3D curve evolution algorithm with tangential redistribution for a fully automatic finding of an ideal camera path in virtual colonoscopy

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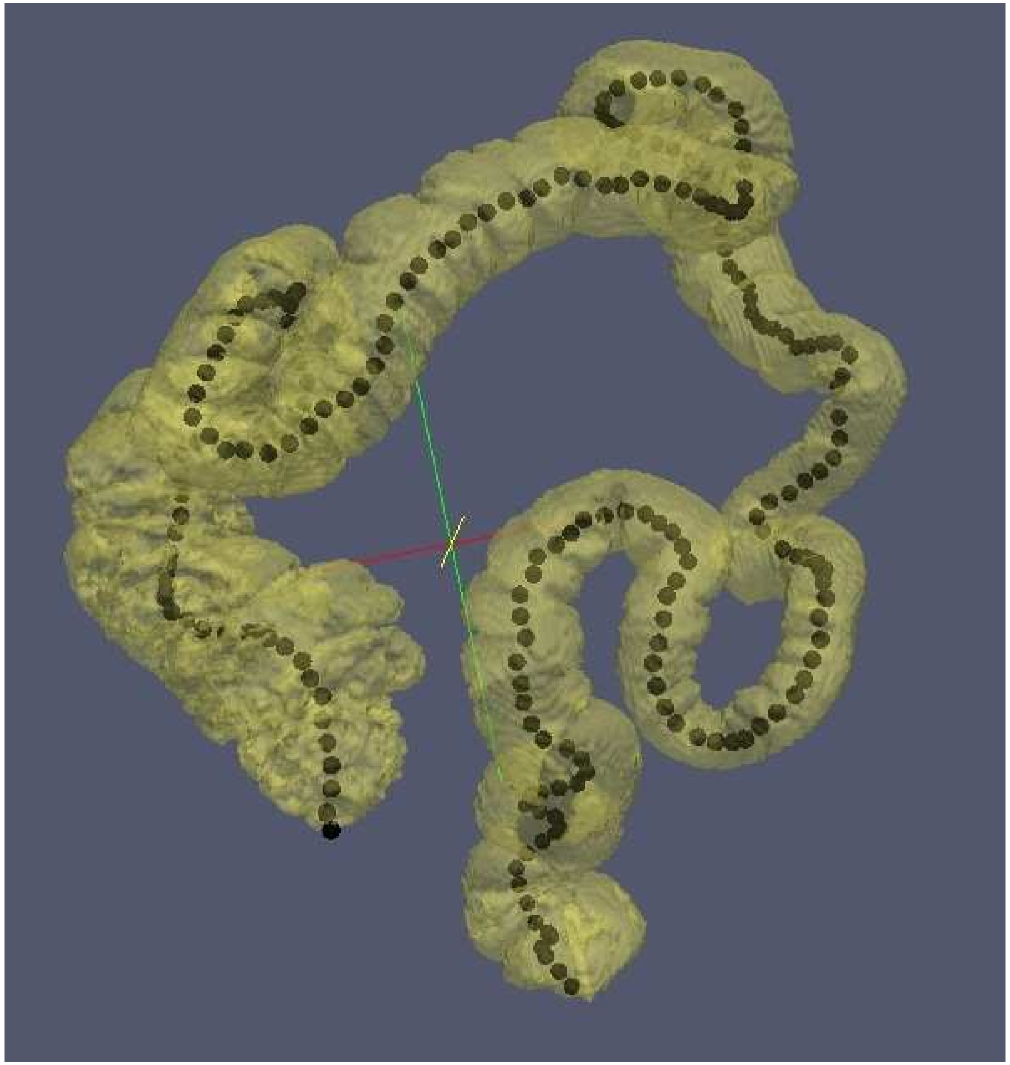


- the cancer of colon - the third most spread cancer disease in WHO countries - one of the most dengerous cancers in Central Europe

Classical (optical) colonoscopy

- classical (optical) colonoscopy - device with camera introduced inside a body and moving inside colon in order to detect polyps - tumors
- patient preparation is very complicated and the procedure itself is very painful
- problems in going through narrow (hardly passable) parts of the colon

Virtual colonoscopy

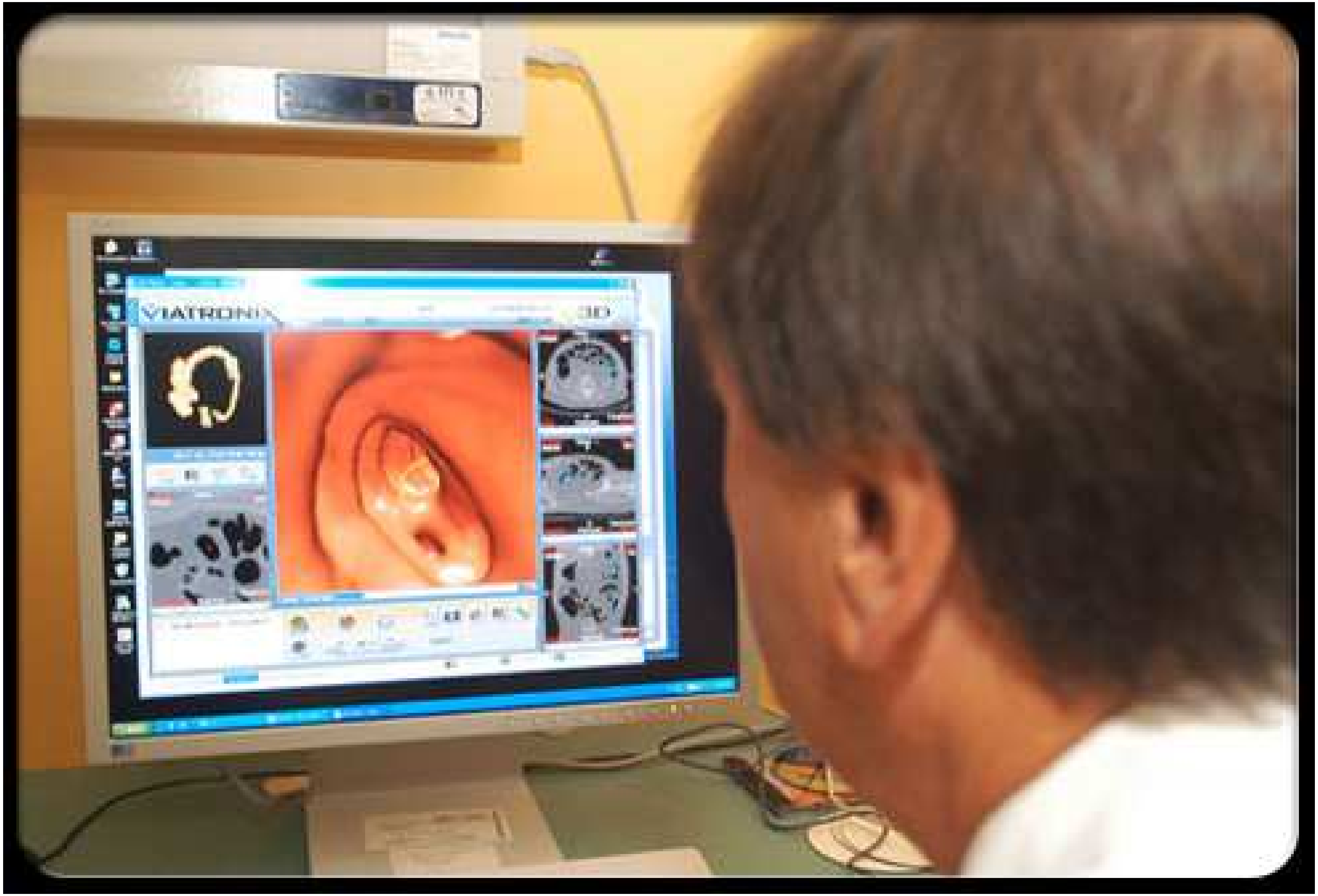
- new technology for the colon diagnosis - the results are comparable with the classical one

- CT (computer tomography) scan of 3D subvolume of a body containg colon followed by analysis of the colon interior borders using computer systems

- simple (and not painful) procedure for patient - diet, application of a contrast substance and inflation of the colon followed by 3D CT scan

- allows diagnosis of any (also hardly passable) colon shapes difficult for the classical colonoscopy

- it needs fast and relialable computer algoritms which mimic the classical optical colonoscopy - **virtual camera path** allowing the physician to check the colon - polyps, tumors - by 3D visualization methods


3D

Virtual colonoscopy

- cooperation with company **TatraMed** Bratislava in development of medical image analysis software

- to be competitive with other systems (Siemens, Philips) they needed to incorporate virtual colonoscopy in their system TomoCon - the core has been to find an optimal camera trajectory - **smooth, uniformly discretized 3D curve going approximately in the middle of colon** - in fully automatic way

- we developed new original and highly competitive concept, based on **evolving 3D curves and numerical solution of nonlinear PDEs** - implemented into the software and must be clinically tested - very fast (8 seconds of CPU) and precise when working with real large 3D CT data sets

