupné zašumené dáta ako počiatočná podmienka:

$$u(x,0) = u_o(x)$$



# Numerické riešenie metódou konečných objemov

## Triangulácia uzavretej plochy

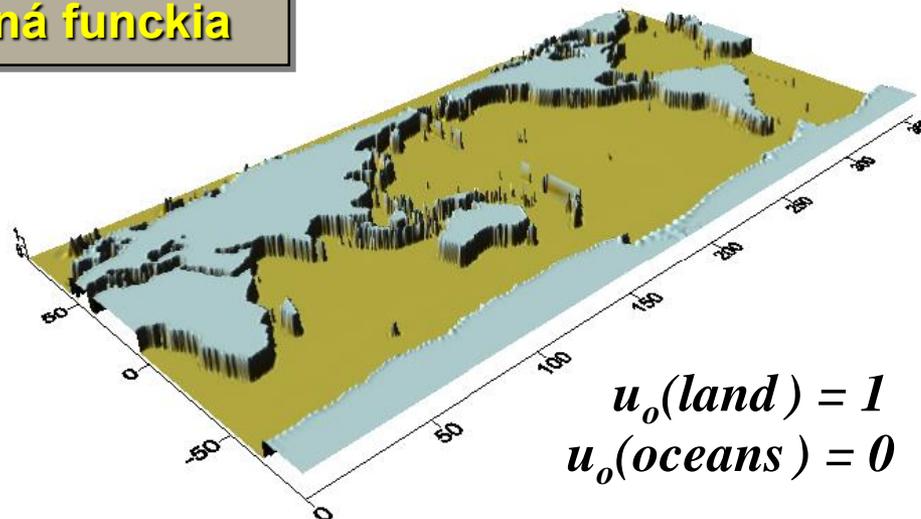
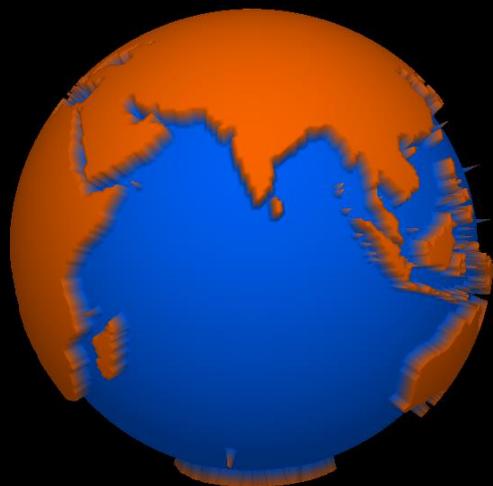


## Riešenie systému lineárnych rovníc

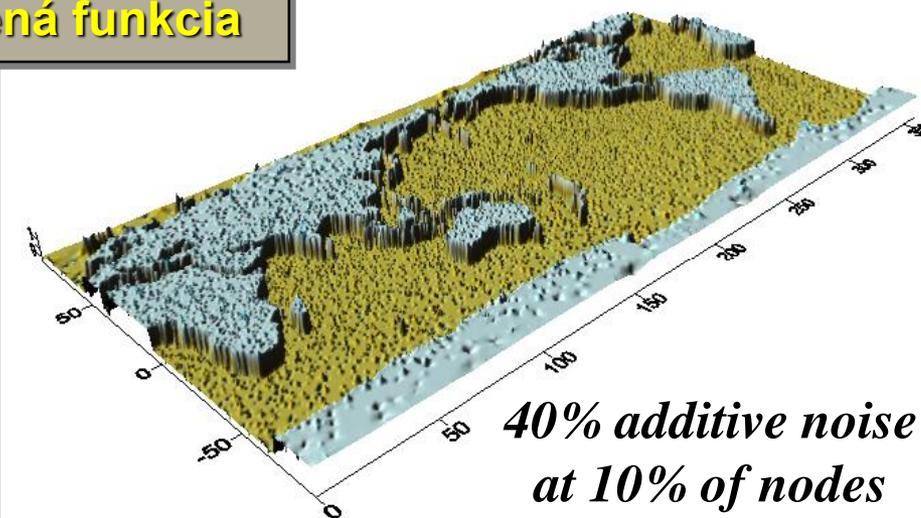
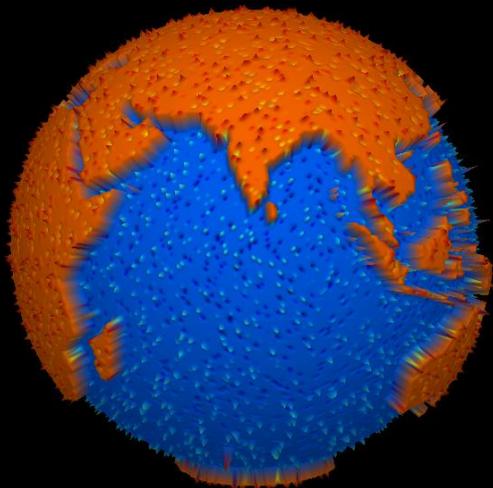
- lineárna difúzia  $\Rightarrow$  riešenia rovnice vedenia tepla (zhladzuje aj dôležité prvky)
- nelineárna difúzia  $\Rightarrow$  riešenia modifikovanej rovnice vedenia tepla (umožňuje zachovávať dôležité prvky)