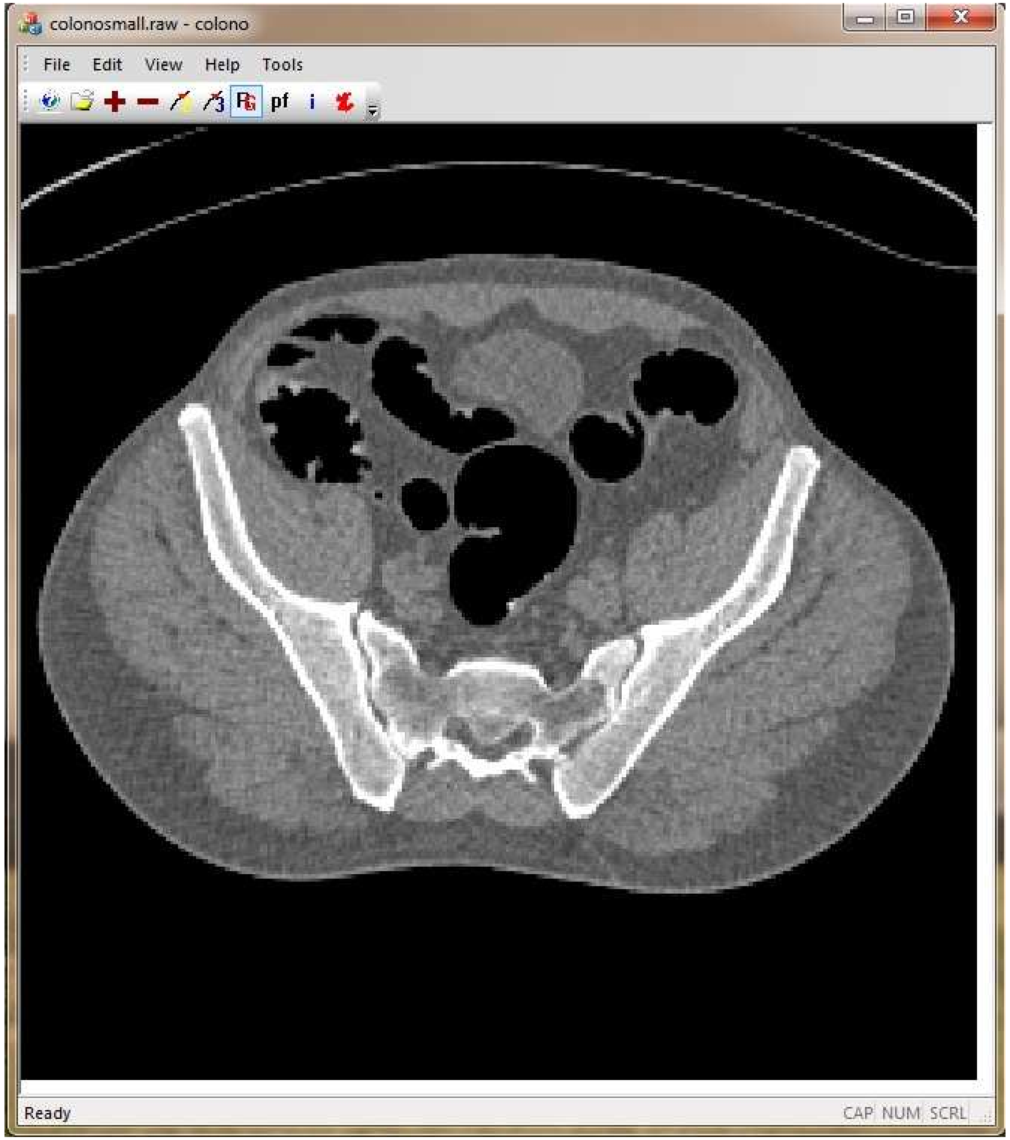
Our concept has three basic steps

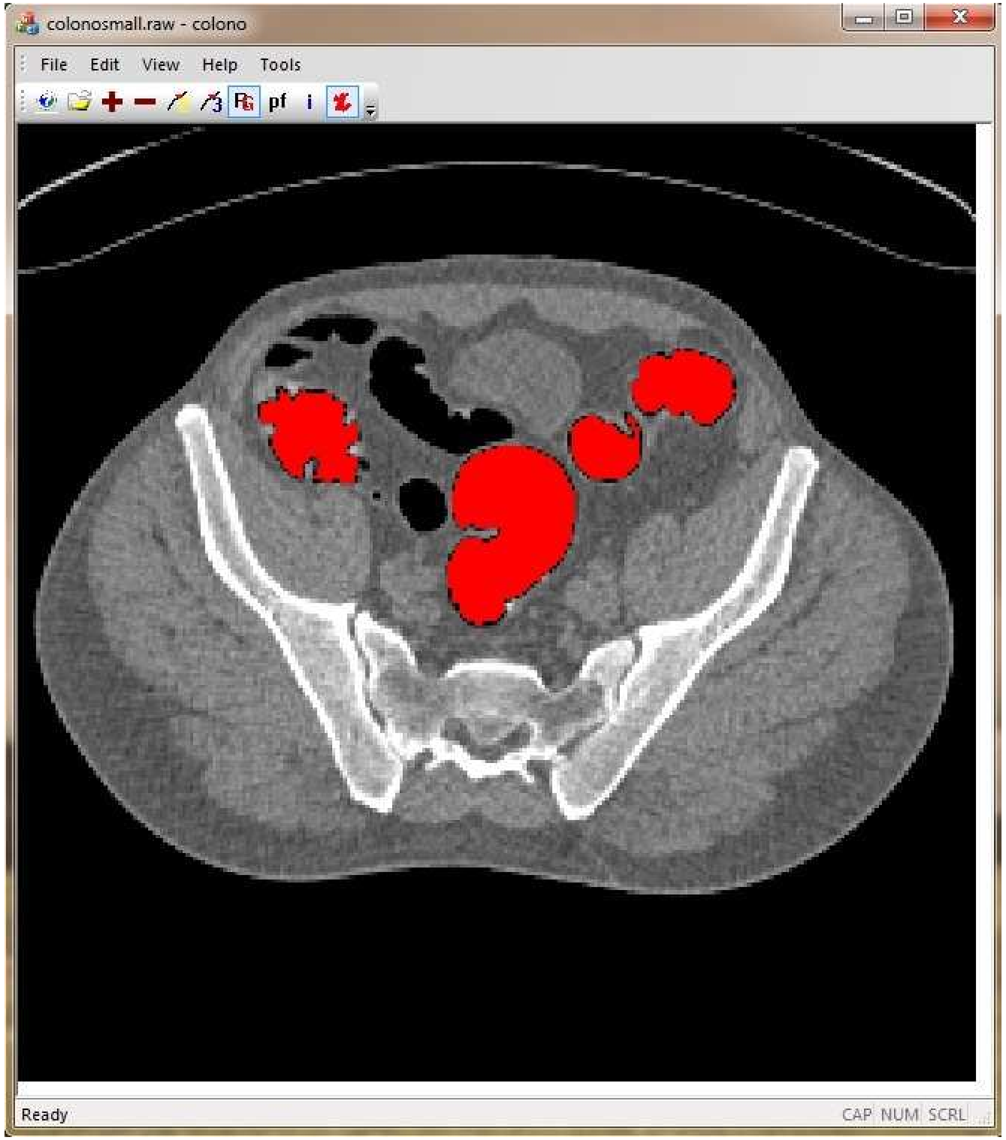
- **segmentation of the colon** from 3D CT data - scan-line seed filling algorithm

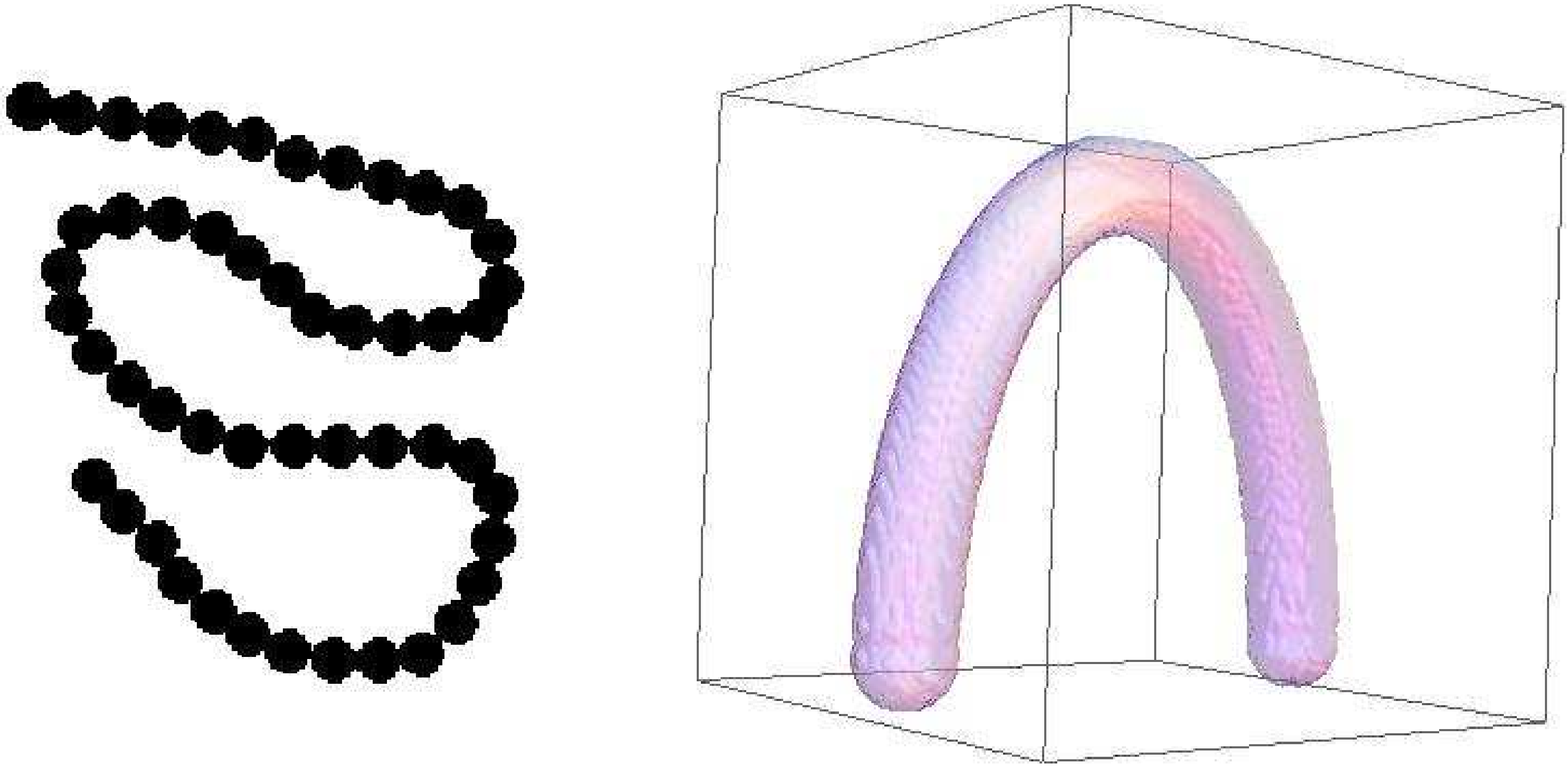
- **finding an initial guess for the ideal path** - computing a **distance function** from a camera starting point inside the segmented object by solving the restricted **Eikonal equation** and backtracking the result in its steepest descent direction

- **finding an optimal 3D curve representing an ideal path** - computing a **distance function** from the boundary of segmented object gives **vector field** for moving 3D curve - **regularization by curvature** gives smoothness and suitable **tangential velocity** gives centered uniformly distributed curve

Segmentation by seed filling



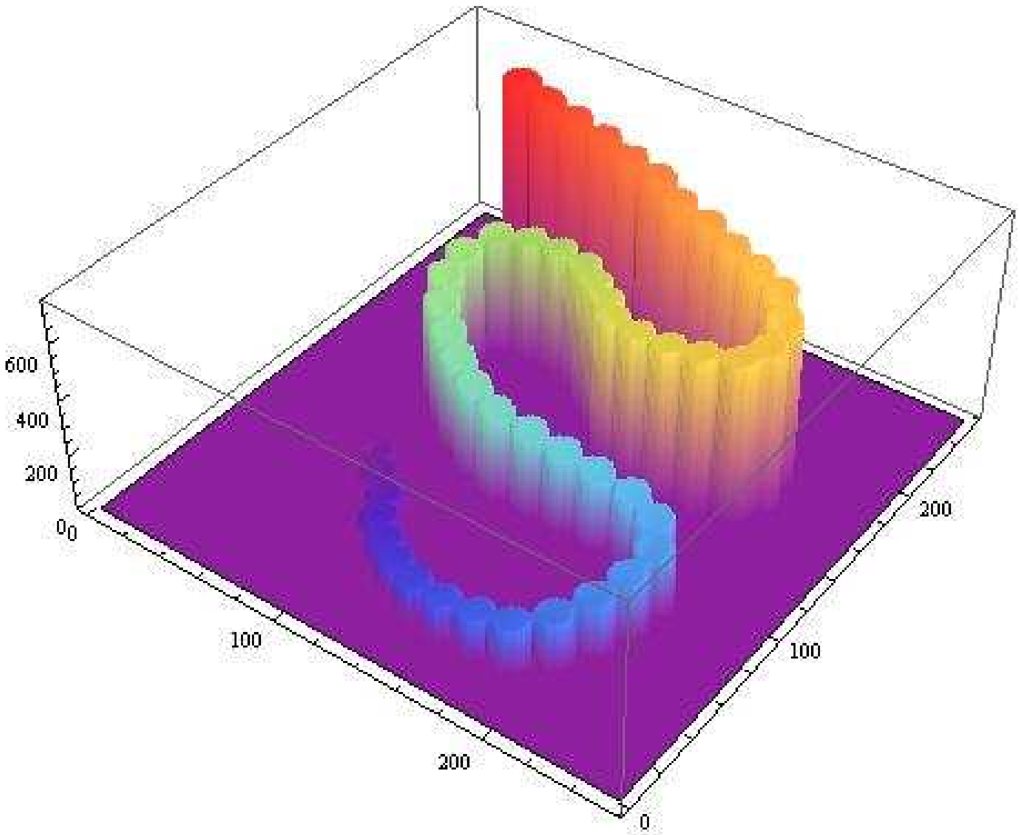


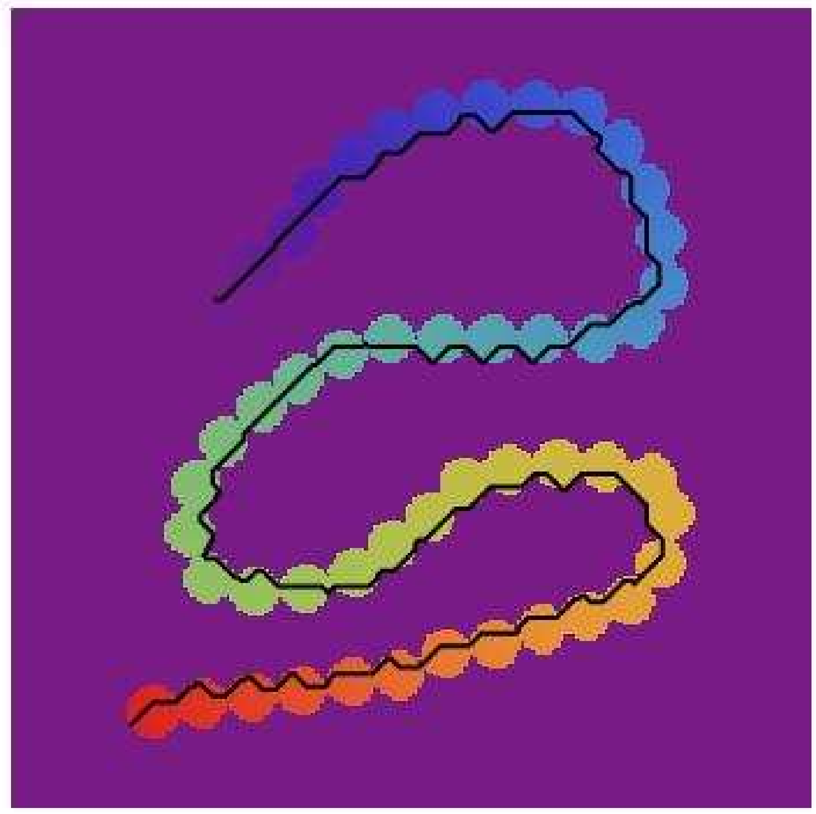


Artificial 2D and 3D testing data

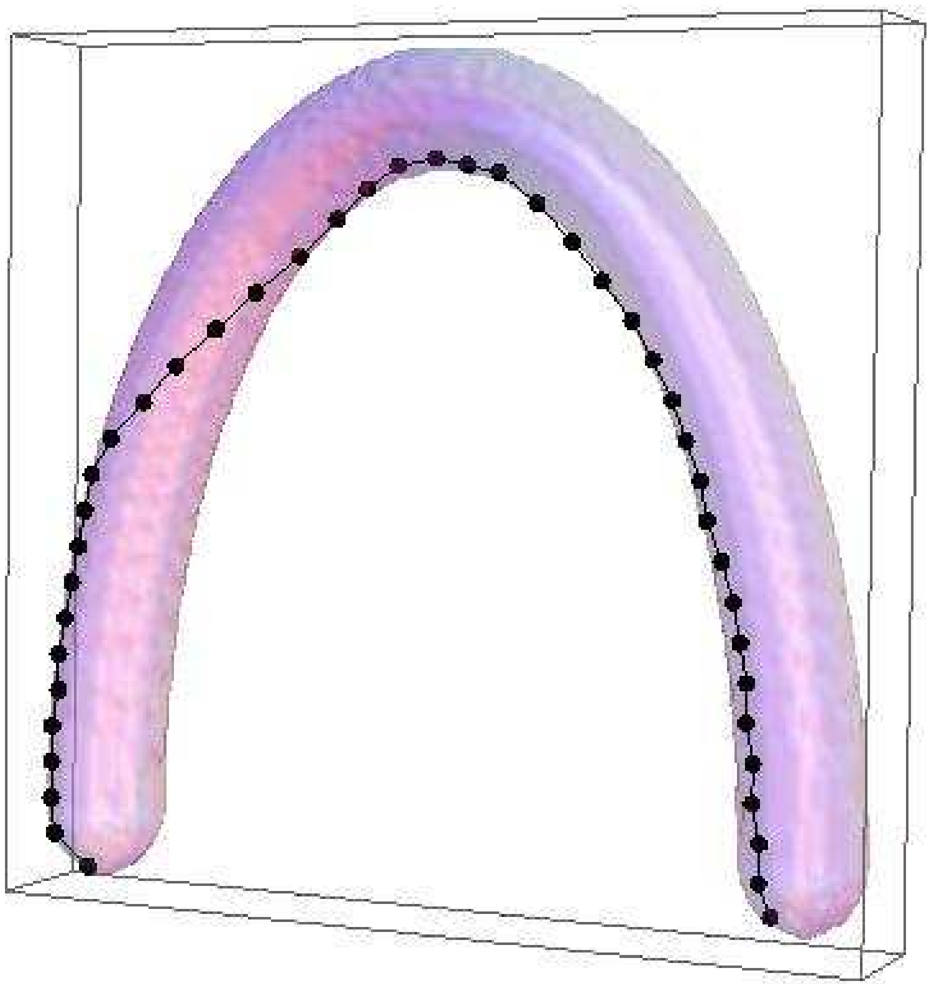
Initial curve for the optimal path search