# Testovací príklad

Pôvodná funkcia

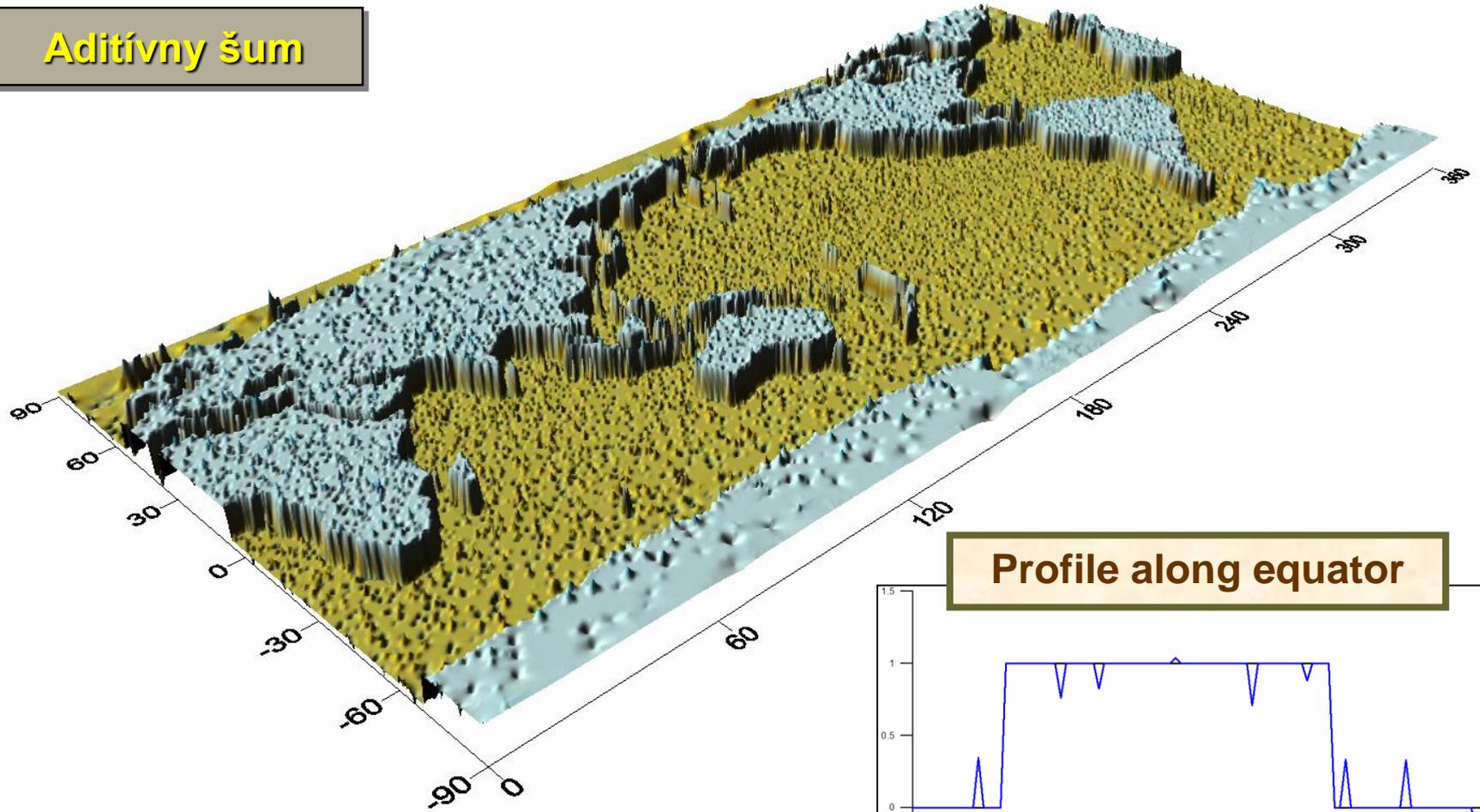


Zašumená funkcia

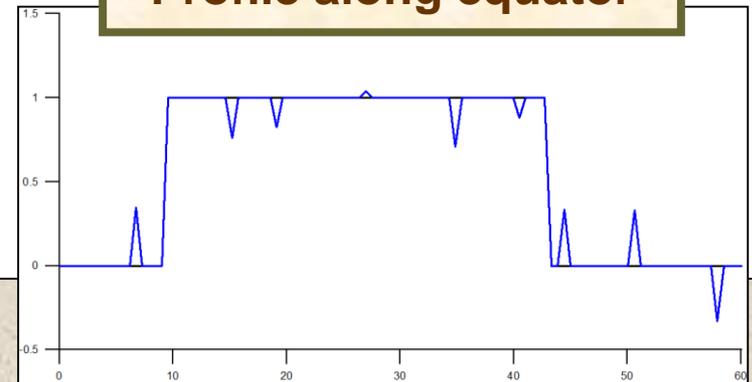


# Lineárna difúzia – vstupné dáta

Aditívny šum

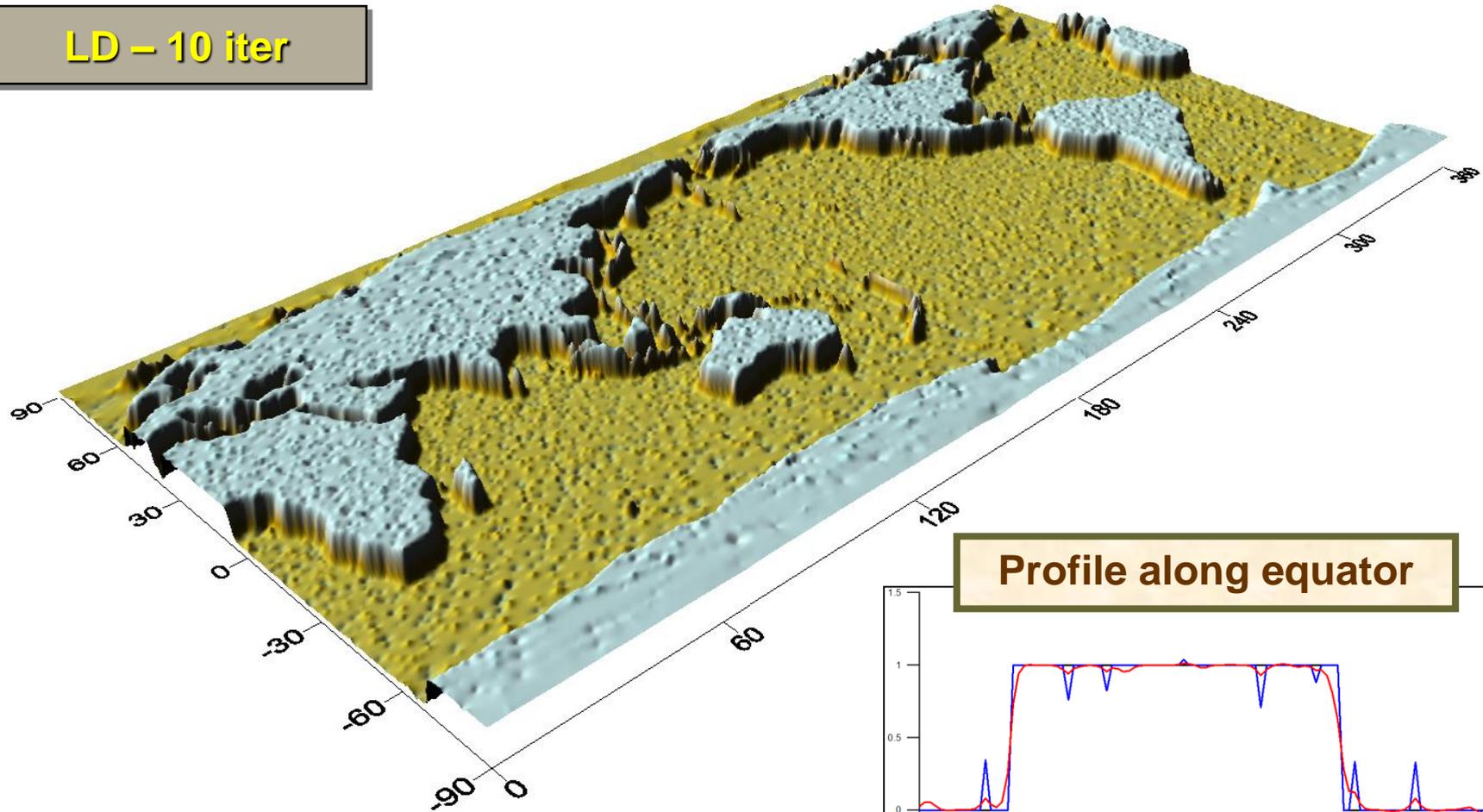


Profile along equator

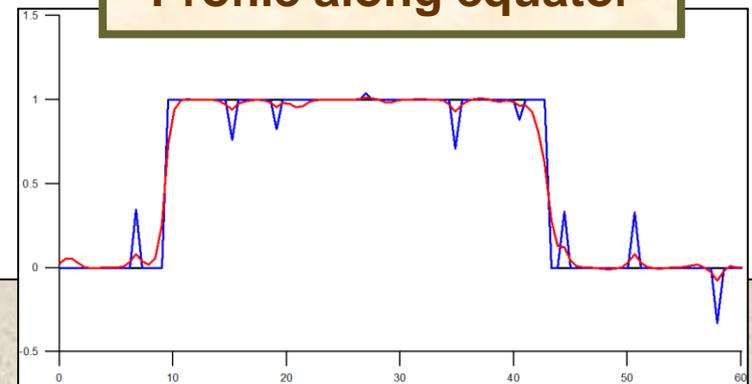


# *Linear diffusion – 10 iterations*

LD – 10 iter

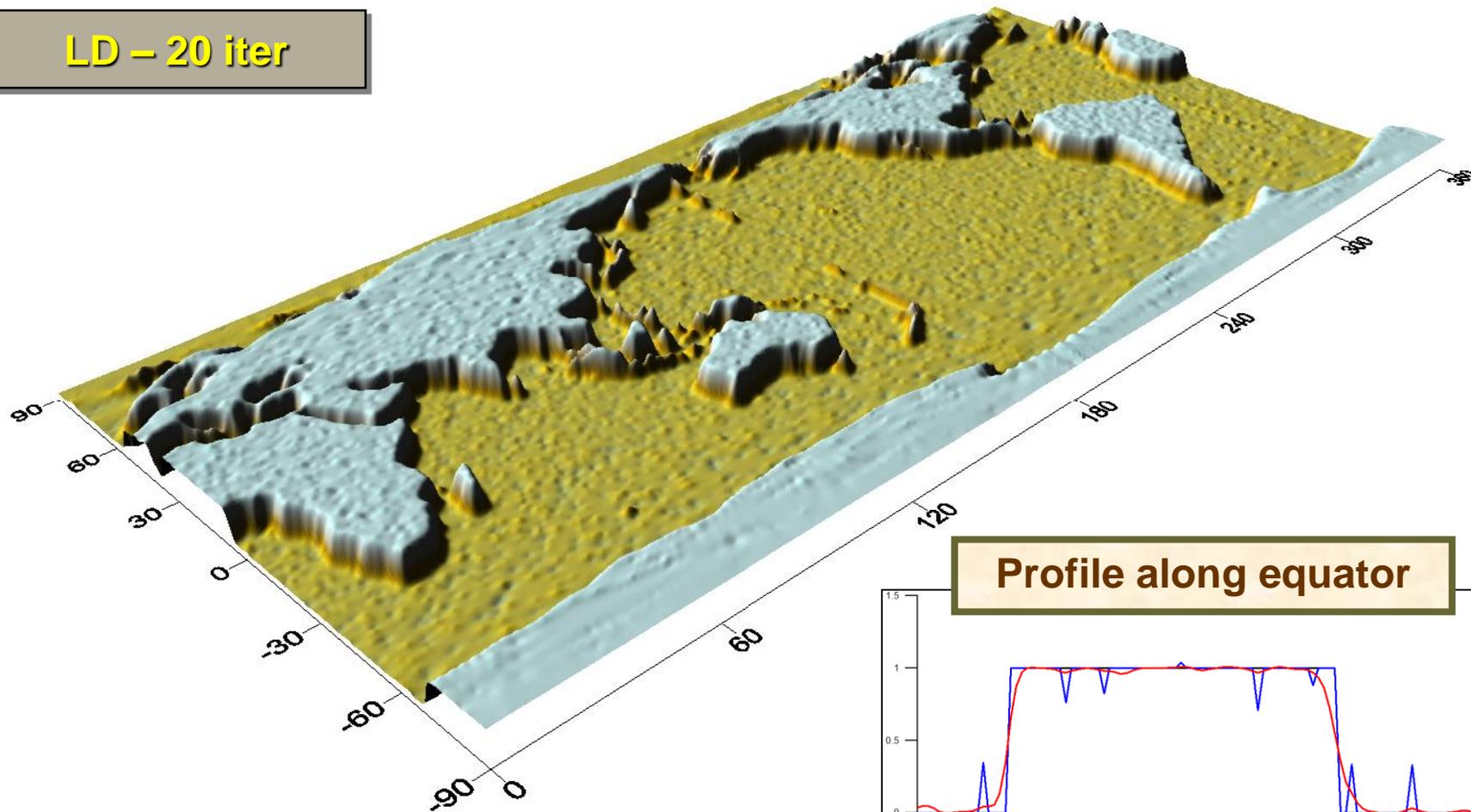


Profile along equator

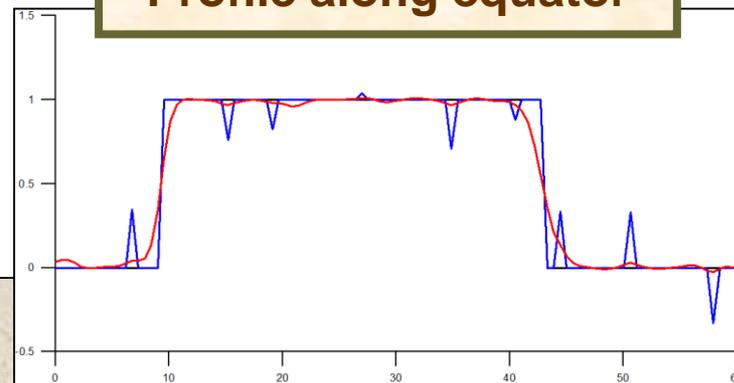


# *Linear diffusion – 20 iterations*

LD – 20 iter

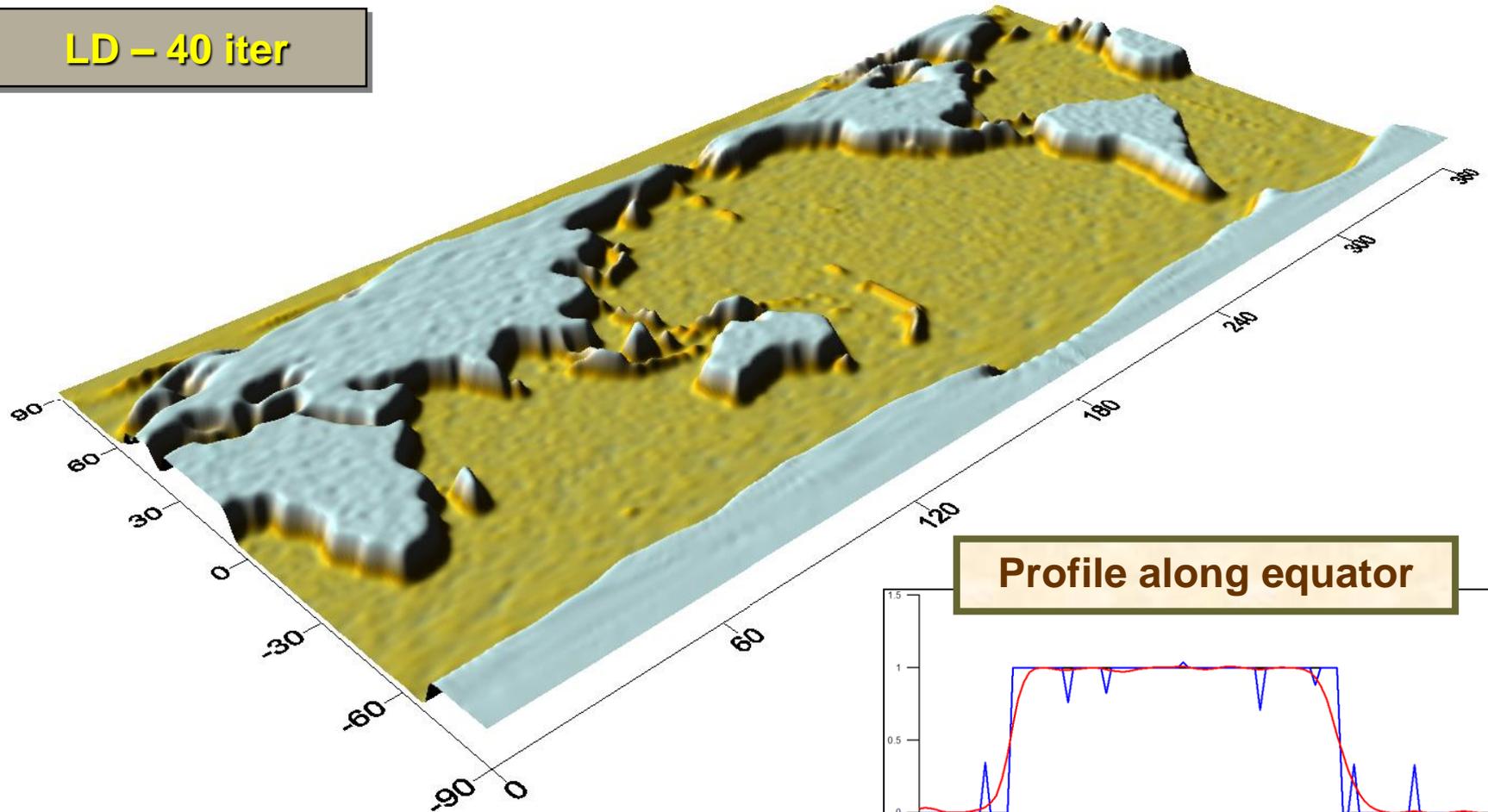


Profile along equator

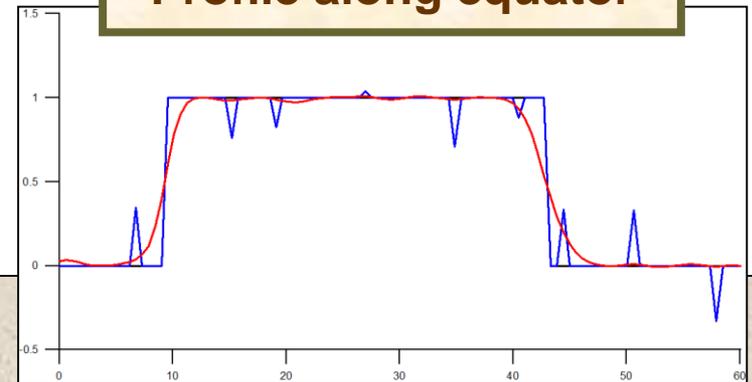


# Linear diffusion – 40 iterations

LD – 40 iter

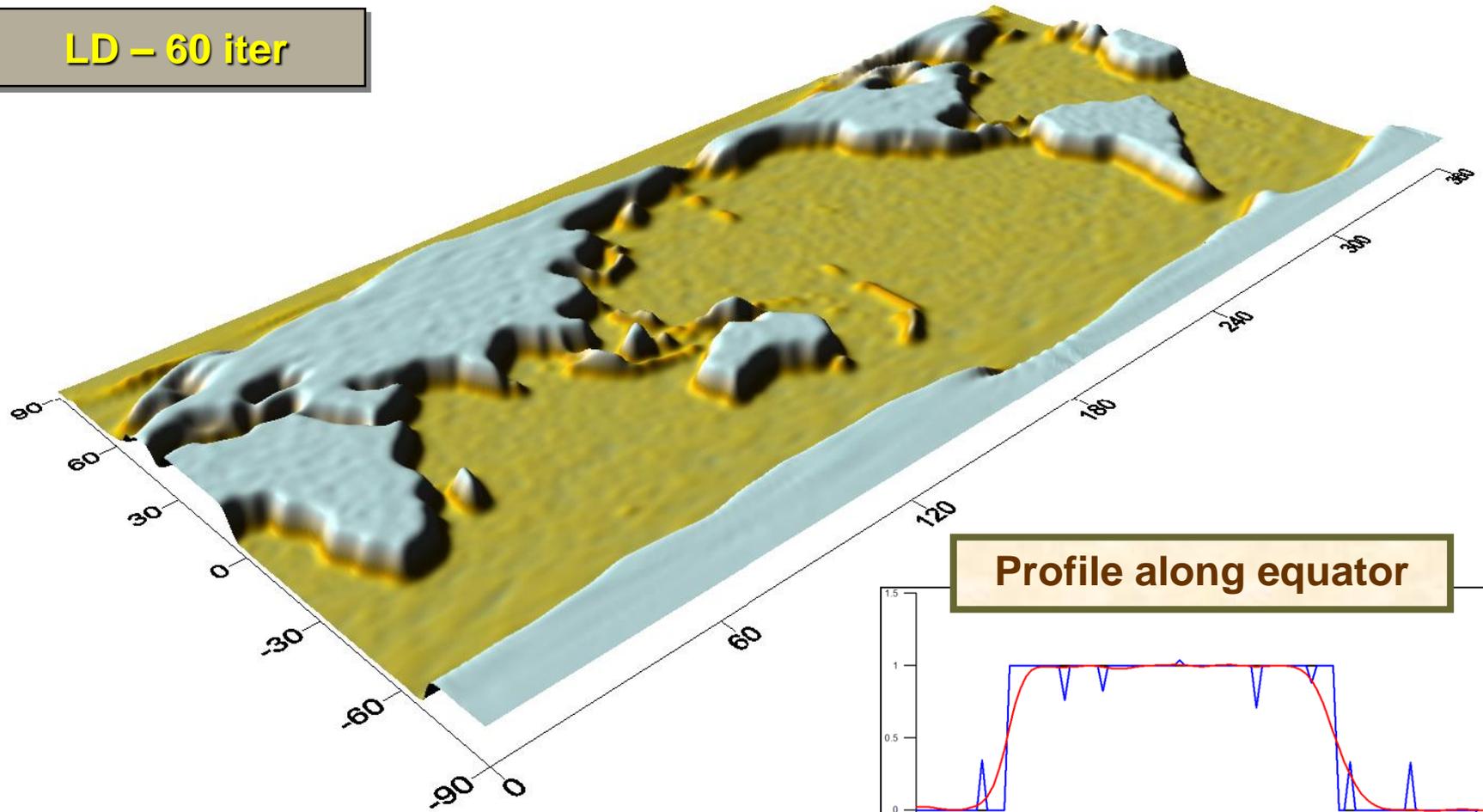


Profile along equator

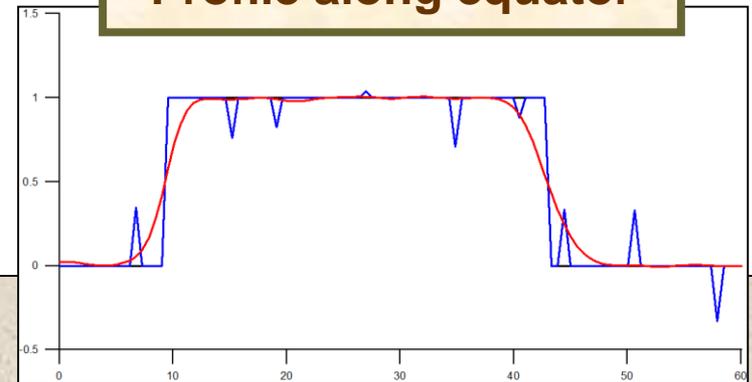


# *Linear diffusion – 60 iterations*

LD – 60 iter

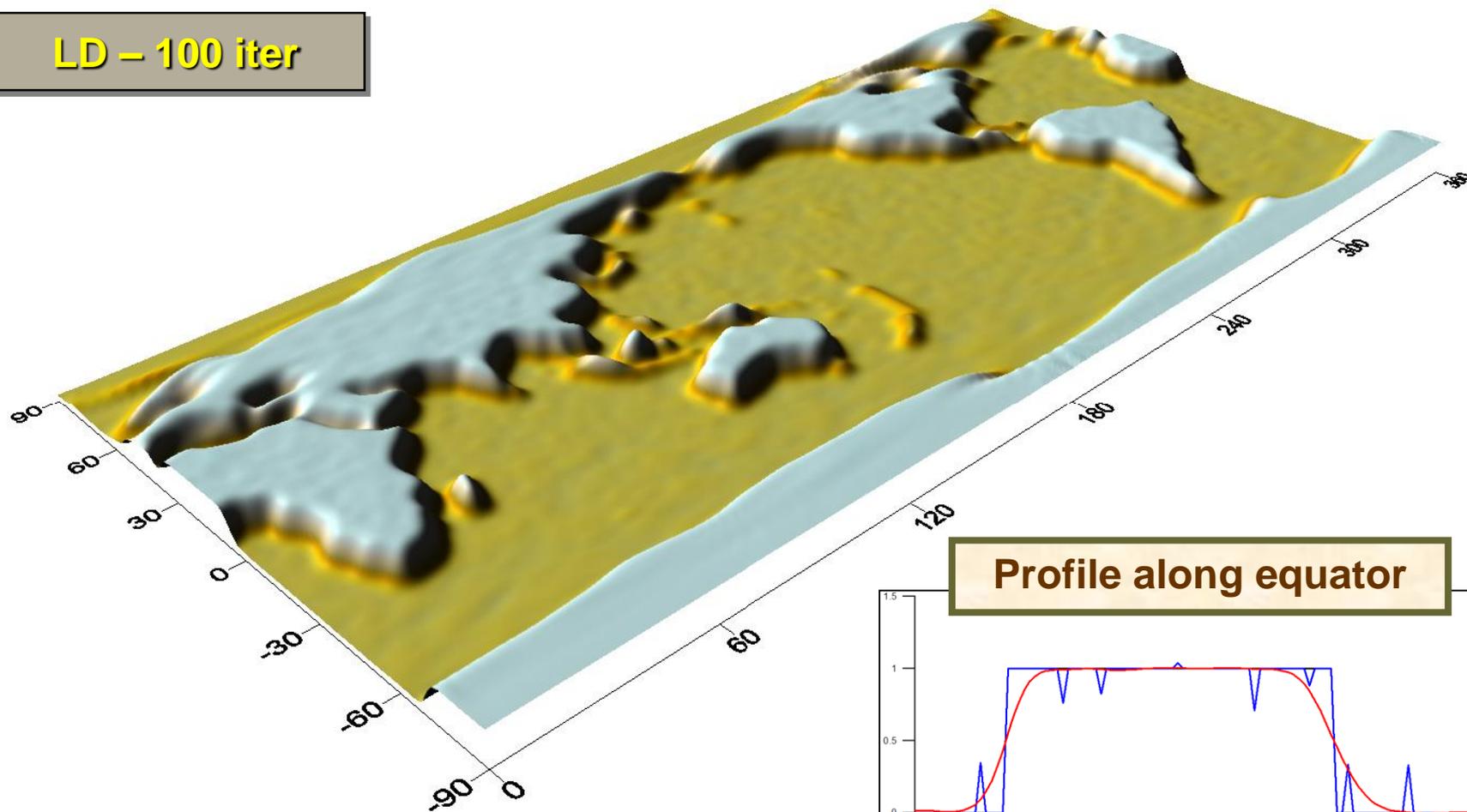


Profile along equator

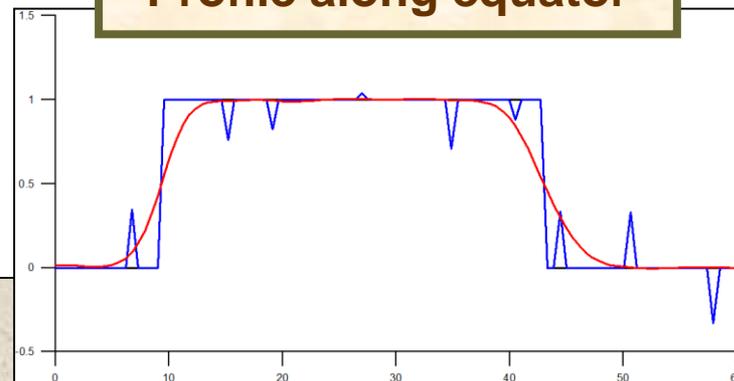


# Linear diffusion – 100 iterations

LD – 100 iter



Profile along equator

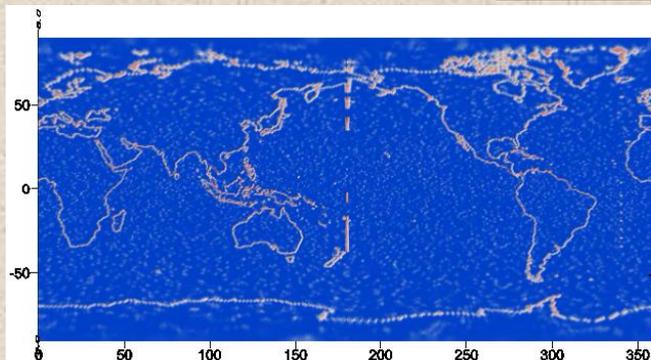


# Nelinárna difúzia – hranový detektor

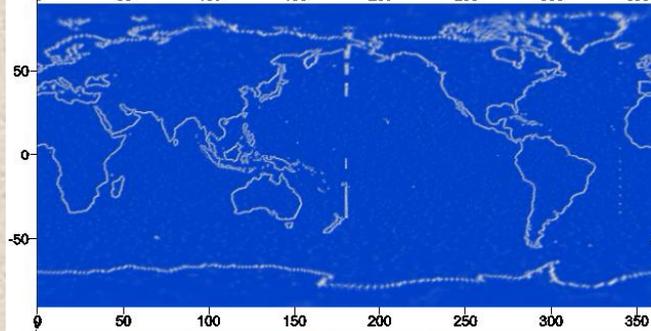
Hranový detektor

$$g(v) = \frac{1}{1 + H|\nabla_s u^\sigma|^2}$$

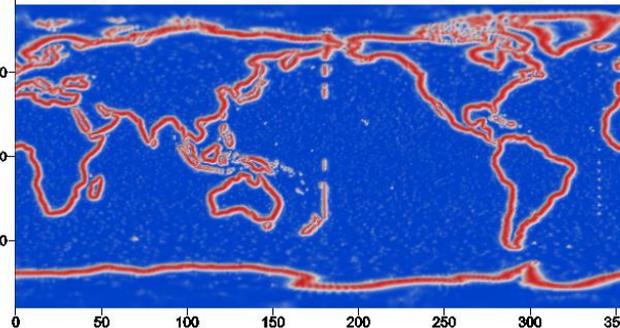
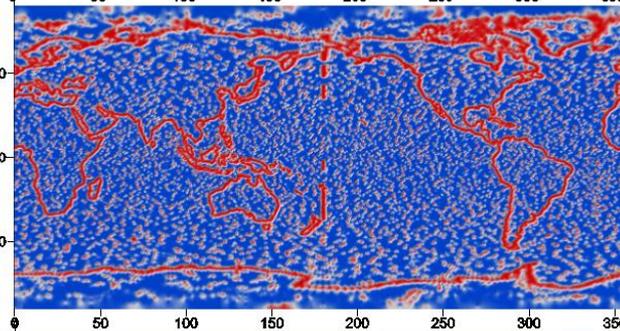
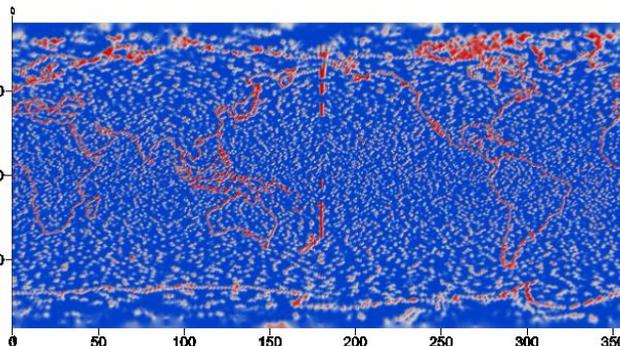
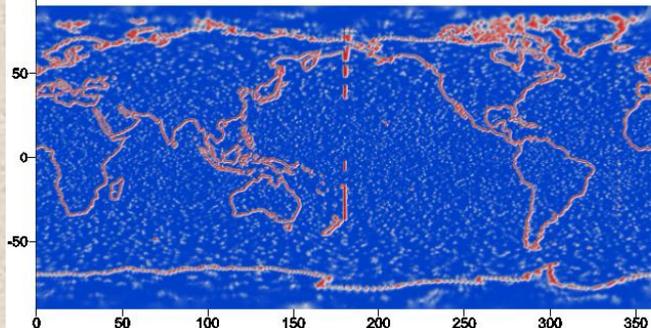
$\sigma = \tau$   
 $H = 10^{10}$