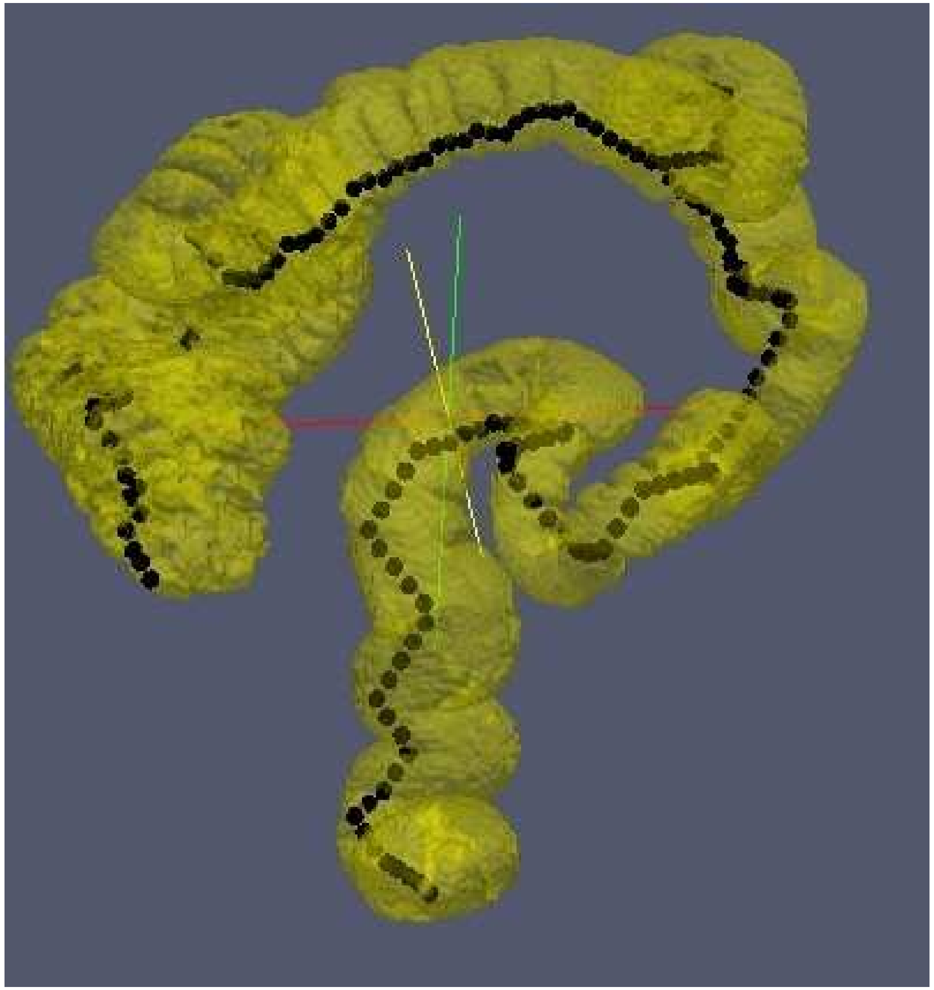
• distance function from the starting point in 2D testing data and the initial curve found by backtracking in steepest descent direction (Dechamps, 2001)





- initial 3D curve found using distance function and backtracking





Computing the distance function

- time relaxed Eikonal equation for $d(x,t)$ restricted to the interior of segmented object setting $d(x_{init},t)=0$ in initialization point x_{init}

$$d_t + |\nabla d| = 1.$$

- discretization by the Rouy-Tourin scheme with fixing - d^n_{ijk} - approximate values of distance function at time step n in voxel center (i,j,k), h_D - voxel size, τ_D - time step size

$$M^{pqr}_{ijk} = (\min(d^n_{i+p,j+q,k+r} - d^n_{ijk}, 0))^2, \;\; p,q,r \in \{-1,0,1\}, |p|+|q|+|r| = 1$$

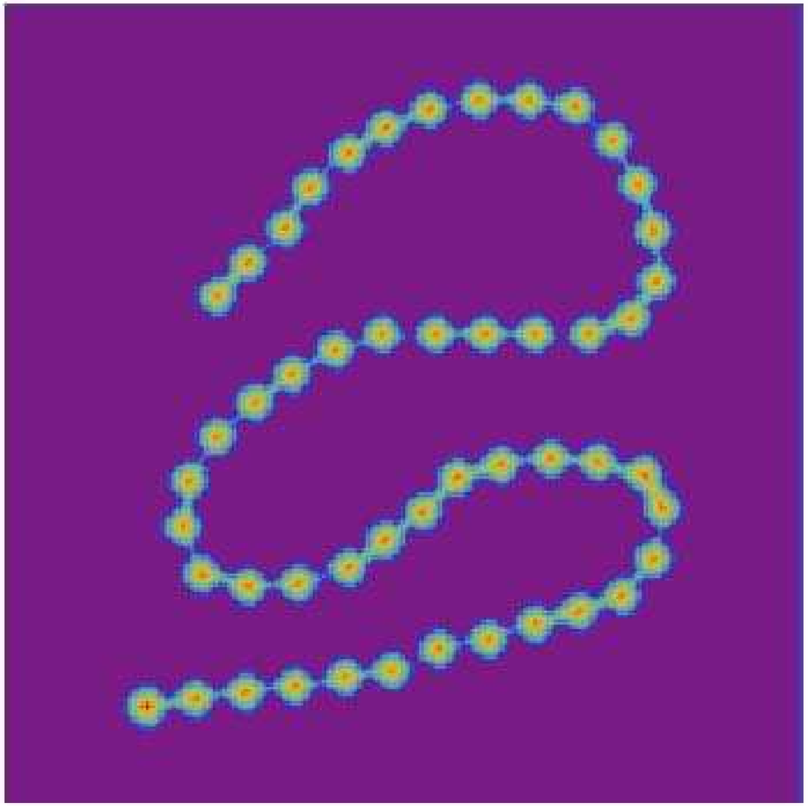
$$d^{n+1}_{ijk} = d^n_{ijk} + \tau_D - \frac{\tau_D}{h_D}$$

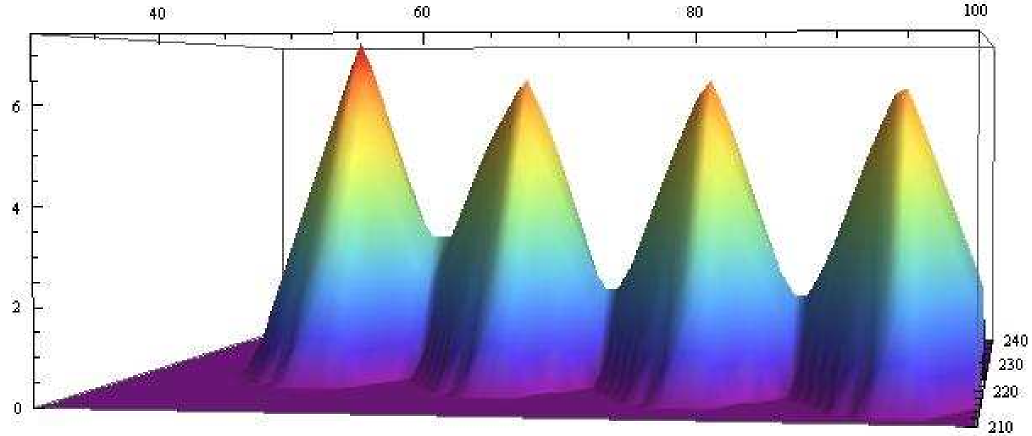
$$\sqrt{\max(M^{-1,0,0}_{ijk}, M^{1,0,0}_{ijk}) + \max(M^{0,-1,0}_{ijk}, M^{0,1,0}_{ijk}) + \max(M^{0,0,-1}_{ijk}, M^{0,0,1}_{ijk})}$$

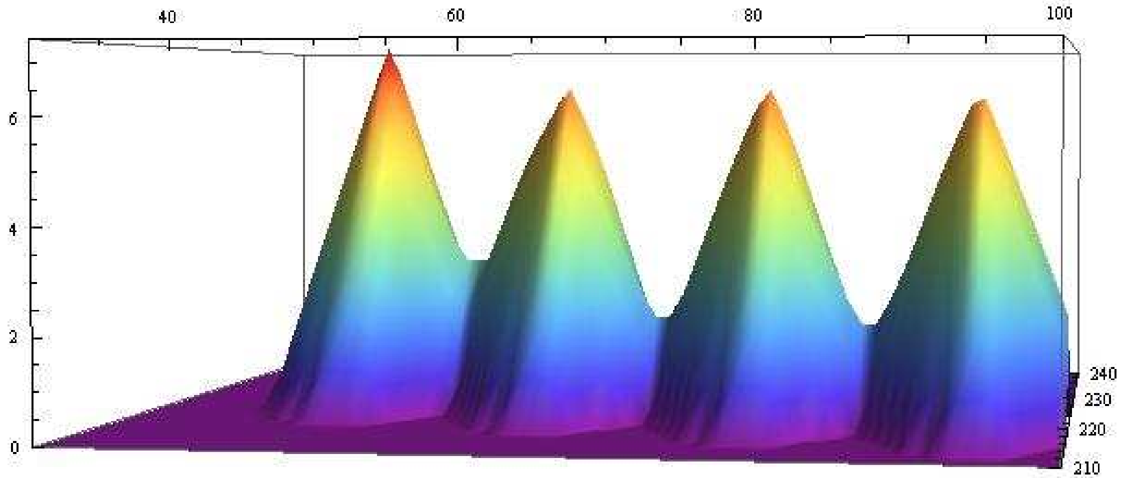
- speed-up by omitting computations in voxels where values are not changed in subsequent time steps

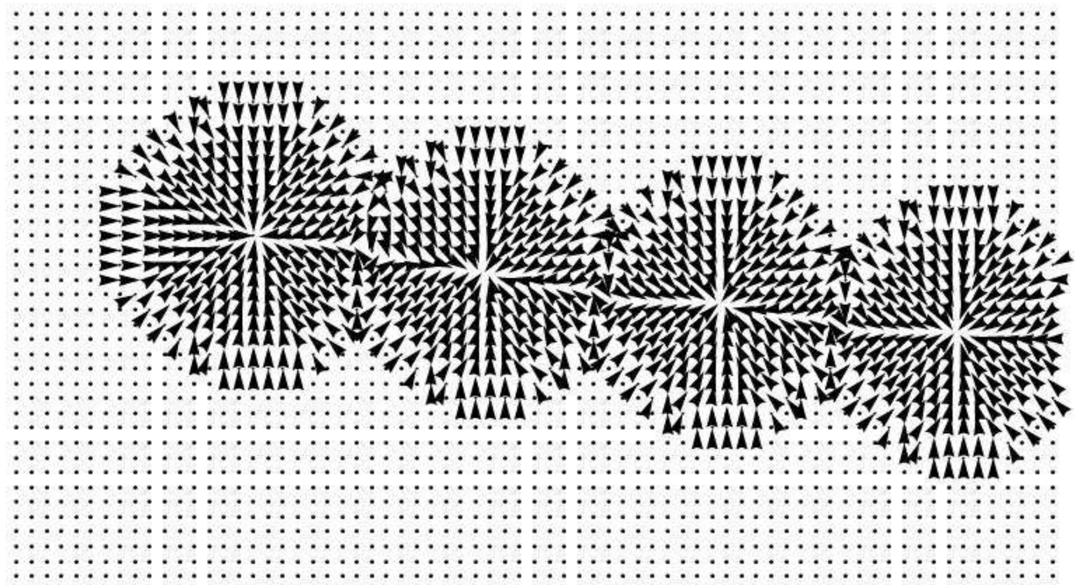
Construction of a vector field driving curve to optimal position

- distance function to the boundary of segmented 2D testing data









- vector field given by the gradient of computed distance function

$$\mathbf{v}(x,y,z) = \nabla d(x,y,z) = \left(\frac{\partial d}{\partial x}, \frac{\partial d}{\partial y}, \frac{\partial d}{\partial z}\right)^T .$$

$$\mathbf{v}_{ijk} = \left(\frac{d_{i+1jk} - d_{i-1jk}}{2\,h_D}, \frac{d_{ij+1k} - d_{ij-1k}}{2\,h_D}, \frac{d_{ijk+1} - d_{ijk-1}}{2\,h_D}\right)$$

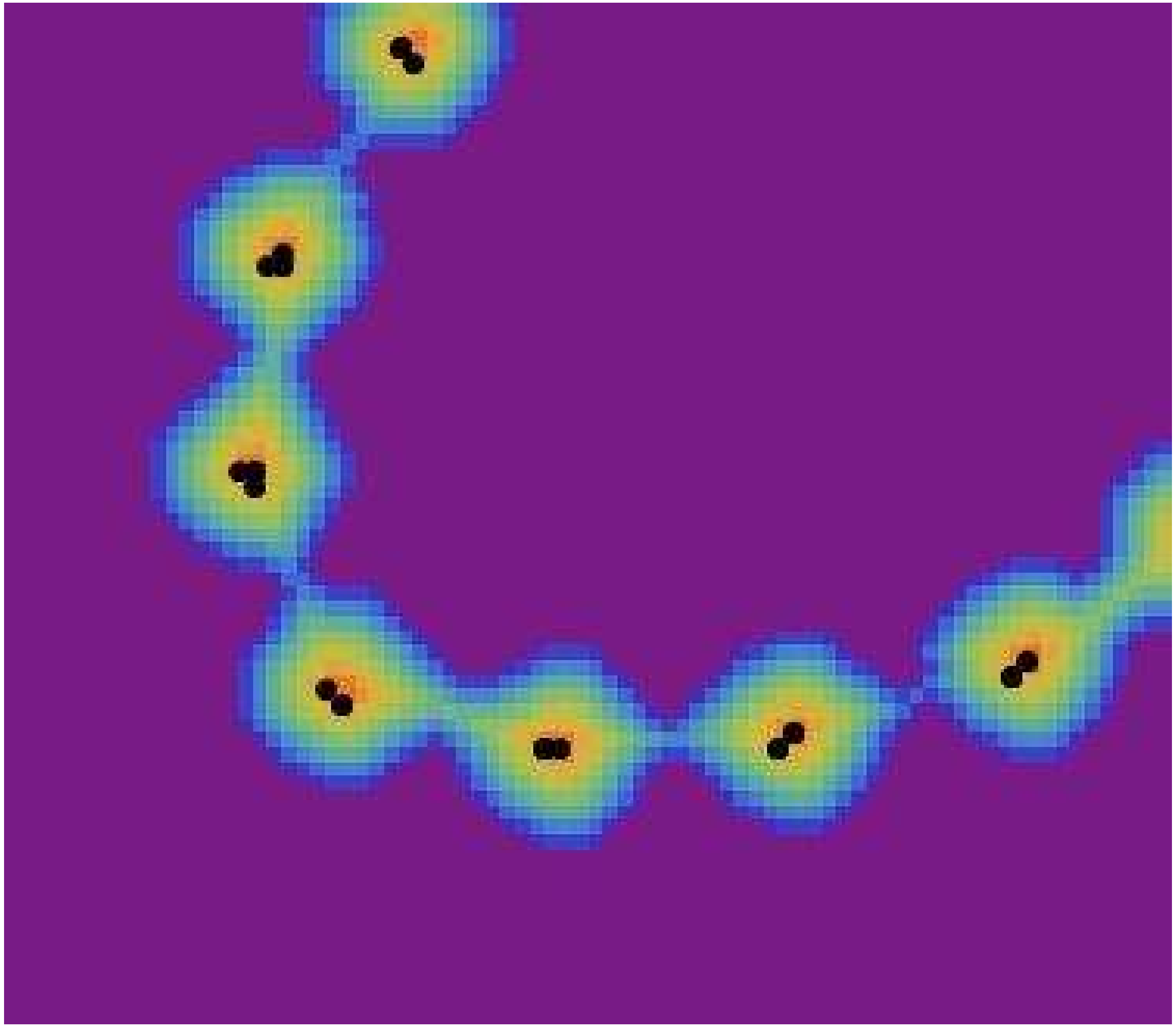
- the evolving curve is represented by its position vector $\mathbf{r}$, vector field $\mathbf{v}$ gives the velocity of motion

$$\partial_t \mathbf{r} = \mathbf{v}$$

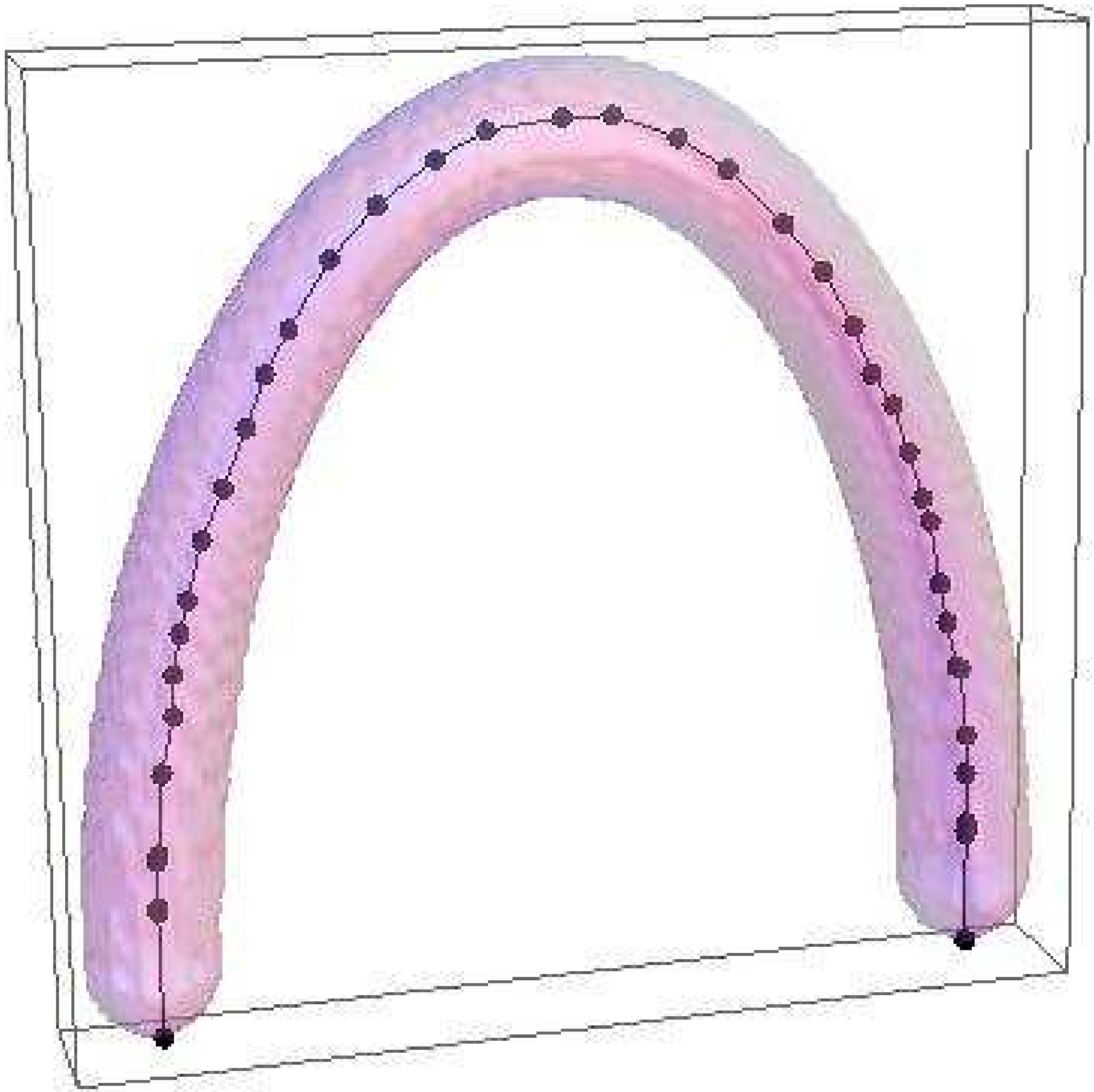
$$\mathbf{r}_i^{n+1} = \mathbf{r}_i^n + \tau \mathbf{v}(\mathbf{r}_i^n)$$

- τ is time step, $\mathbf{r}_i^n$ - position of the i-th point at time step n, $i = 0, ..., m$, endpoints are fixed