$\sigma = 10\tau$   
 $H = 10^{10}$



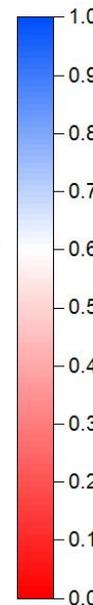
$\sigma = 10\tau$   
 $H = 10^{11}$



$\sigma = \tau$   
 $H = 10^{11}$

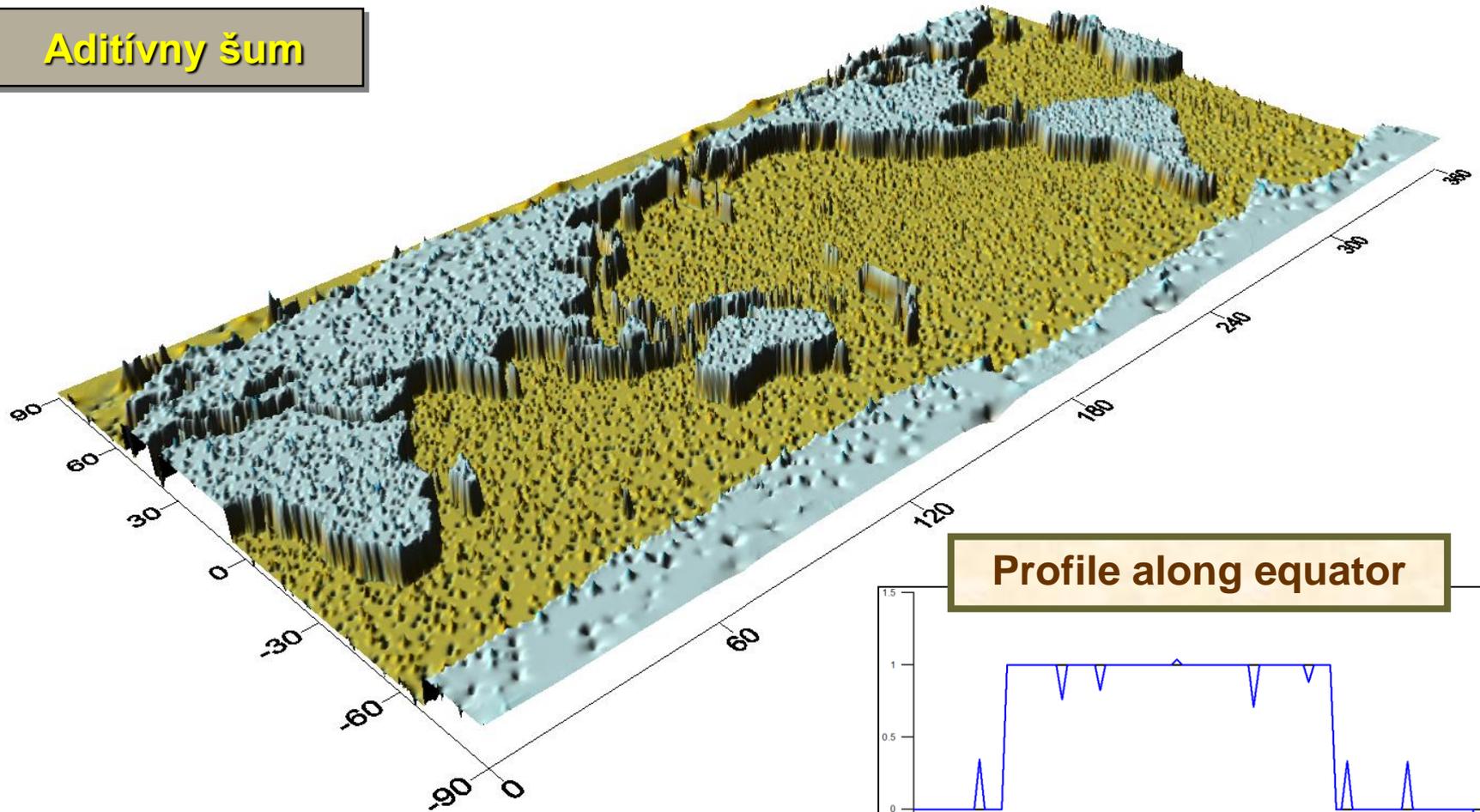
$\sigma = 10\tau$   
 $H = 10^{12}$

$\sigma = 100\tau$   
 $H = 10^{12}$

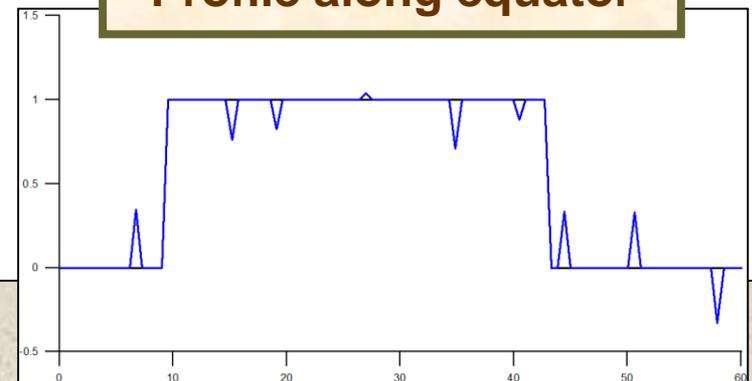


# *Nelineárna difúzia – vstupné dáta*

**Aditívny šum**

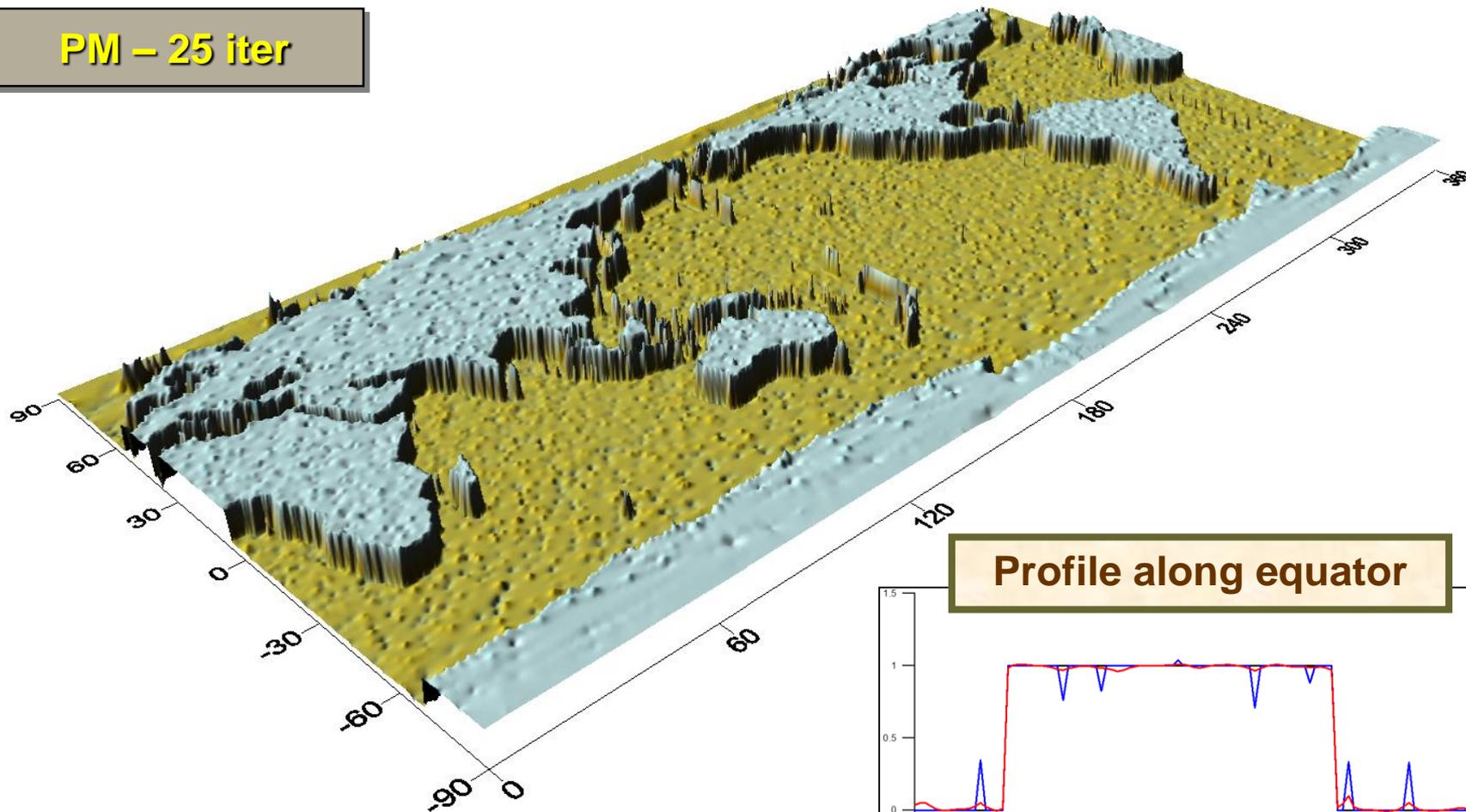


**Profile along equator**

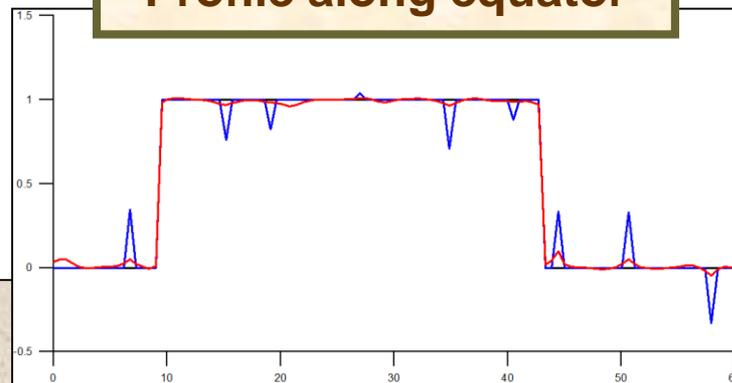


# *Nonlinear diffusion – 25 iterations*

PM – 25 iter

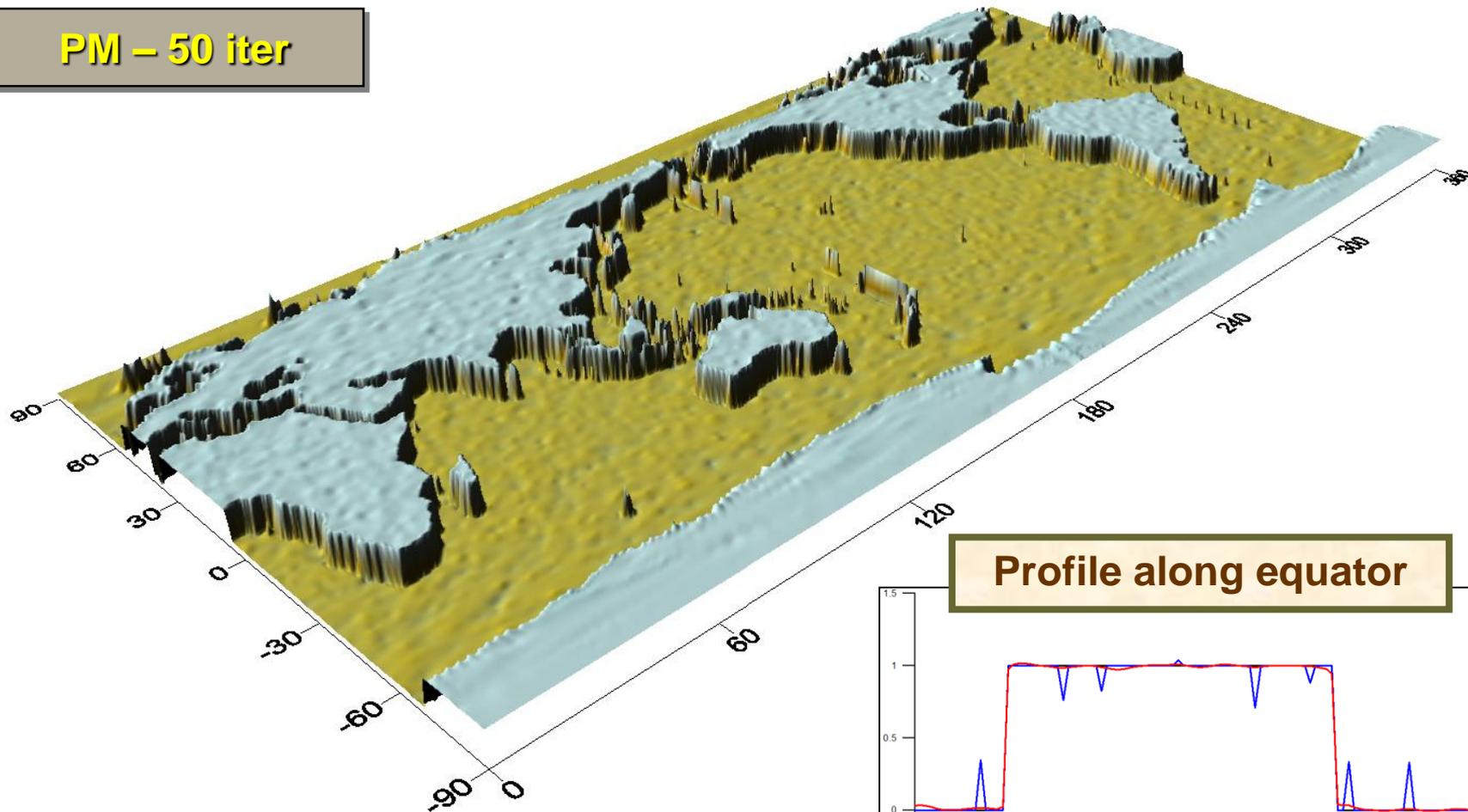


Profile along equator

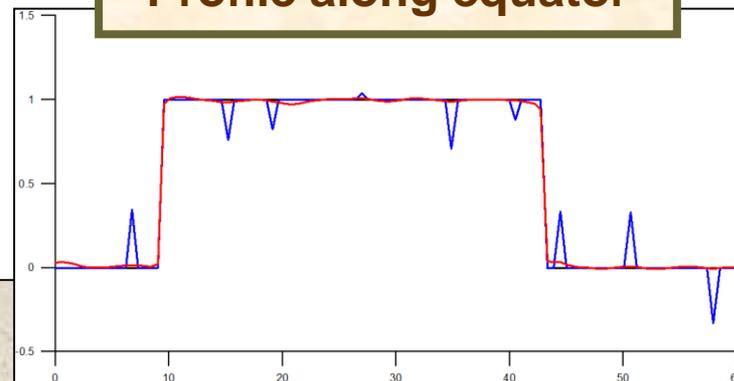


# *Nonlinear diffusion – 50 iterations*

PM – 50 iter

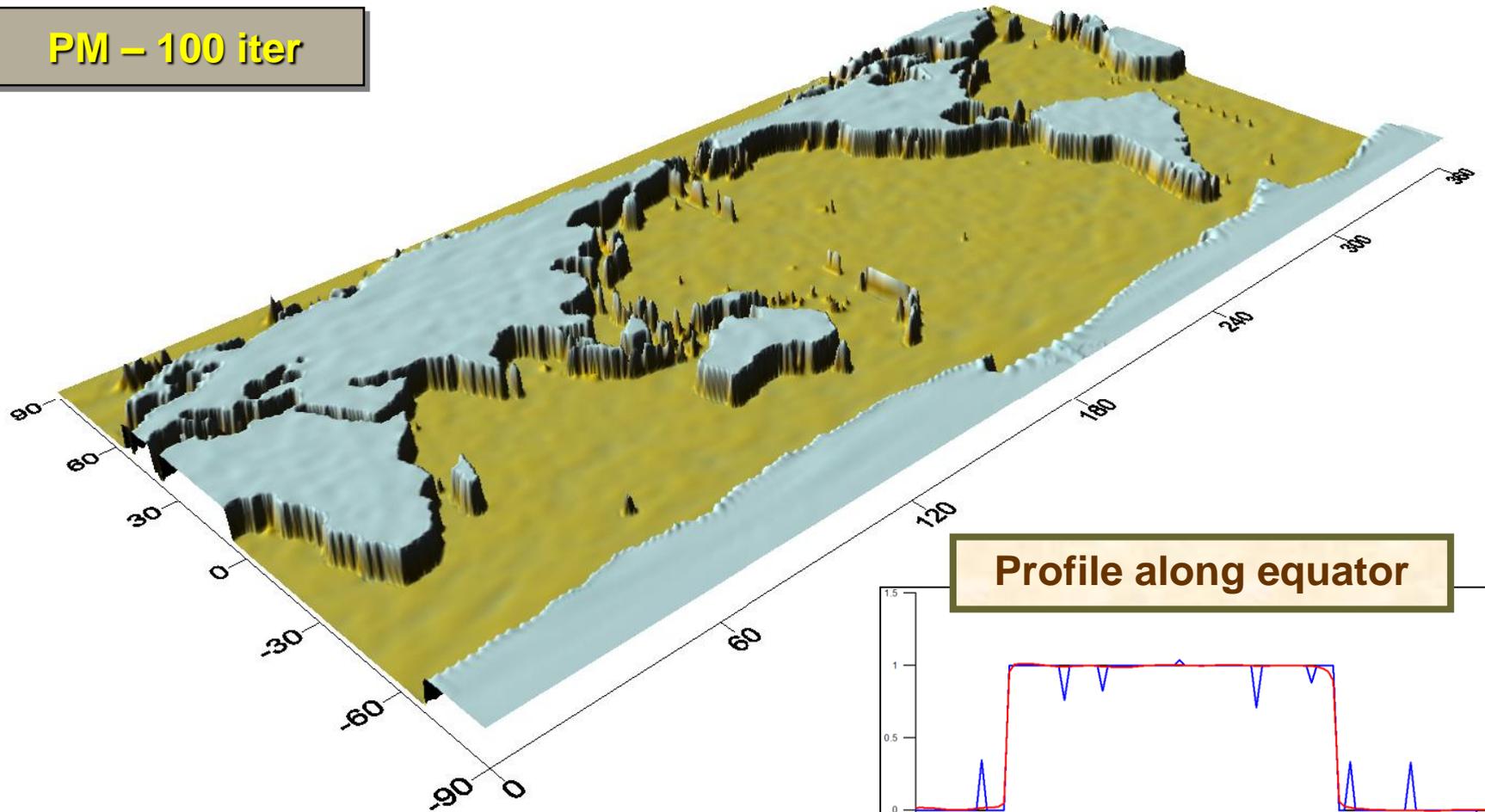


Profile along equator

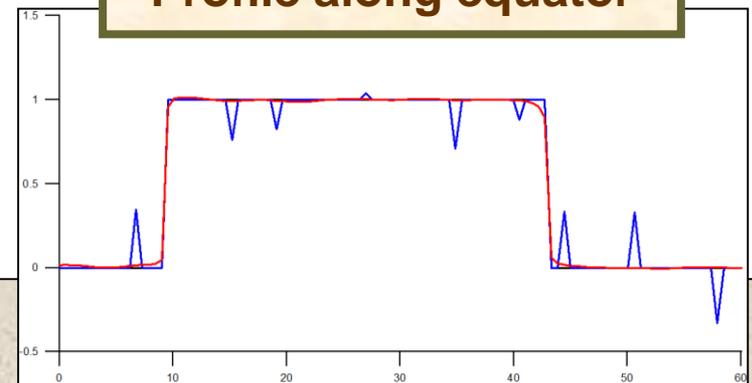


# *Nonlinear diffusion – 100 iterations*

PM – 100 iter

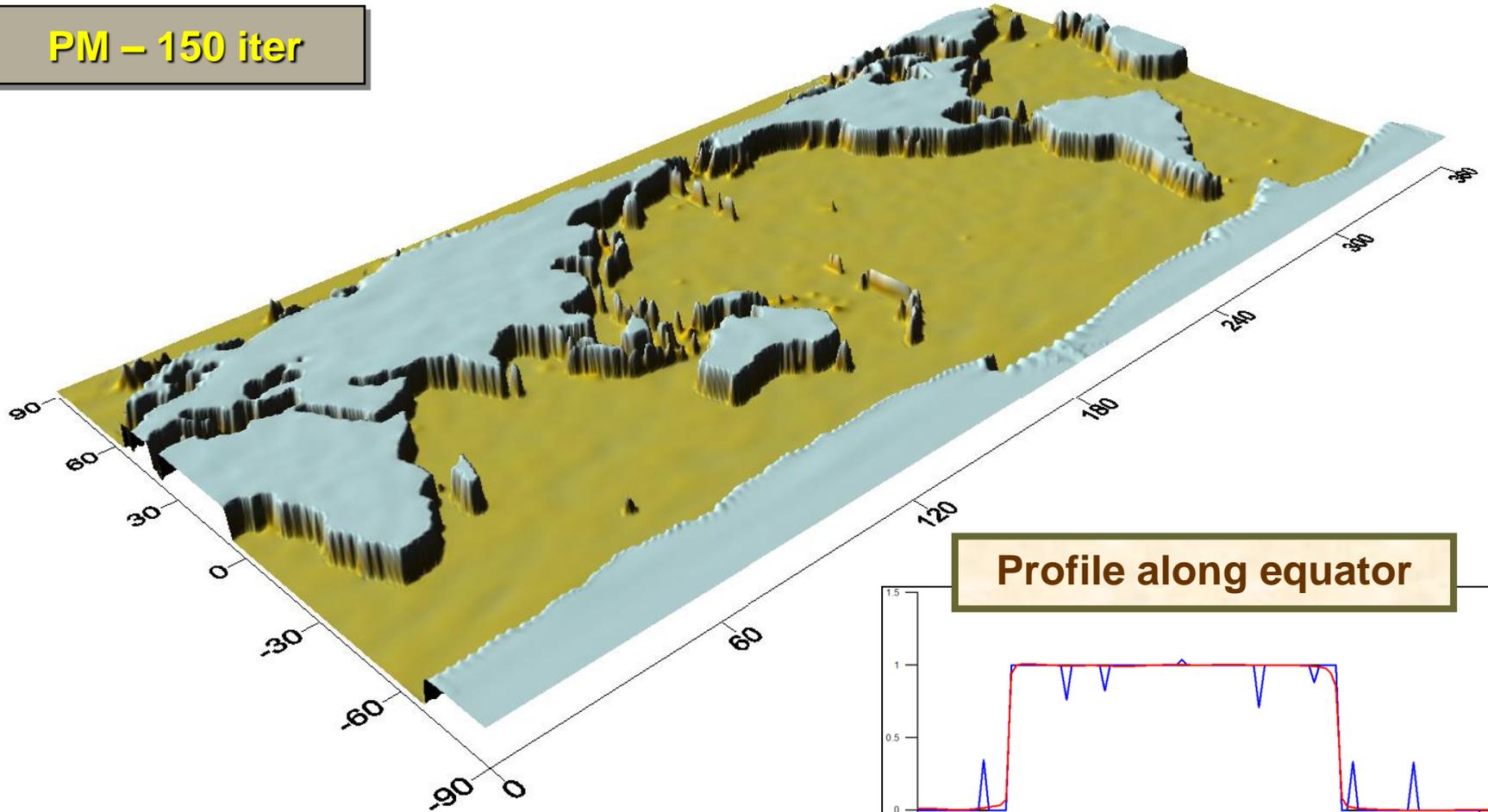


Profile along equator

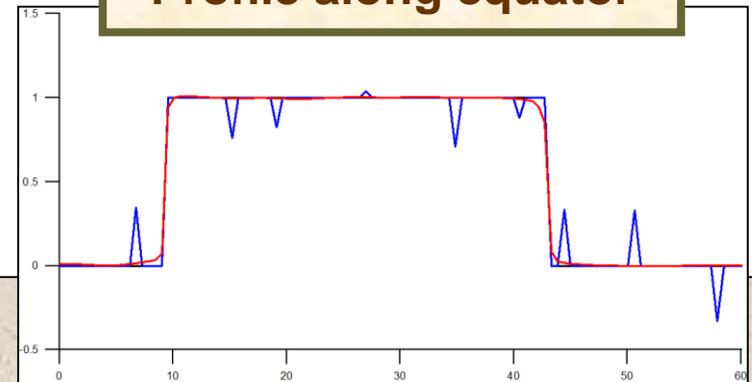


# *Nonlinear diffusion – 150 iterations*

PM – 150 iter

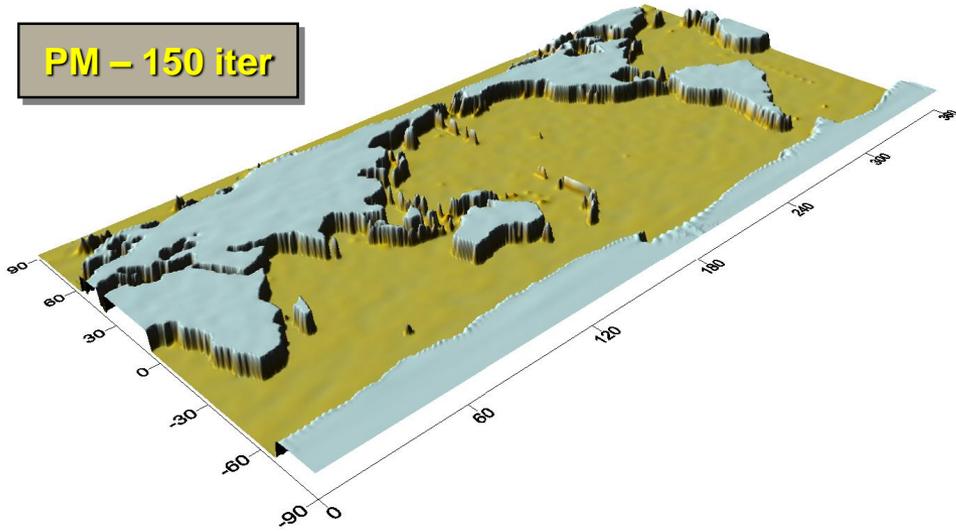


Profile along equator

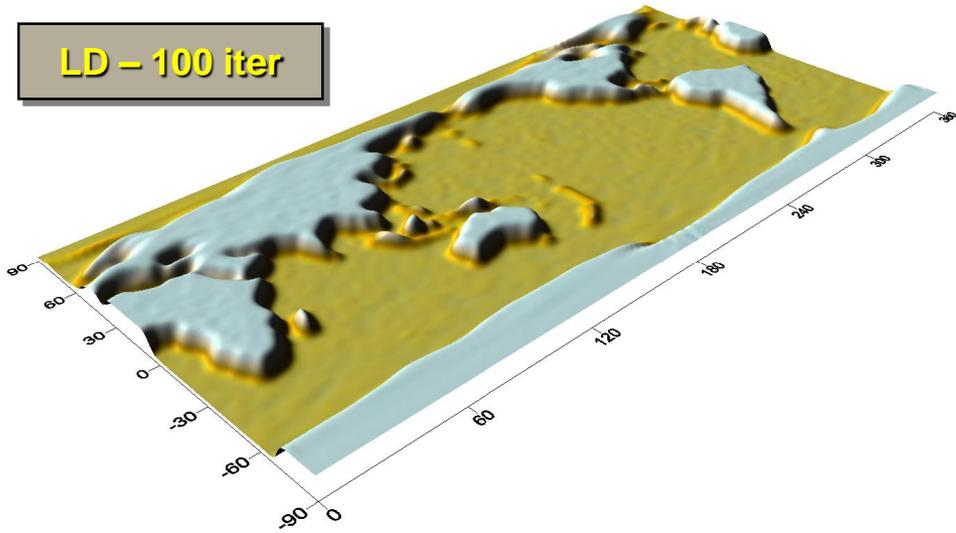


# Porovnanie: lineárna ↔ nelineárna difúzia

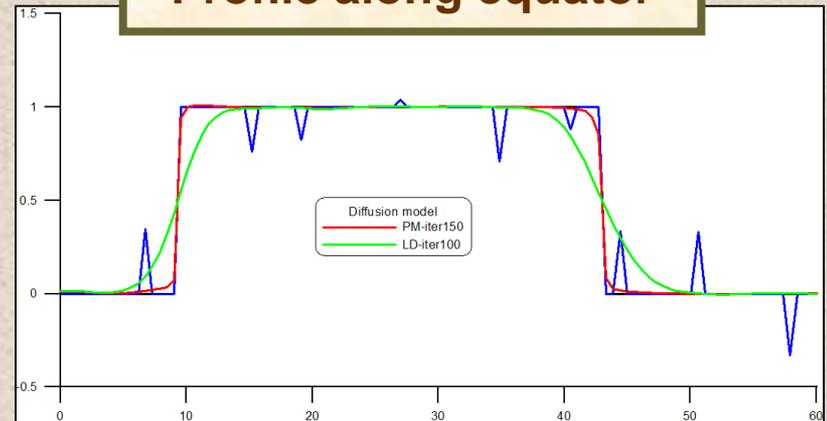
PM – 150 iter



LD – 100 iter

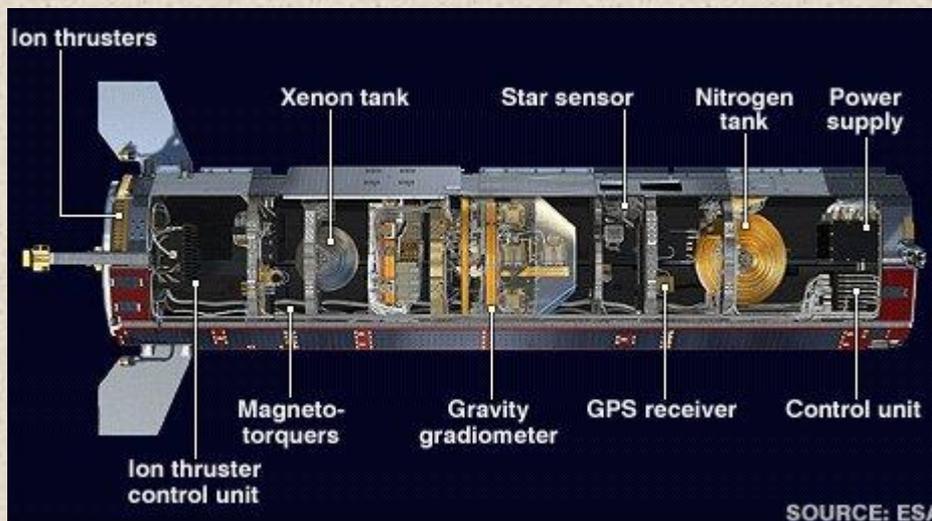


Profile along equator



# Družicová misia GOCE

## Gravity Field and Stady-State Ocean Circulation Explorer



$$\text{grad}(\text{grad}V) = \text{grad}(\bar{g}) =$$

$$= \begin{bmatrix} V_{xx} & V_{xy} & V_{xz} \\ & V_{yy} & V_{yz} \\ & & V_{zz} \end{bmatrix}$$

### Vypustenie družice

(17 marec 2009)



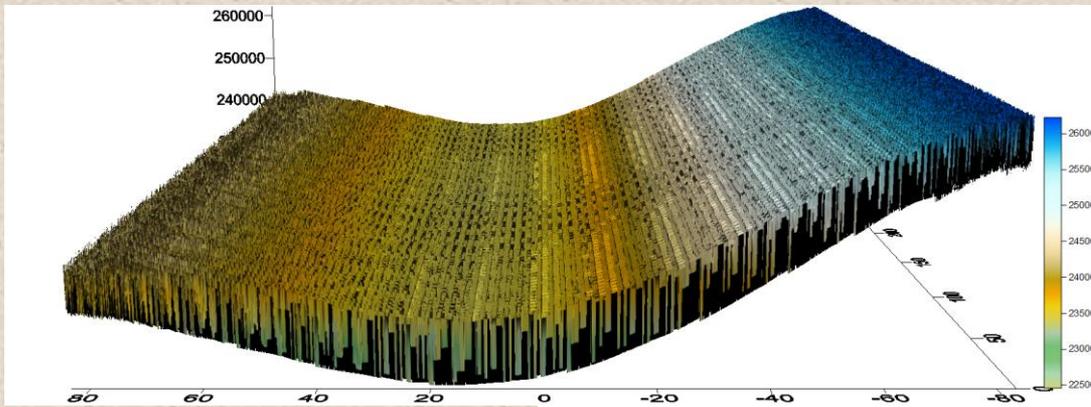
### Koniec misie

(11 november 2013)



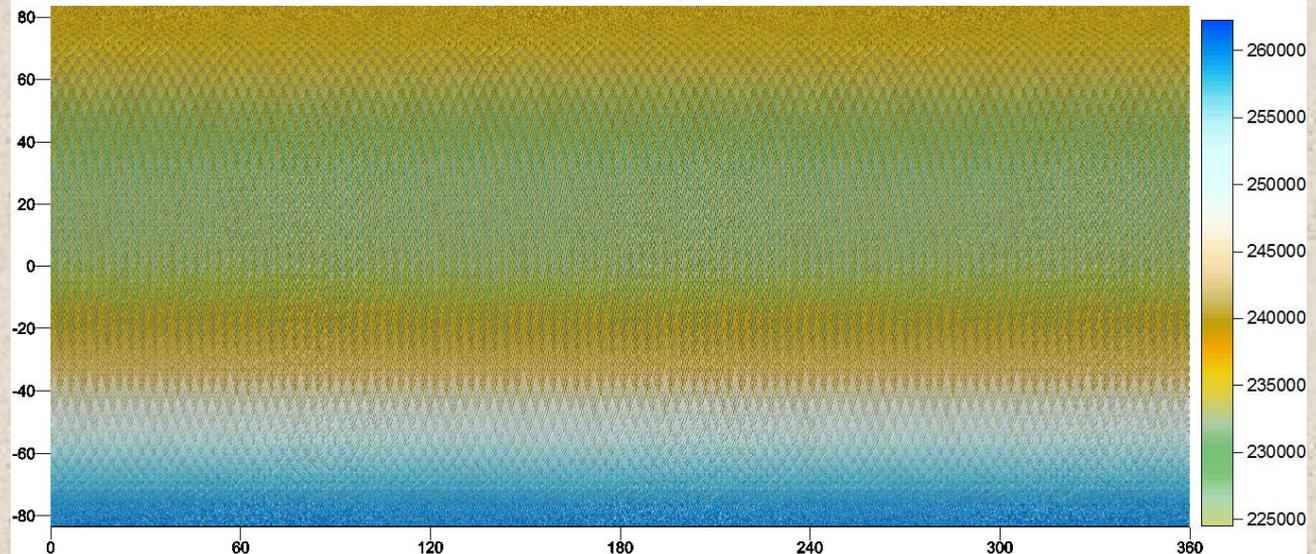
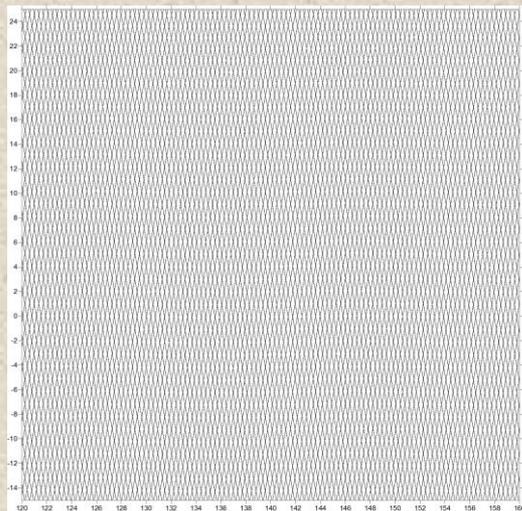
# Orbity družicovej misie GOCE