- motion of initial 2D curve in such vector field



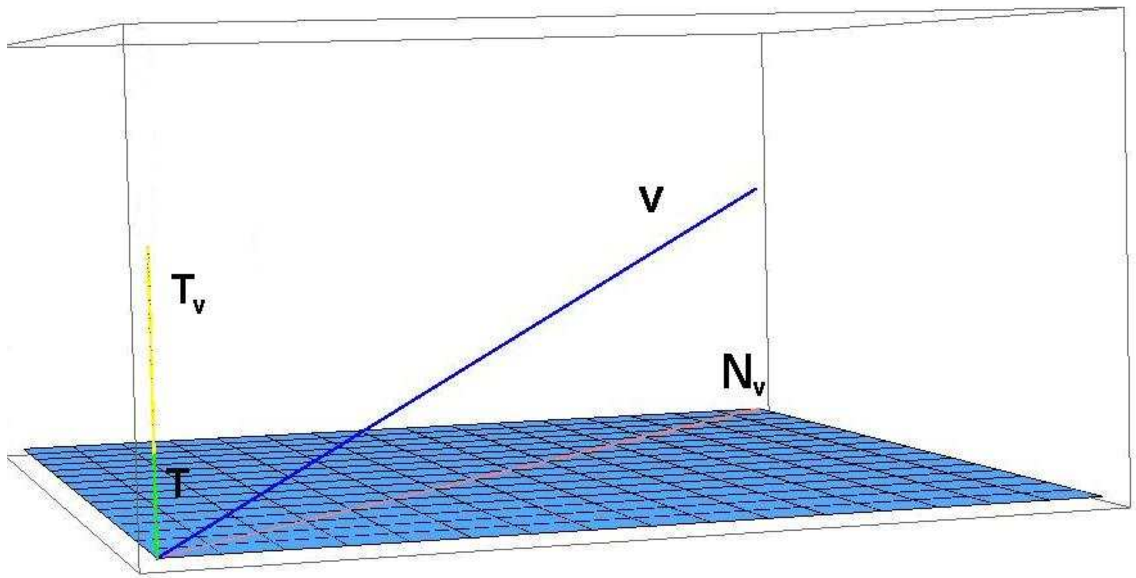
- motion of initial 3D curve in such vector field



Adjustments of the vector field

- removing the unwanted tangential part of the motion by projection of the vector field $\mathbf{v}$ to the normal plane to the curve

$$\partial_t \mathbf{r} = \mu\ \mathbf{N_v}\ + \epsilon k \mathbf{N}$$



$$\mathbf{T_v} = (\mathbf{T}.\mathbf{v})\mathbf{T}, \quad \mathbf{N_v} = \mathbf{v} - \mathbf{T_v}$$

- regularization of the motion in normal plane by curvature adding the curvature vector $k\mathbf{N}$

Numerical discretization

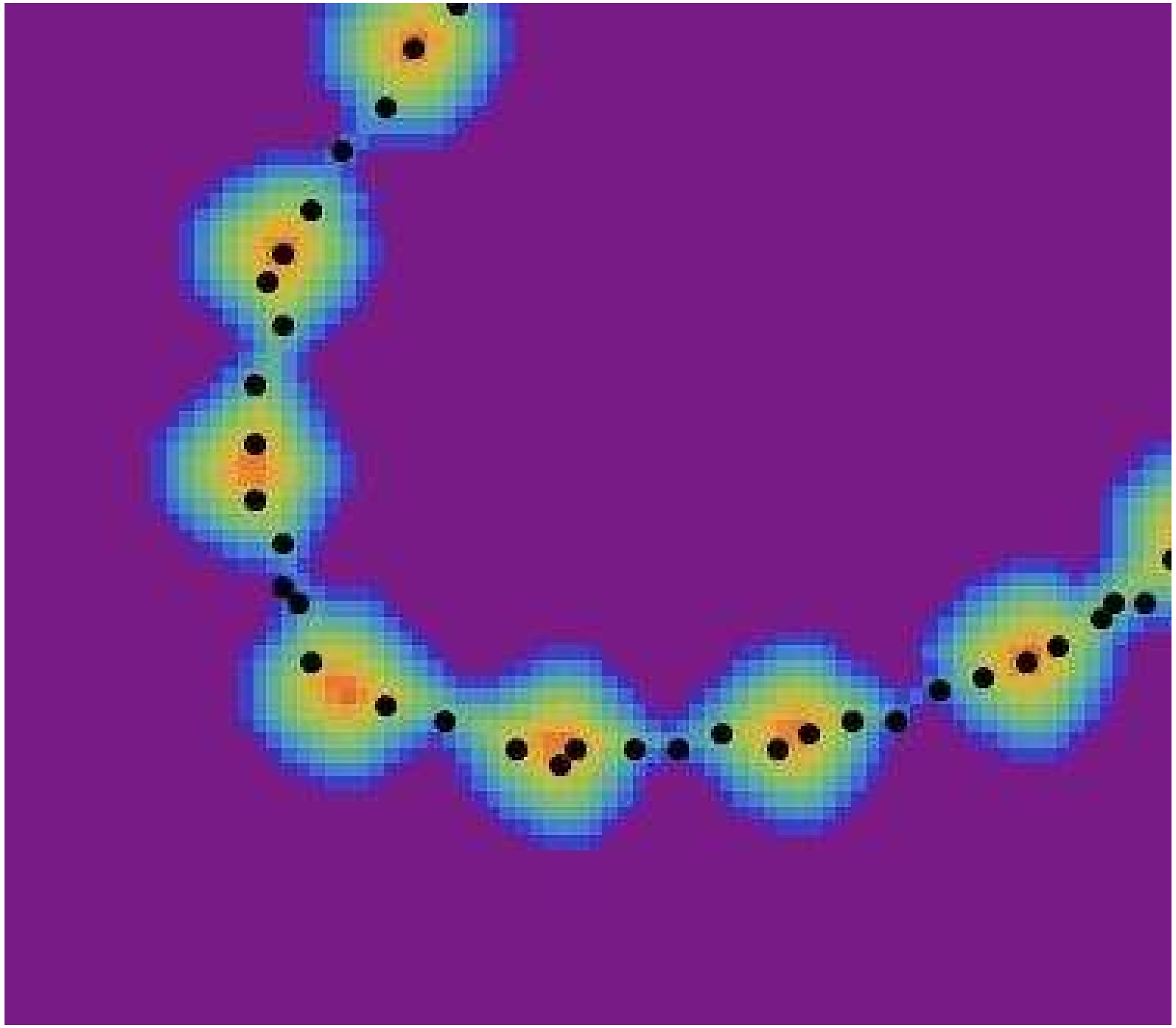
$$\partial_t \mathbf{r} = \mu \; \mathbf{N_v} \; + \epsilon k \mathbf{N}$$

$$\frac{\mathbf{r}_i^{n+1} - \mathbf{r}_i^n}{\tau} = \mu \, (\mathbf{N_v})_i^n + \epsilon \frac{2}{h_{i+1}^n + h_i^n} \left(\frac{\mathbf{r}_{i+1}^{n+1} - \mathbf{r}_i^{n+1}}{h_{i+1}^n} - \frac{\mathbf{r}_i^{n+1} - \mathbf{r}_{i-1}^{n+1}}{h_i^n} \right)$$

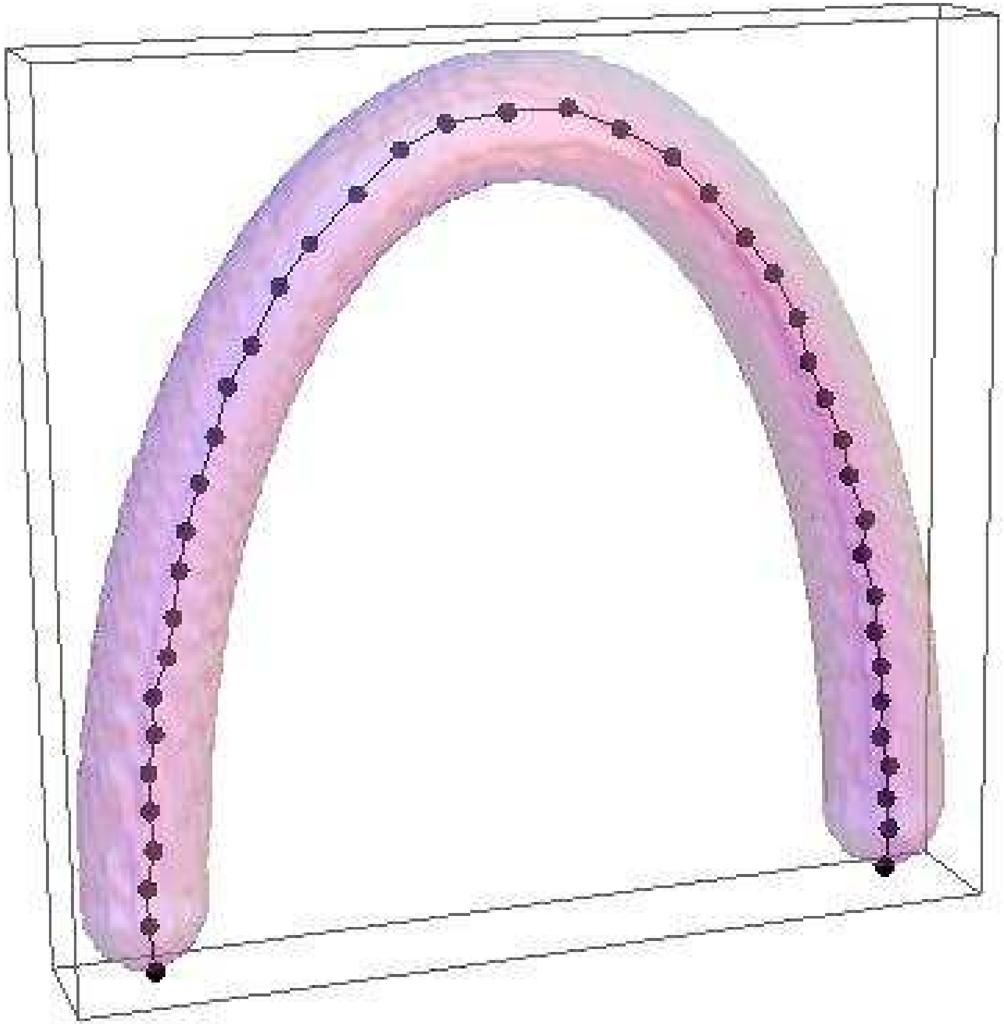
$$h_i^n = \sqrt{(x_i^n - x_{i-1}^n)^2 + (y_i^n - y_{i-1}^n)^2 + (z_i^n - z_{i-1}^n)^2}$$

$$\mathbf{r}_i^n = (x_i^n, y_i^n, z_i^n).$$

- motion of 2D curve using projection of the vector field to the normal plane and the regularization by curvature