## Výška letu nad elipsoidom



## Hustota meraní (61 dní)

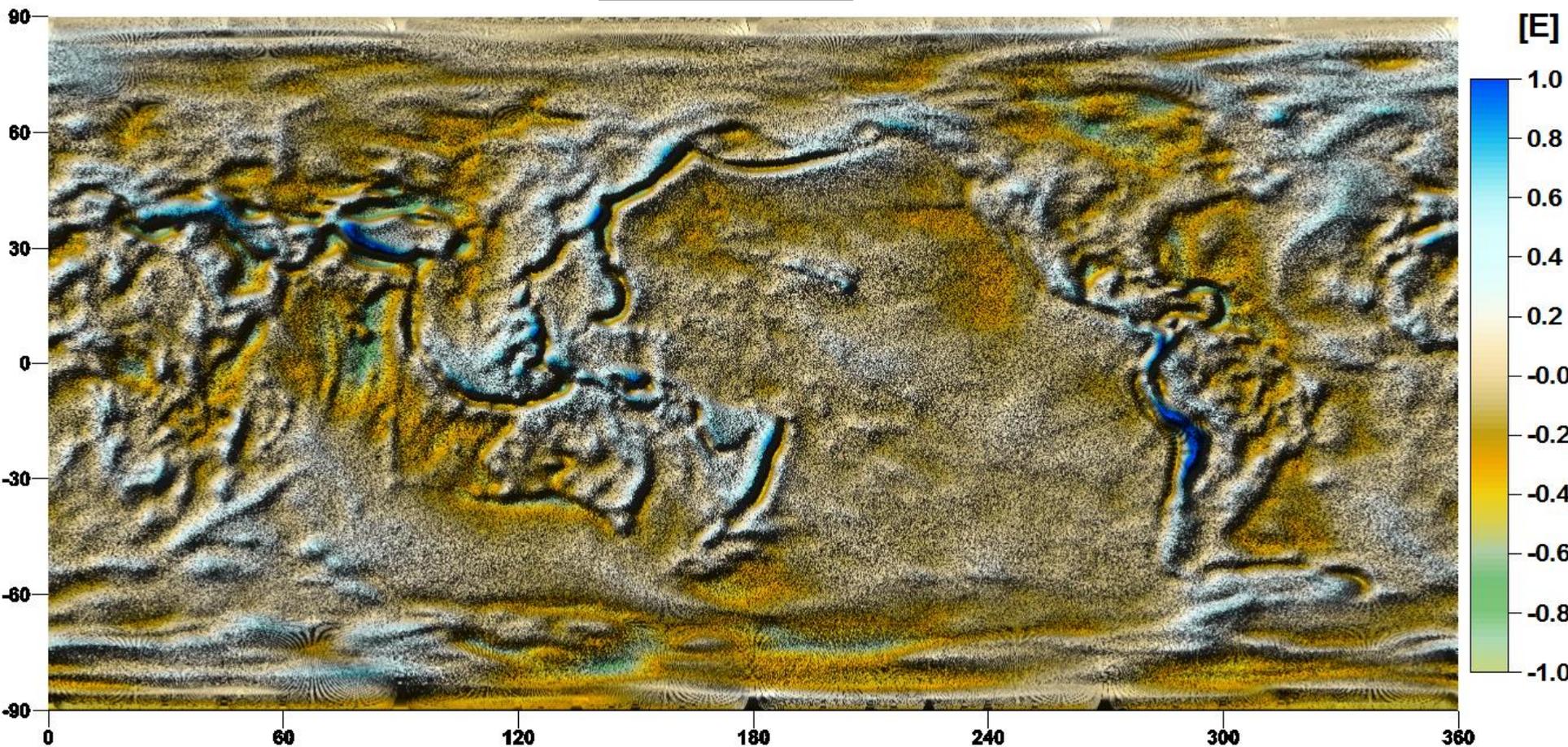
06 -07-2013



# Priame merania GOCE

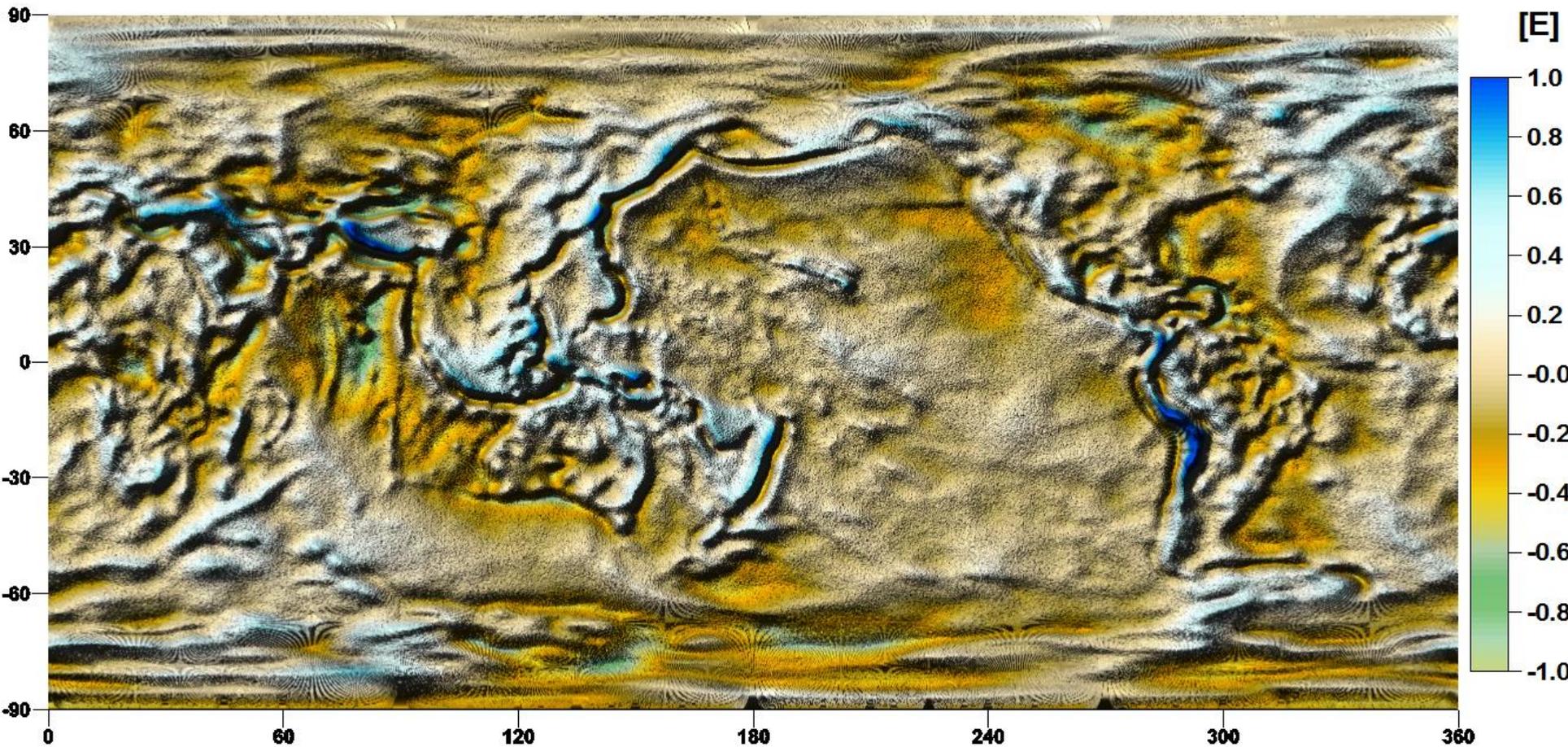
$T_{zz}$  komponent tiažového tenzora

$$T_{zz} = V_{zz} - U_{zz}$$



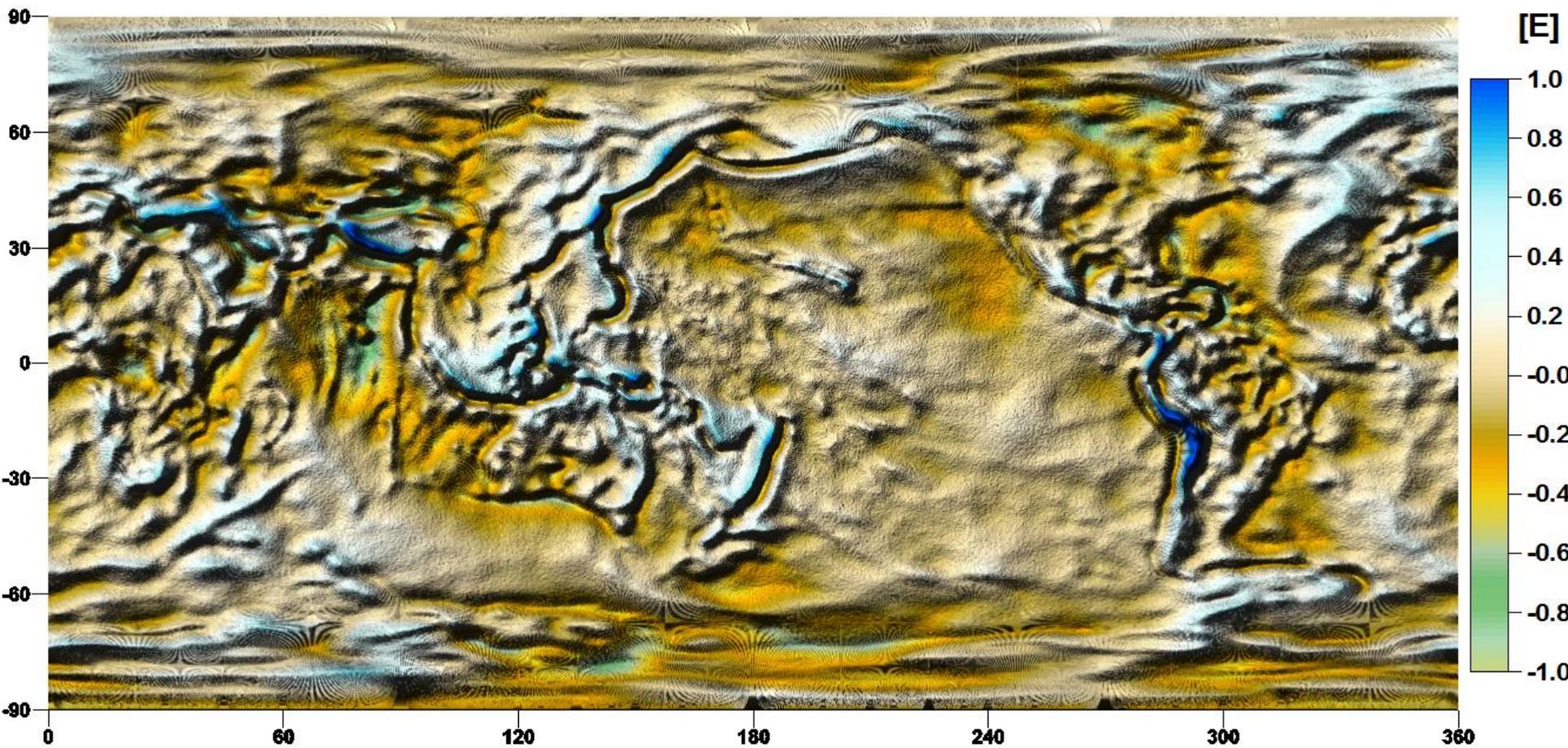
# *Nonlinear diffusion*

10 iterations



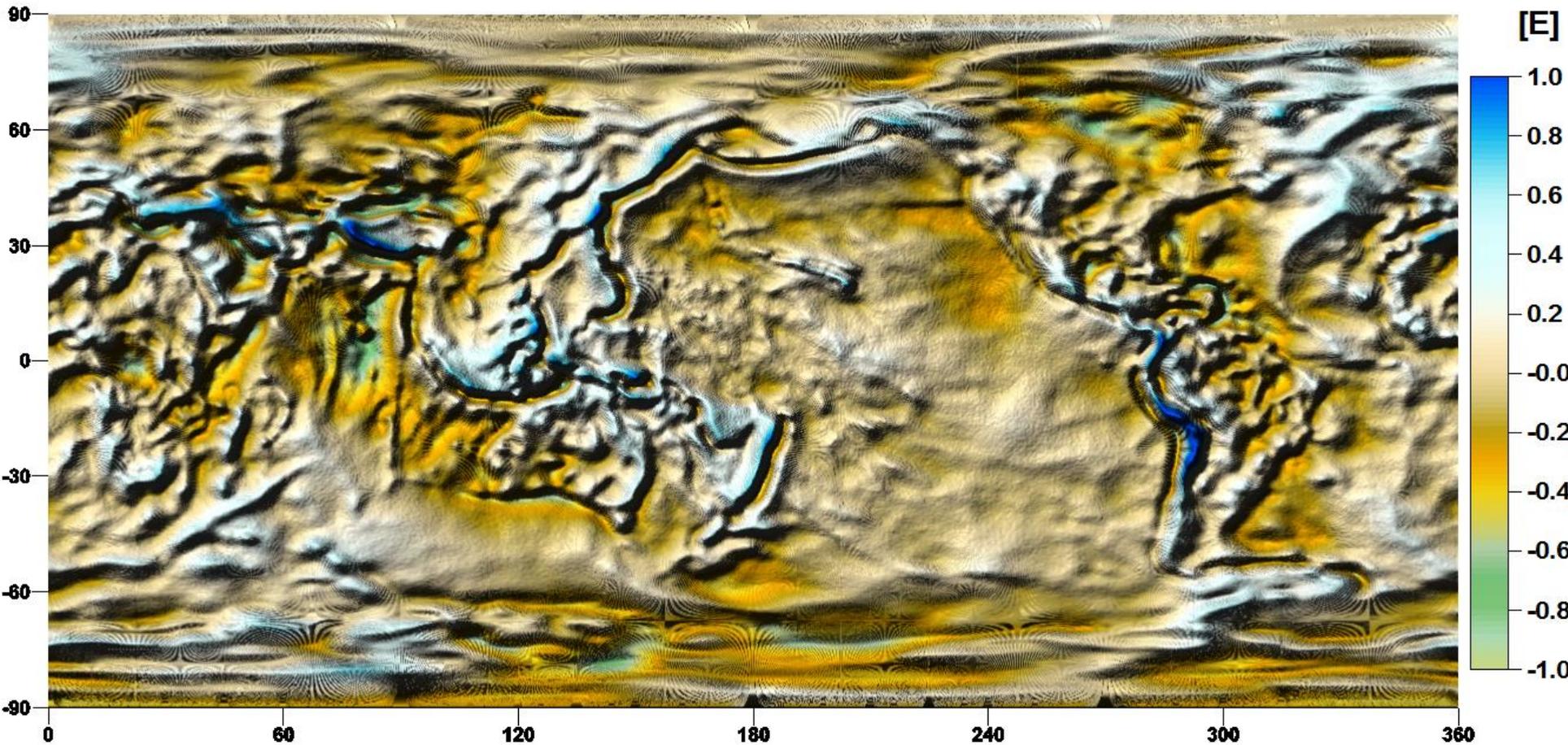
# *Nonlinear diffusion*

20 iterations



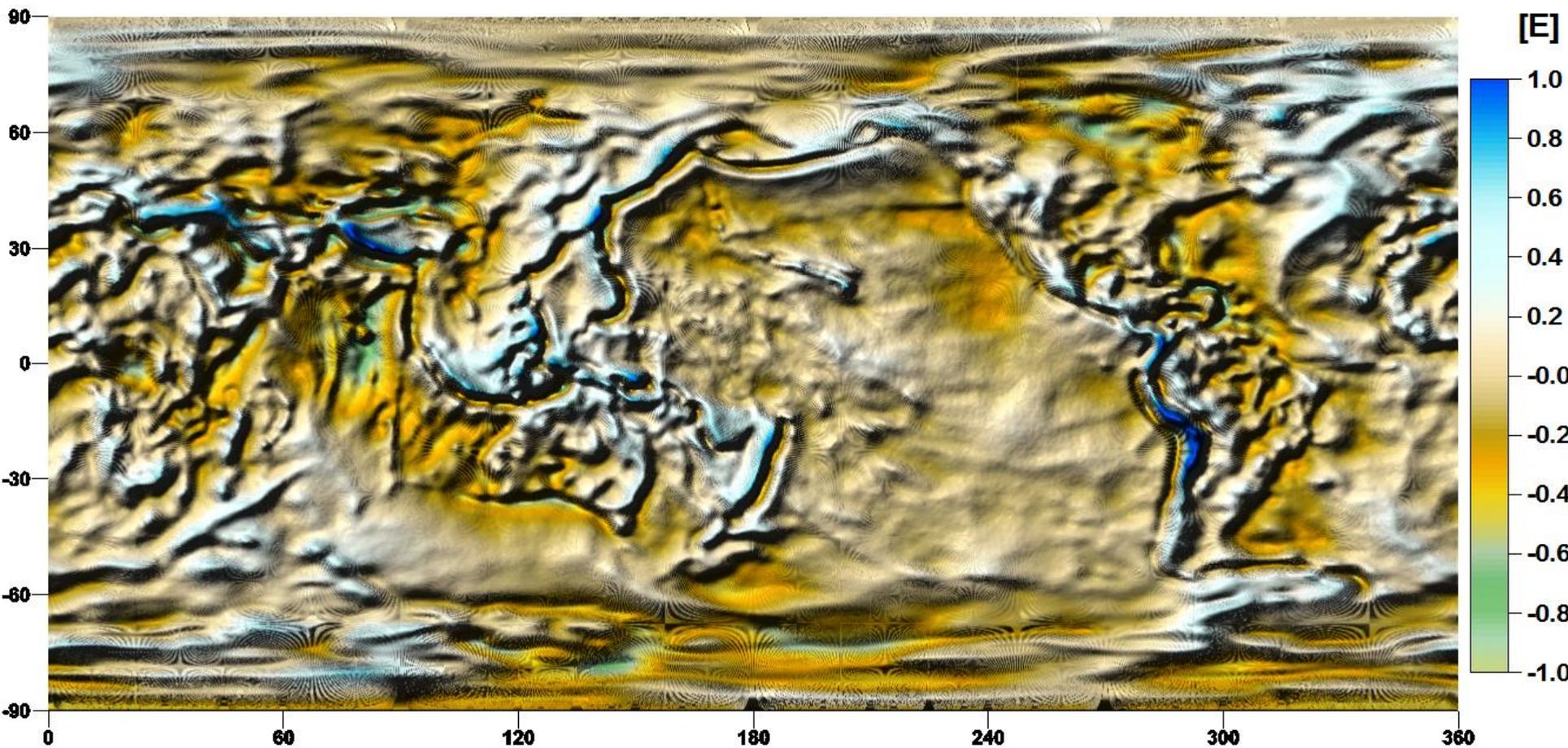
# *Nonlinear diffusion*

50 iterations



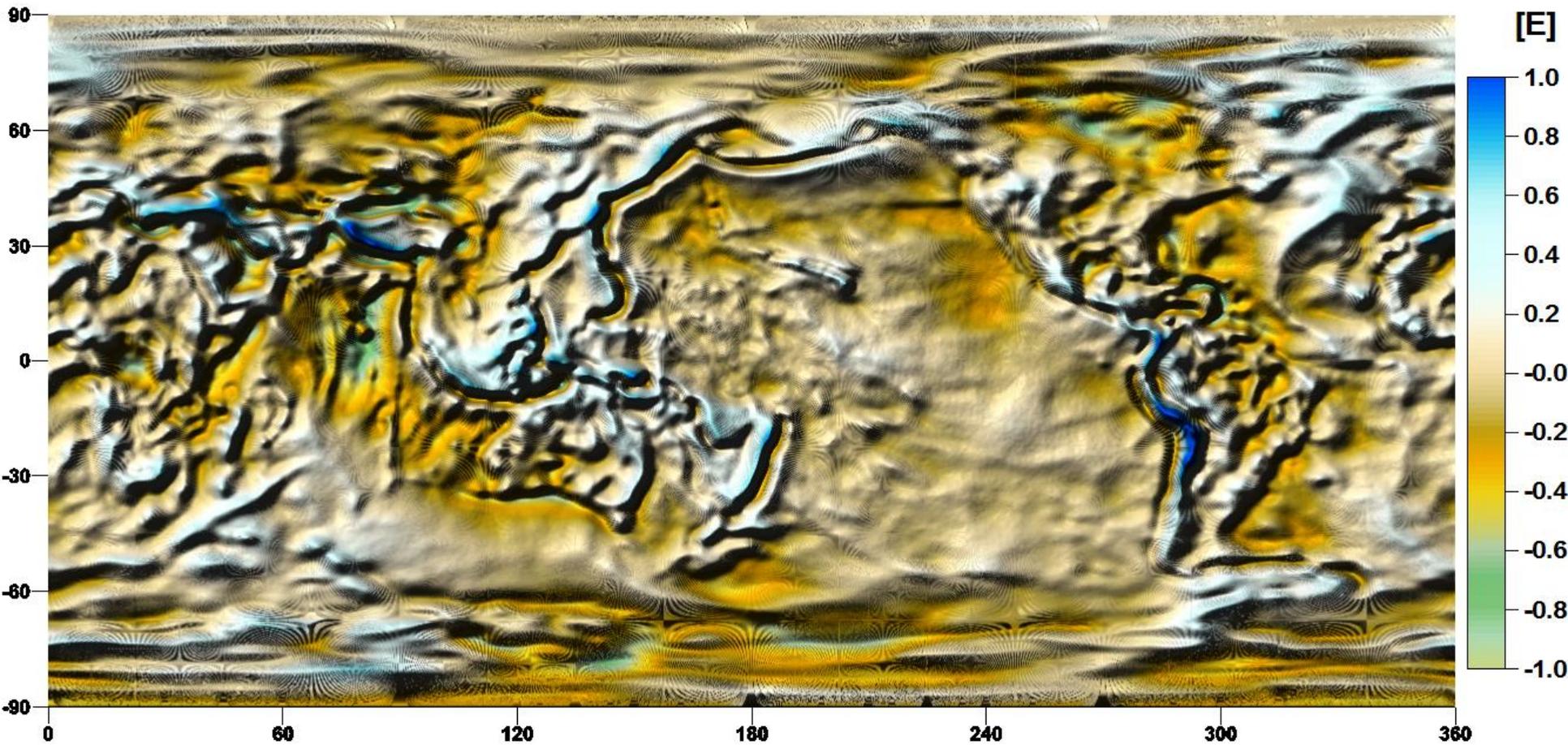
# *Nonlinear diffusion*

100 iterations



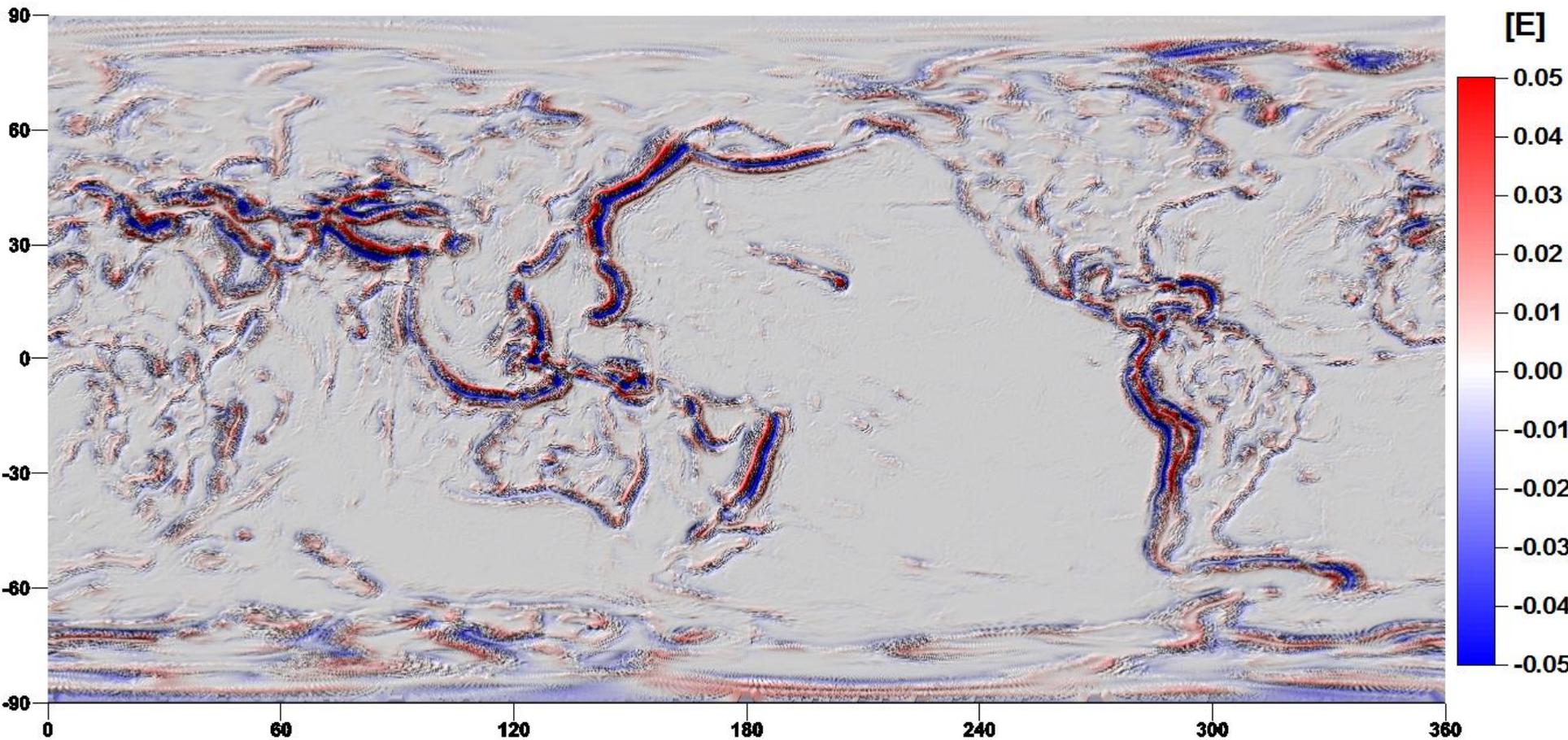
# *Linear diffusion*

100 iterations



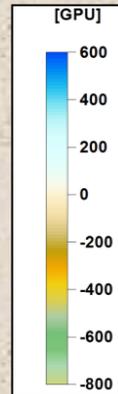
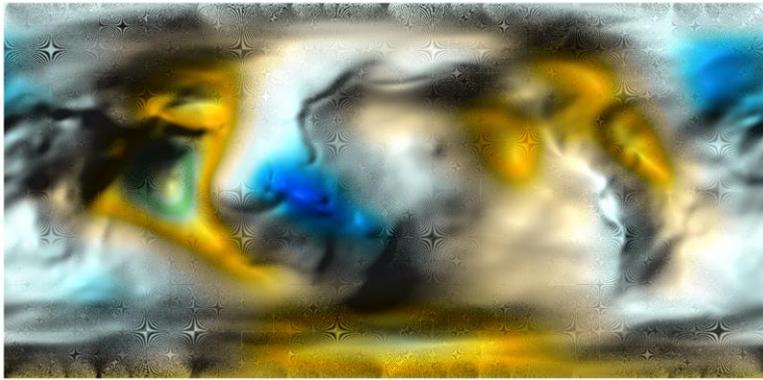
# *Nelineárna – lineárna difúzia*

100 iterations

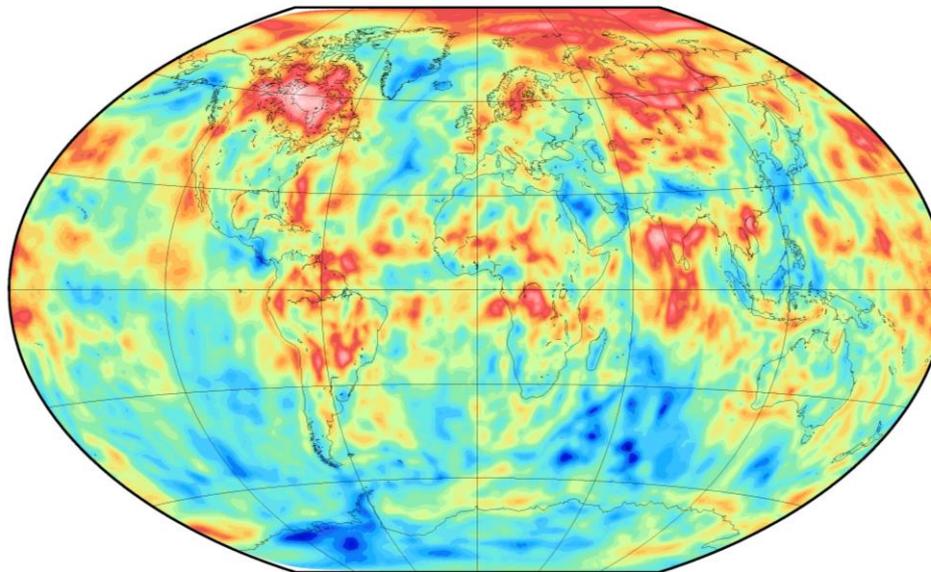
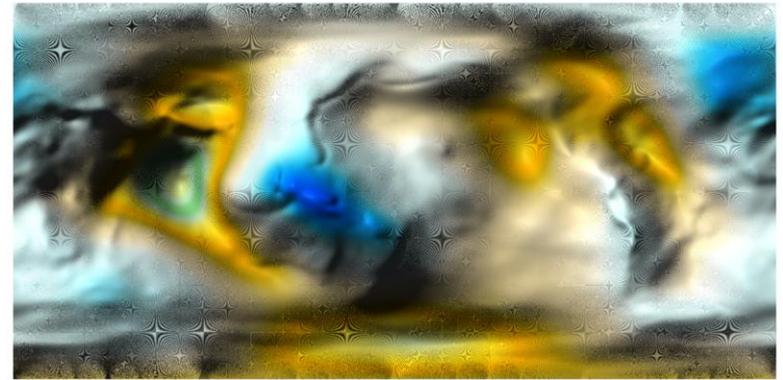


# Model tiažového poľa z filtrovaných dát

MFS



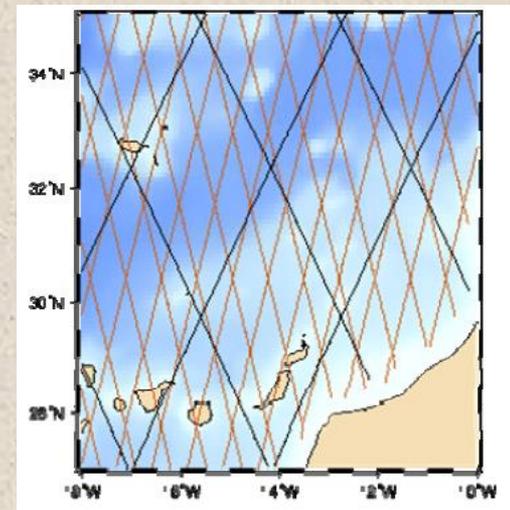
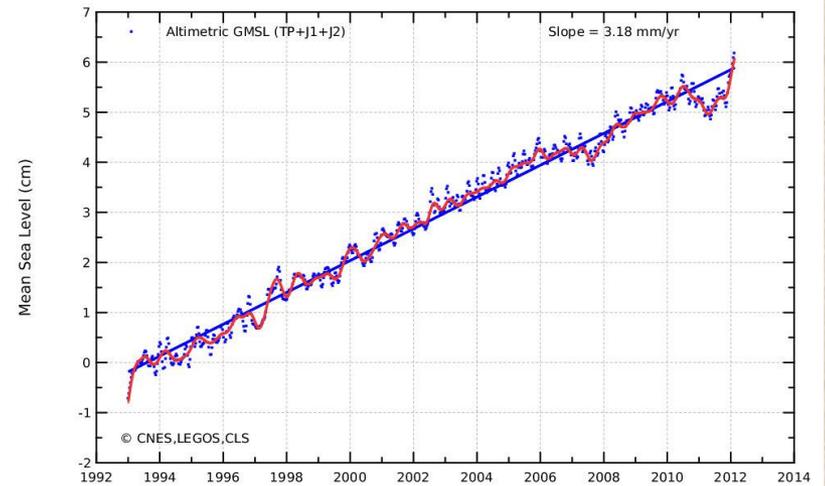
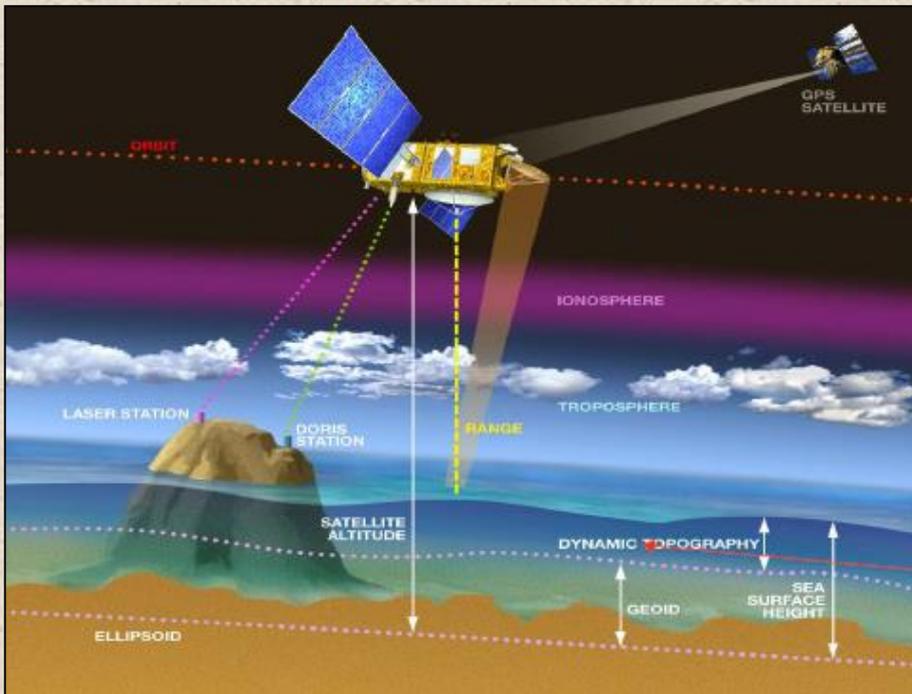
GOCO03S



## STATISTICS

Nodes	5 760 002
Mean	<b>-0.004 m<sup>2</sup>s<sup>-2</sup></b>
Max	0.268 m <sup>2</sup> s <sup>-2</sup>
MIN	-0.231 m <sup>2</sup> s <sup>-2</sup>
St. Dev.	<b>0.069 m<sup>2</sup>s<sup>-2</sup></b>