- motion of 3D curve using projection of the vector field to the normal plane and the regularization by curvature



Asymptotically uniform tangential redistribution for 3D curves

- we consider an orthogonal basis $\mathbf{T}$, $\quad \mathbf{N_1} = \frac{\mathbf{N_v}}{|\mathbf{N_v}|}, \quad \mathbf{N_2} = \mathbf{N_1} \times \mathbf{T}$

$$k_1 = k\mathrm{N}.\mathrm{N}_1, \quad k_2 = k\mathrm{N}.\mathrm{N}_2, \quad k\mathrm{N} = k_1\mathbf{N_1} + k_2\mathbf{N_2}$$

$$\partial_t \mathbf{r} = \mu \ \mathbf{N_v} \ + \epsilon k \mathrm{N}, \quad U = \epsilon k_1 + \mu|\mathbf{N_v}|, \quad V = \epsilon k_2$$

$$\partial_t \mathbf{r} = U\mathbf{N_1} + V\mathbf{N_2} + \alpha\mathbf{T}$$

- for the local length $g = |\frac{\partial \mathbf{r}}{\partial u}| \approx \frac{\mathbf{r}_i - \mathbf{r}_{i-1}}{h}$, $h = \frac{1}{m}$ we have

$$\partial_t g = g\partial_s \alpha - g(Uk_1 + Vk_2)$$

- from the local length equation

$$\partial_t g = g\partial_s \alpha - g(Uk_1 + Vk_2)$$

- we get the total length equation

$$\frac{dL}{dt} = - < Uk_1 + Vk_2 >_\Gamma \ L$$

- asymptotically uniform redistribution

$$\frac{g}{L} \approx \frac{|\mathbf{r}_i - \mathbf{r}_{i-1}|}{Lh} = \frac{|\mathbf{r}_i - \mathbf{r}_{i-1}|}{\left(\frac{L}{m}\right)} = \frac{h_i}{\left(\frac{L}{m}\right)} \quad \rightarrow 1$$

$$\theta = \ln\left(\frac{g}{L}\right) \rightarrow 0, \quad \partial_t \theta = \partial_t \left[\ln\left(\frac{g}{L}\right)\right] = \frac{L}{g}\frac{\partial_t g L - g\partial_t L}{L^2}$$

$$\partial_t \theta = \partial_s \alpha - (Uk_1 + Vk_2) + < Uk_1 + Vk_2 >_\Gamma$$

$$\partial_t \theta = (e^{-\theta} - 1)\omega_r, \quad \partial_s \alpha = Uk_1 + Vk_2 - < Uk_1 + Vk_2 >_\Gamma + \left(\frac{L}{g} - 1\right)\omega_r$$

- our redistribution strategy is based on equation

$$\partial_s \alpha = Uk_1 + Vk_2 - < Uk_1 + Vk_2 >_\Gamma + \left(\frac{L}{g} - 1\right) \omega_r$$

- which is discretized together with our **general equation of motion** including normal and suitable tangential velocity components

$$\partial_t \mathbf{r} = \mu \, \mathbf{N_v} + \epsilon \, \partial_{ss} \mathbf{r} + \alpha \partial_s \mathbf{r}$$

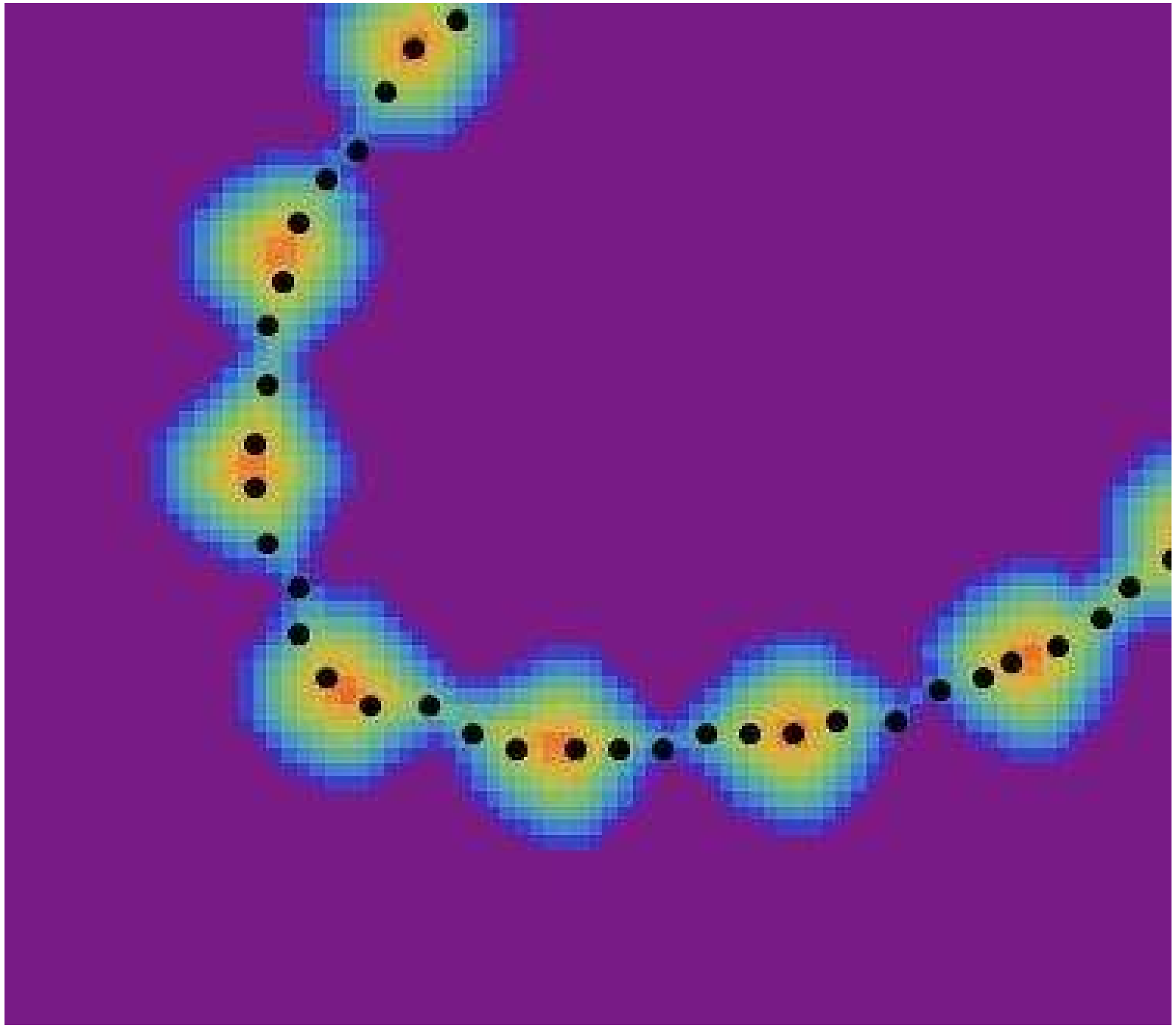
and solved by the semi-implicit scheme in a fast and stable way.

- analogy with asymptotically uniform redistribution for 2D curves evolution - Mikula and Ševčovič, M^2AS, 2004

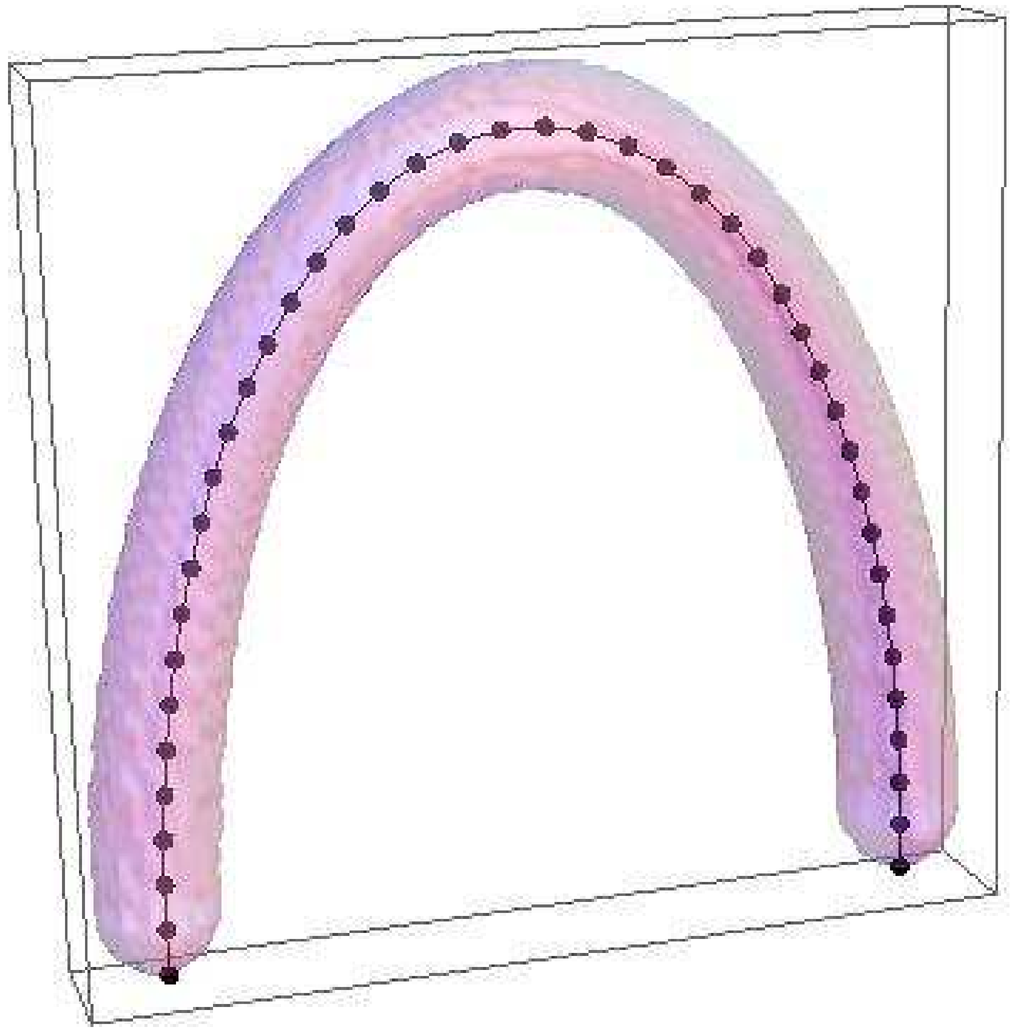
$$\partial_s \alpha = \beta k - < \beta k >_\Gamma + \left(\frac{L}{g} - 1\right) \omega_r$$