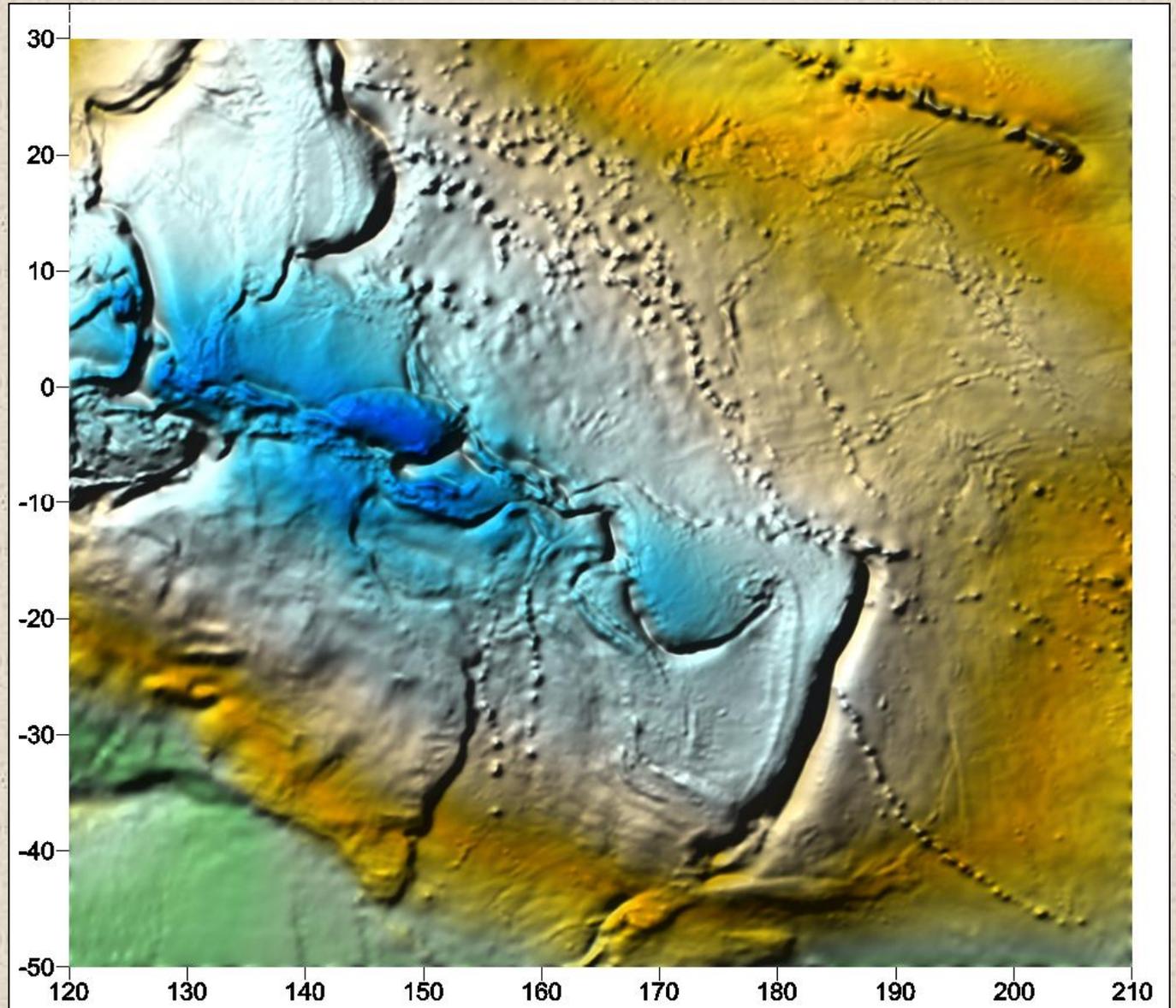
# Družicová altimetria



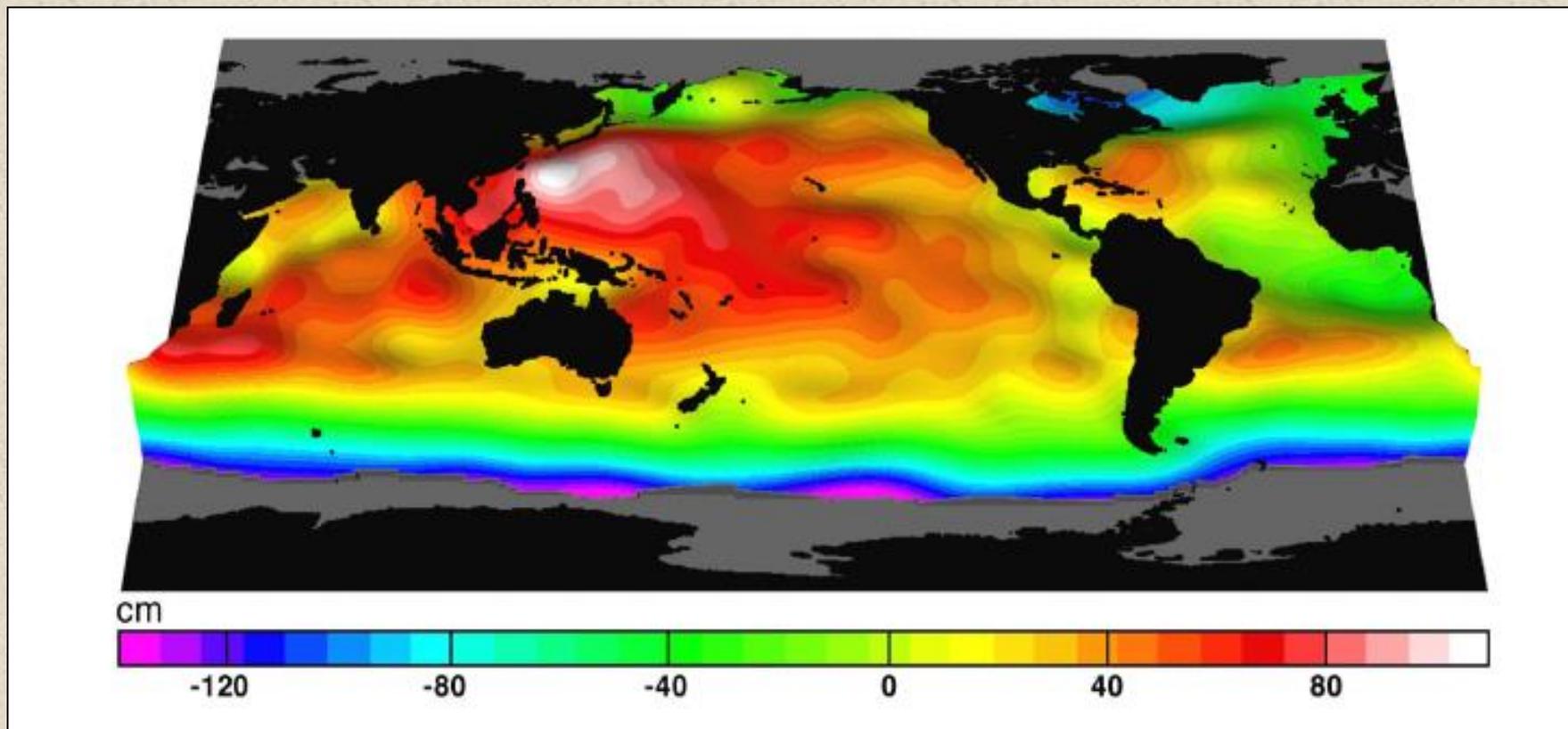
# Družicová altimetria



Stredná  
hladina mora



# *Dynamická topografia oceánov*



*“geodetický prístup”*

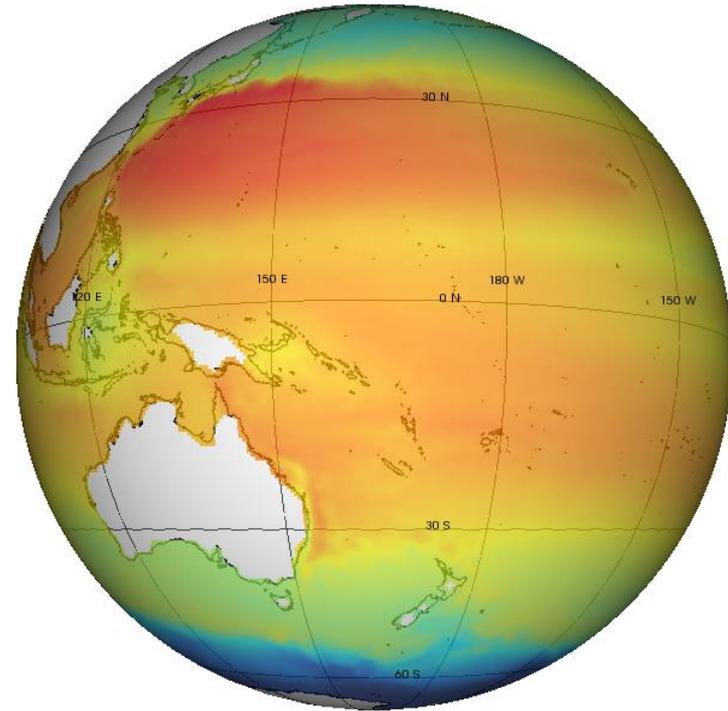
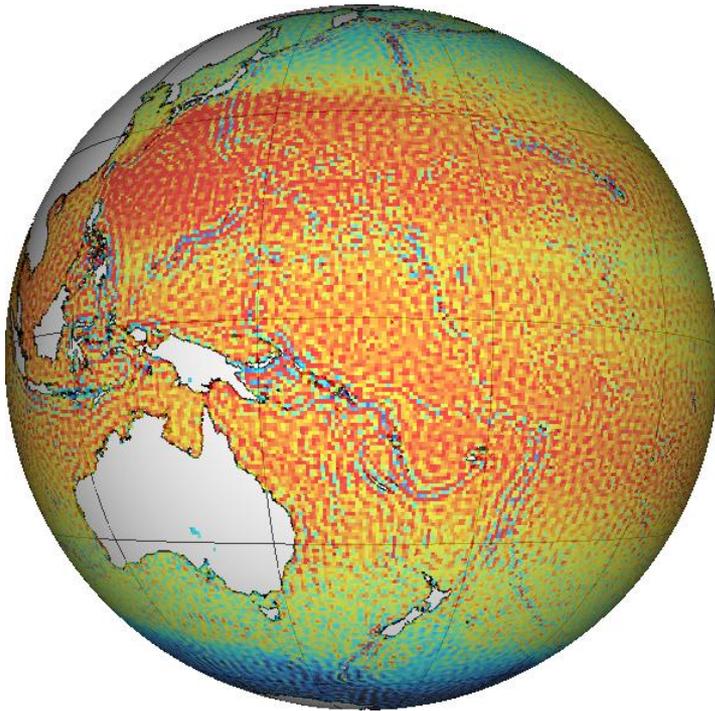
SDTO = SHM - geoid

*SDTO – stredná dynamická topografia oceánov*  
*SHM – stredná hladina morí*

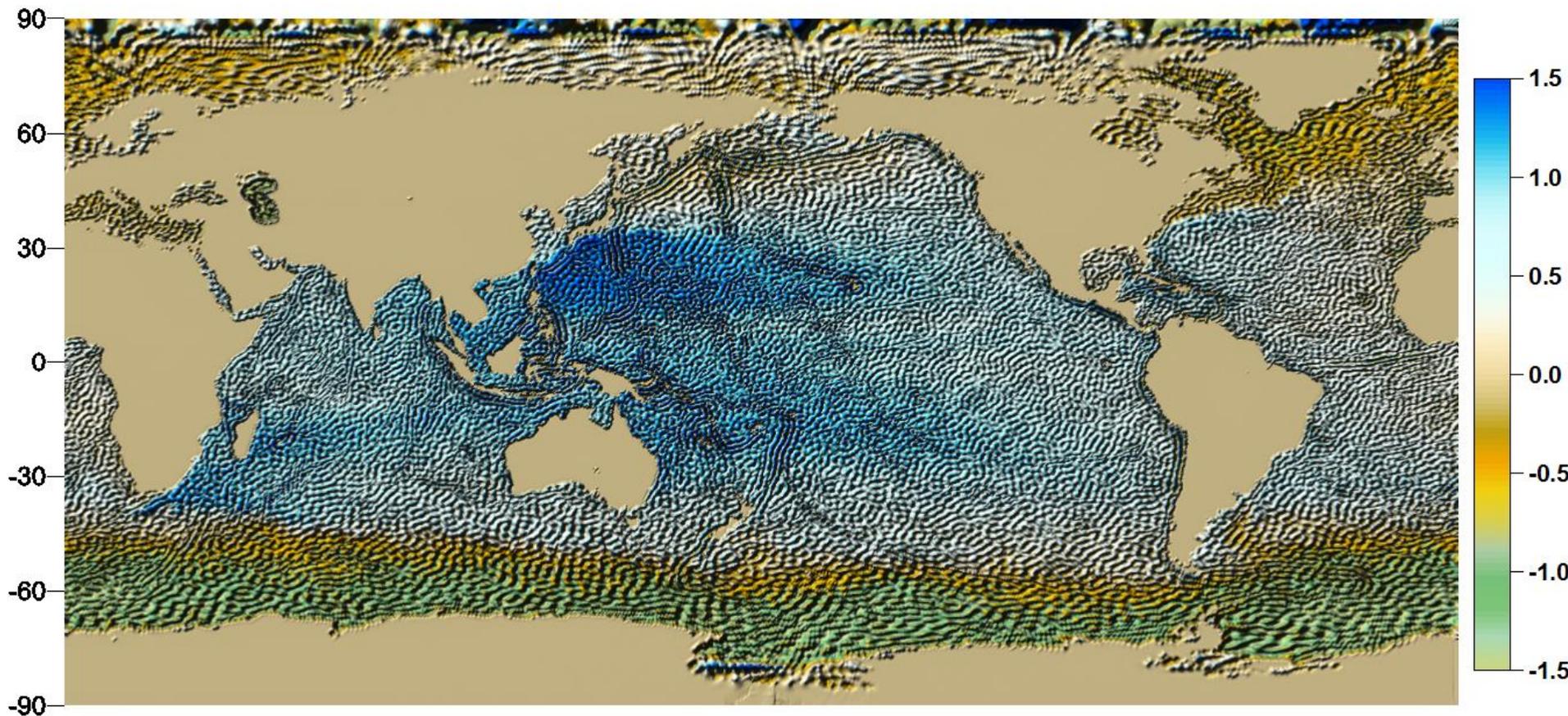
# *Dynamická topografia oceánov*

*Satellite altimetry + GOCE*

*After filtering*

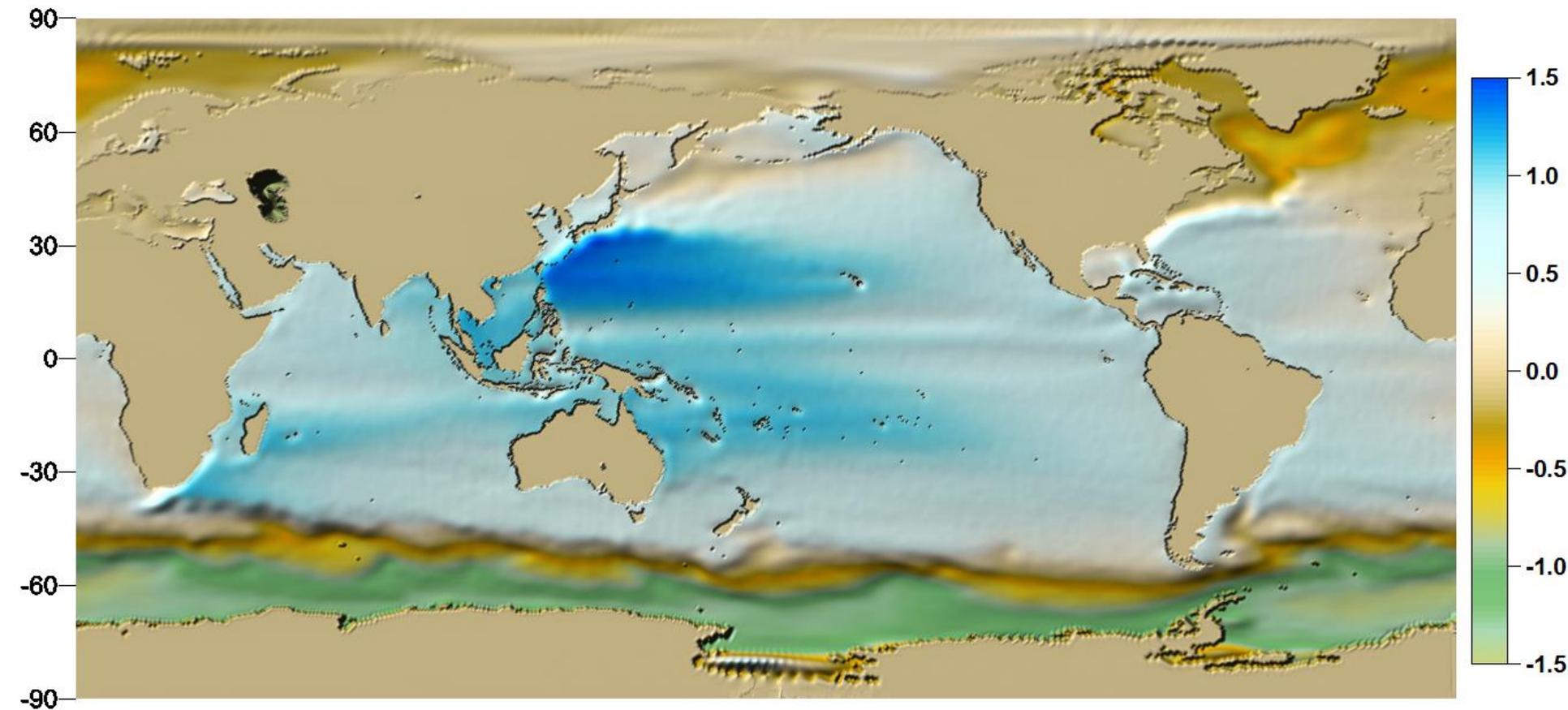


# *SDTO určená iba z družicových dát*



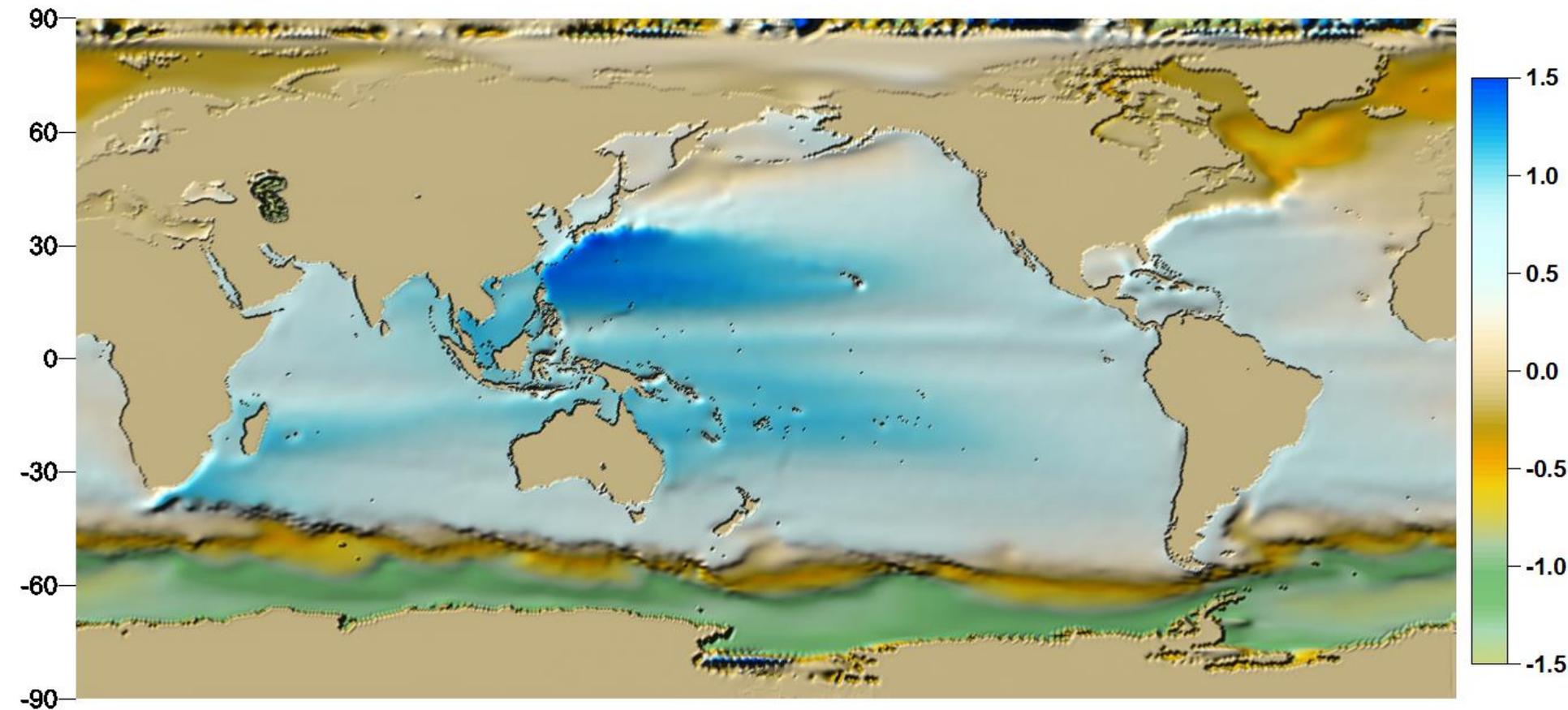
**Initial data**

# *Lineárna difúzia*



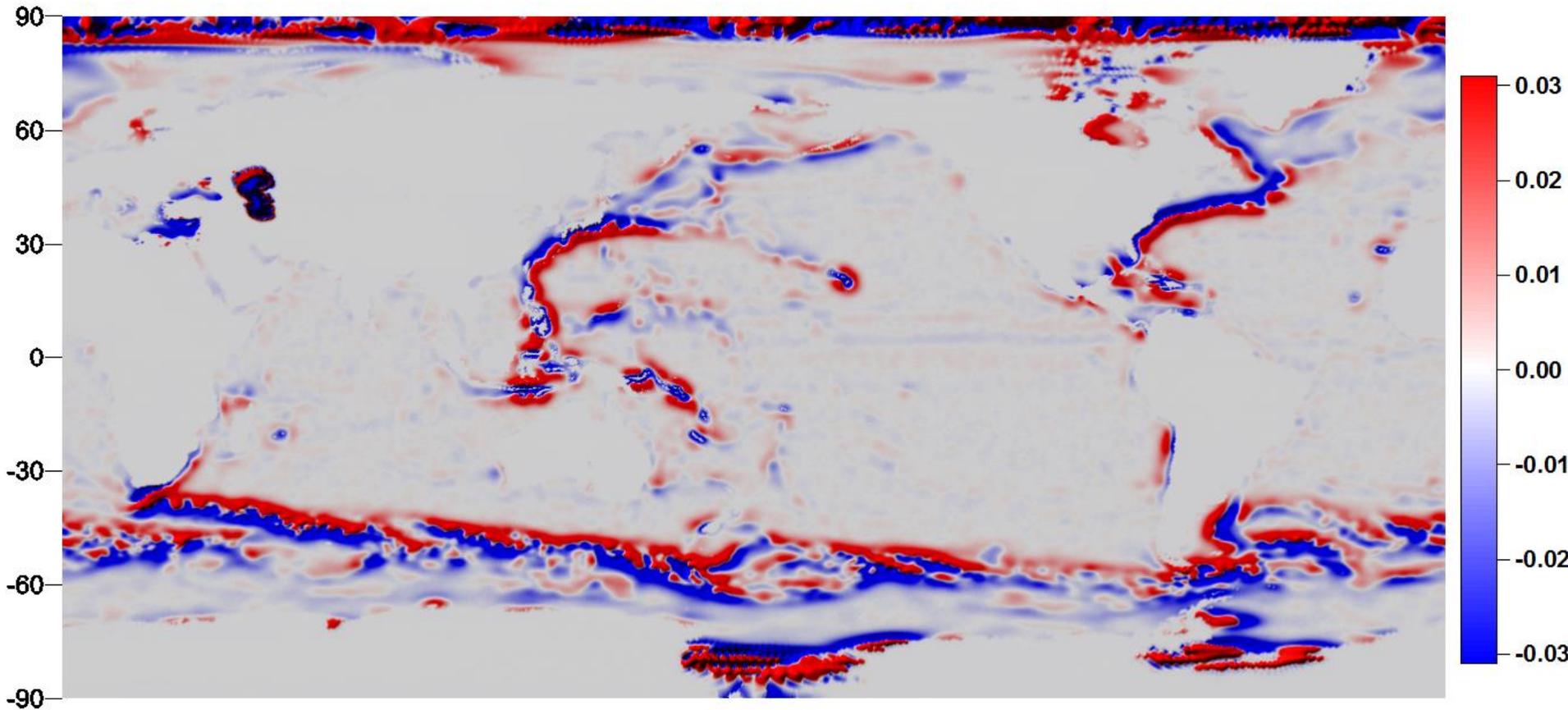
25 iterations

# *Nelineárna difúzia*



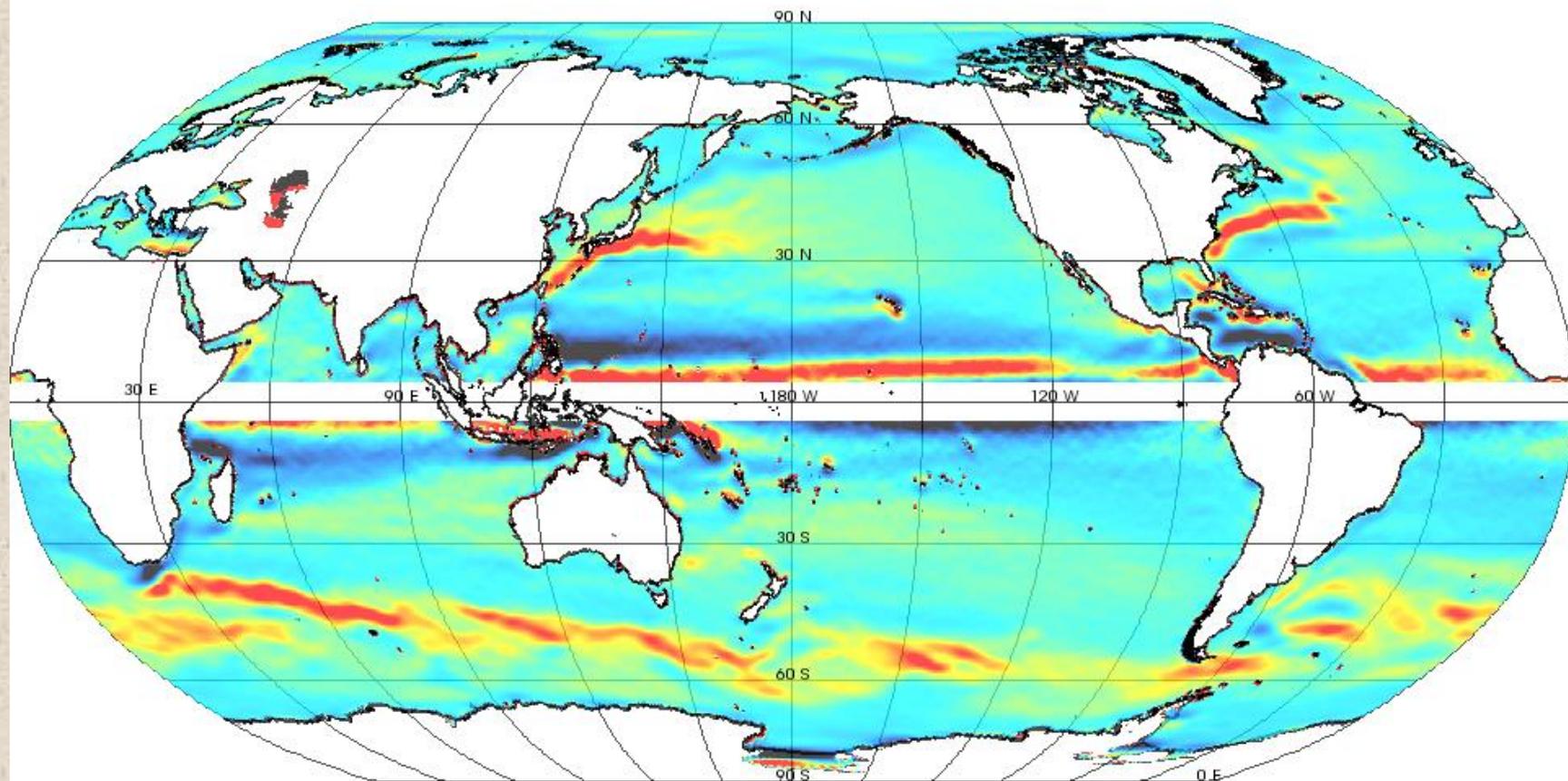
25 iterations

# *Nelineárna - lineárna difúzia*

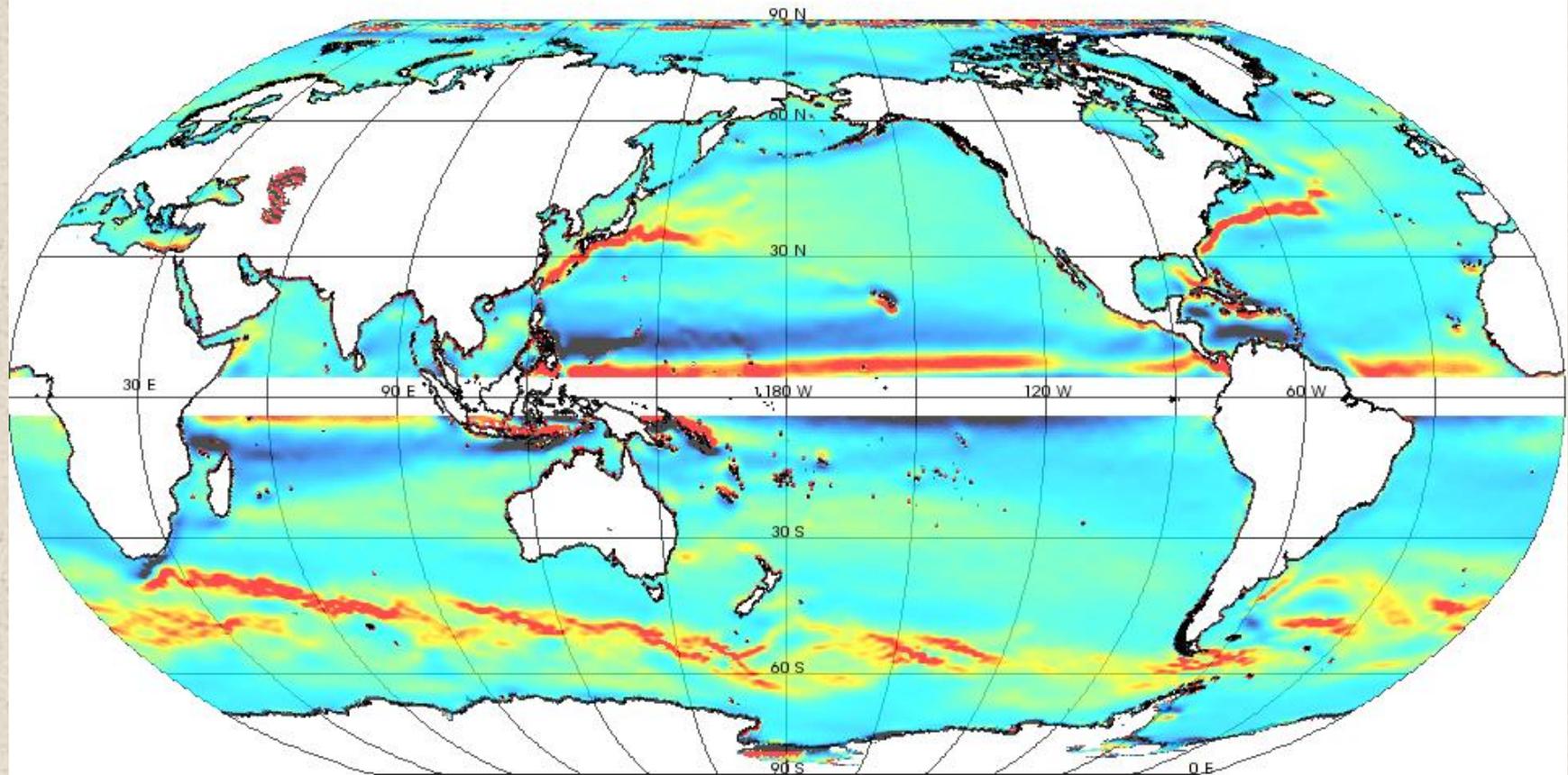


25 iterations

# *Geostrofické rýchlosti – lineárna difúzia*

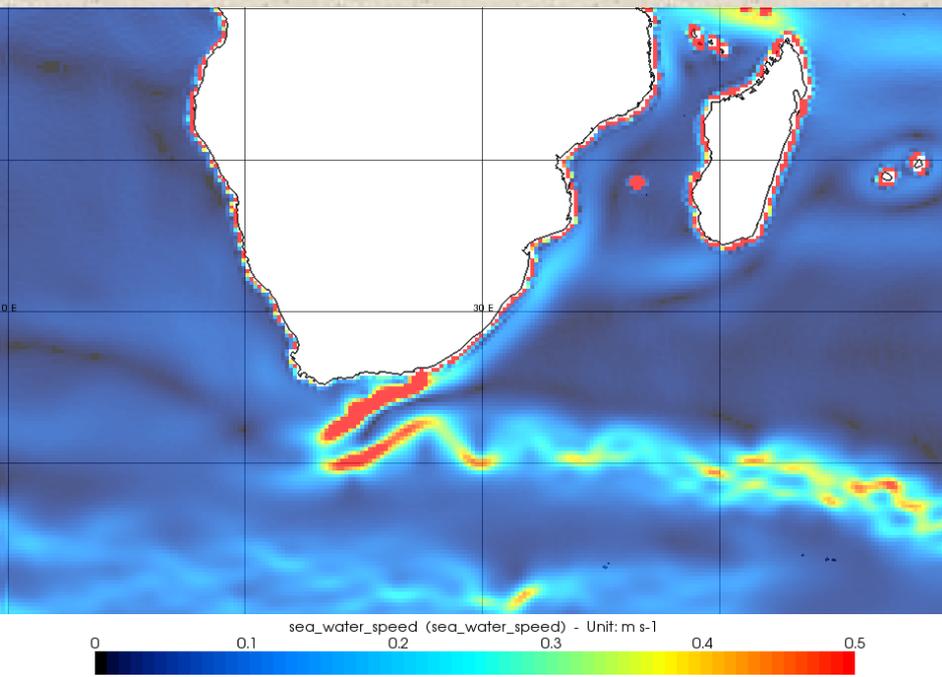


# *Geostrofické rýchlosti – nelineárna difúzia*

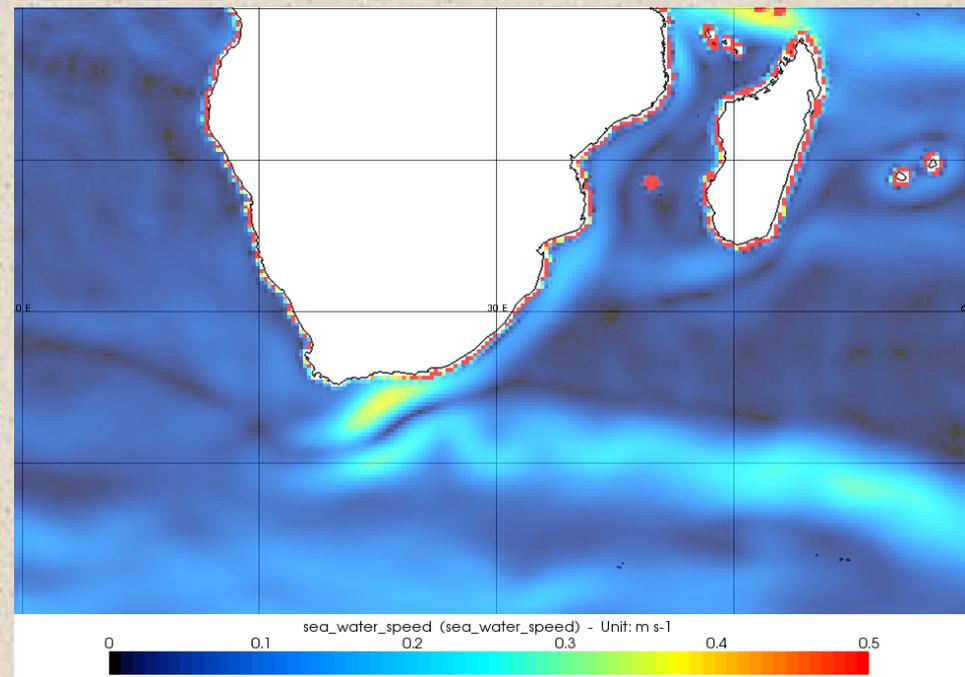


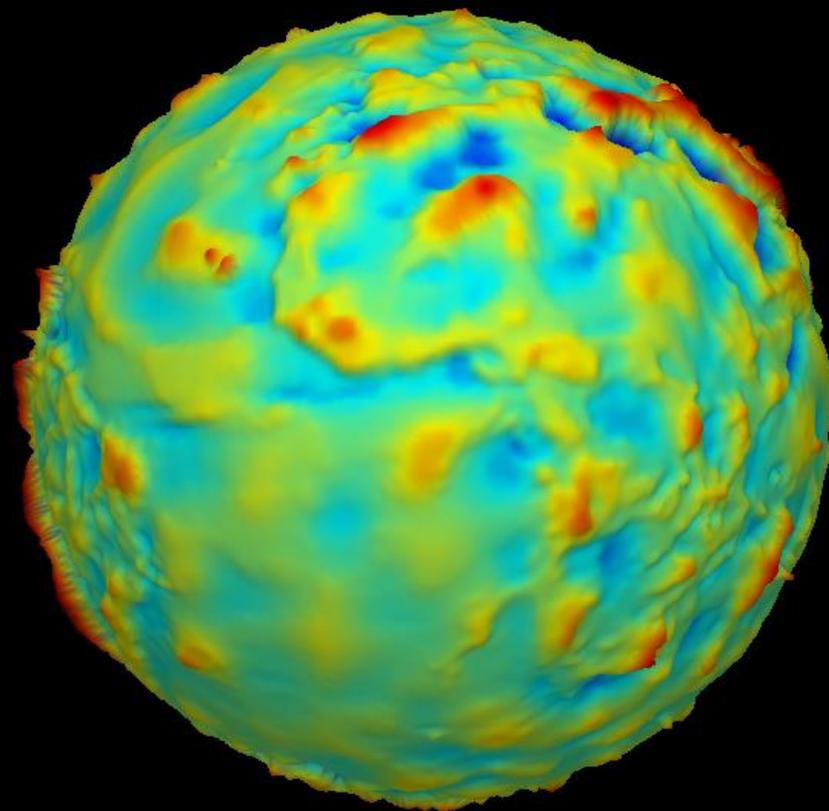
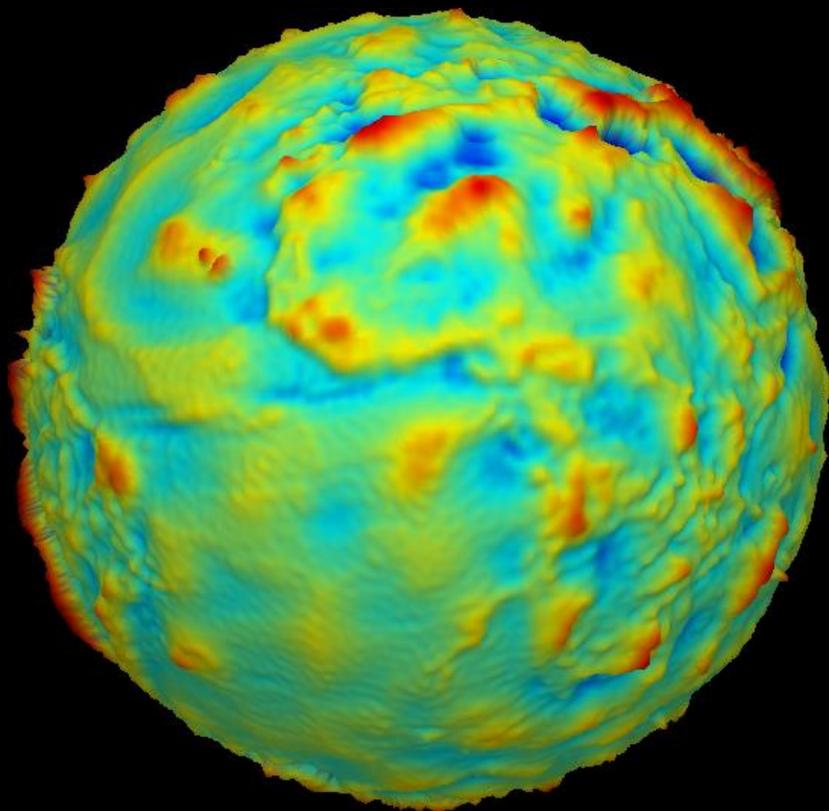
# *Geostrofické rýchlosti prúdov*

*Nelineárna difúzia*



*Lineárna difúzia*





*Vd'aka za pozornost'*