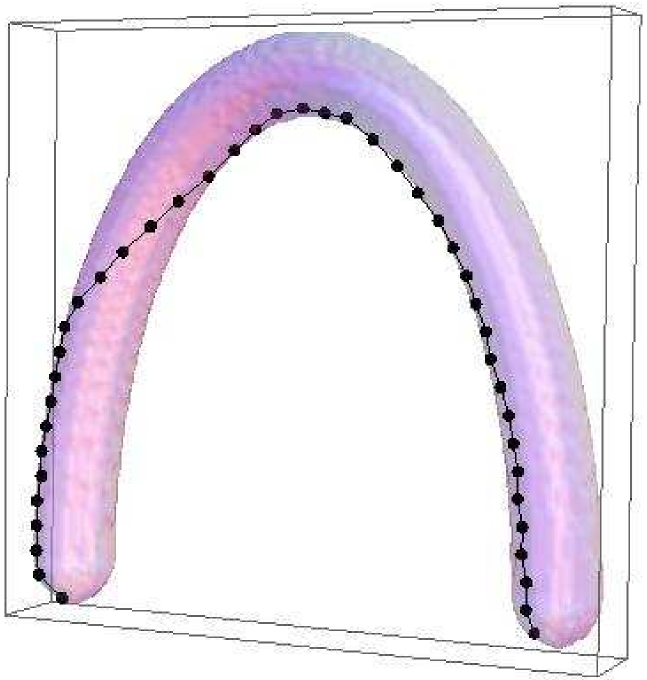
- Hou, Klapper, Si, JCP 1998 ($k_1 - k_2 - \omega - L$ 3D curve evolution formulation with redistribution conserving relative local length)

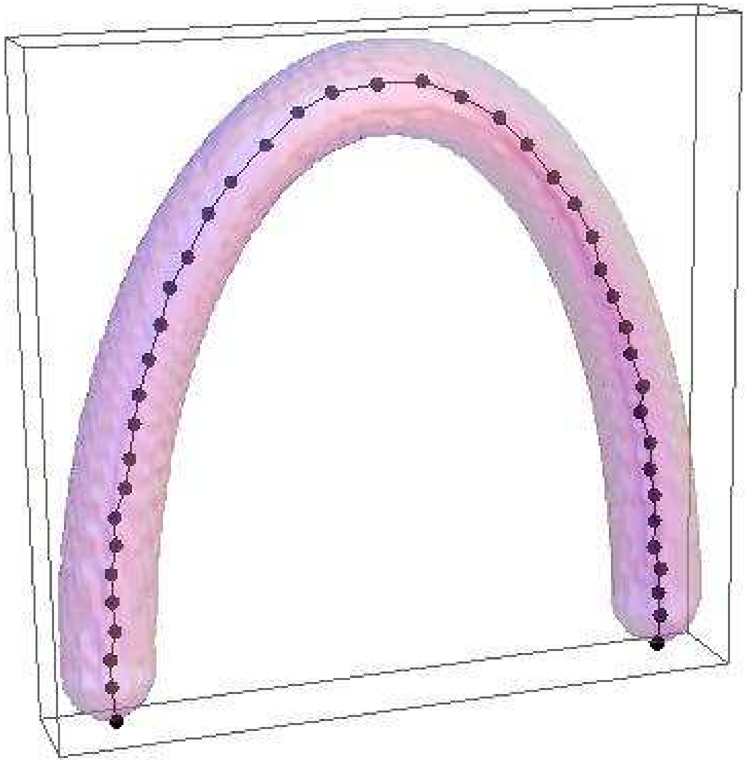
- motion of 2D curve using projection of the vector field to the normal plane, regularization by curvature and asymptotically uniform tangential redistribution

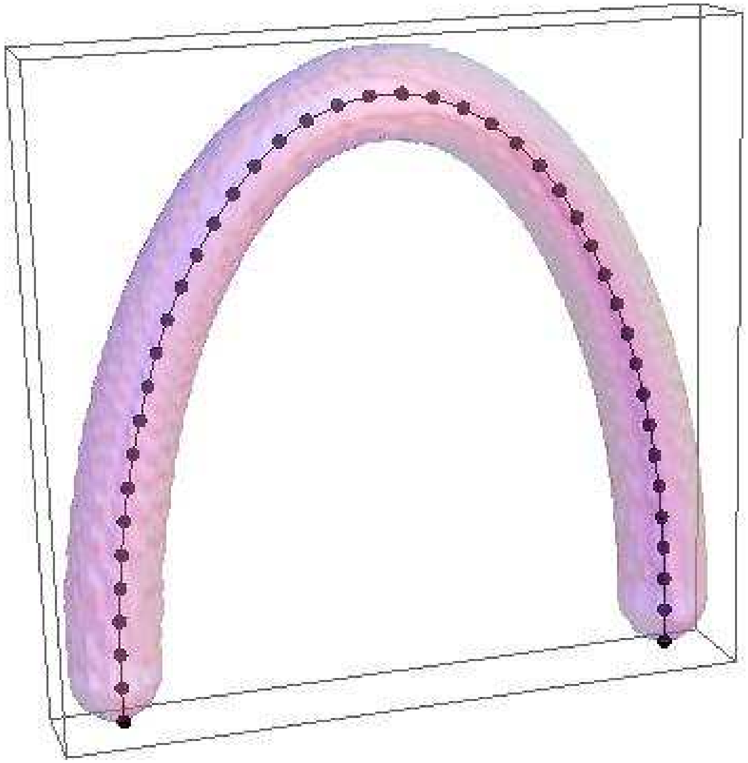


• motion of 3D curve using projection of the vector field to the normal plane, regularization by curvature and asymptotically uniform tangential redistribution

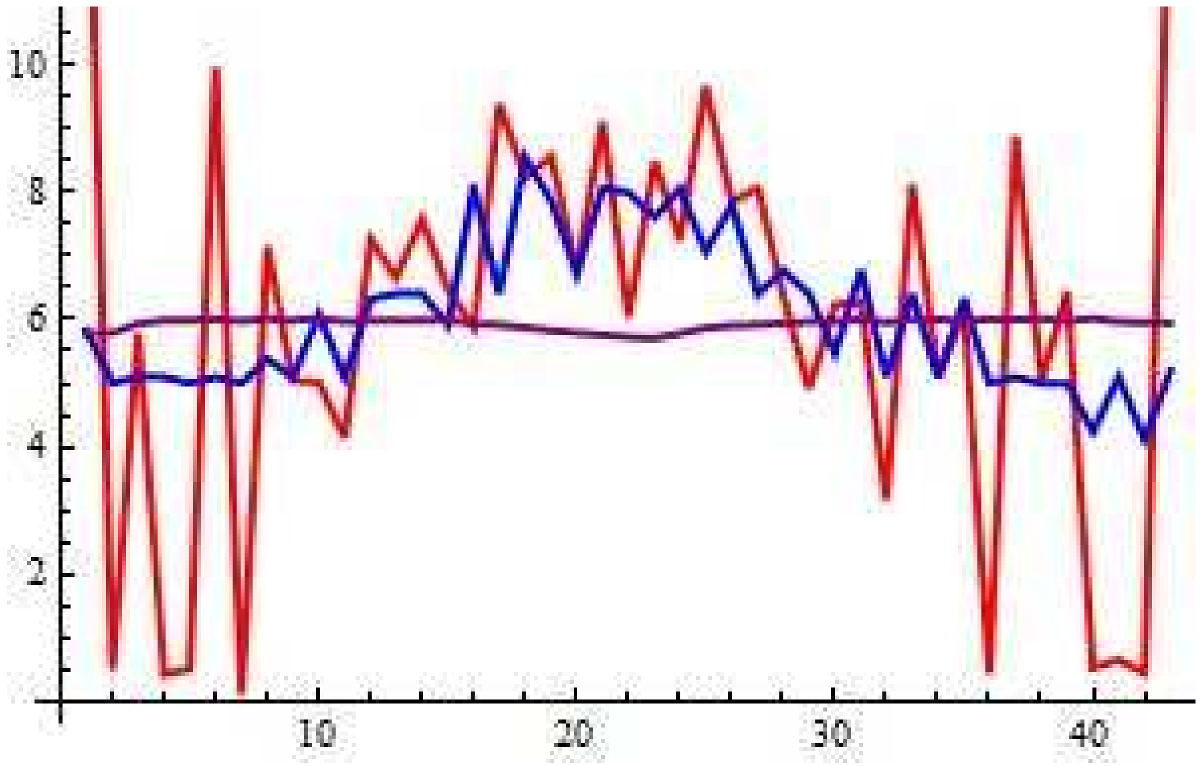


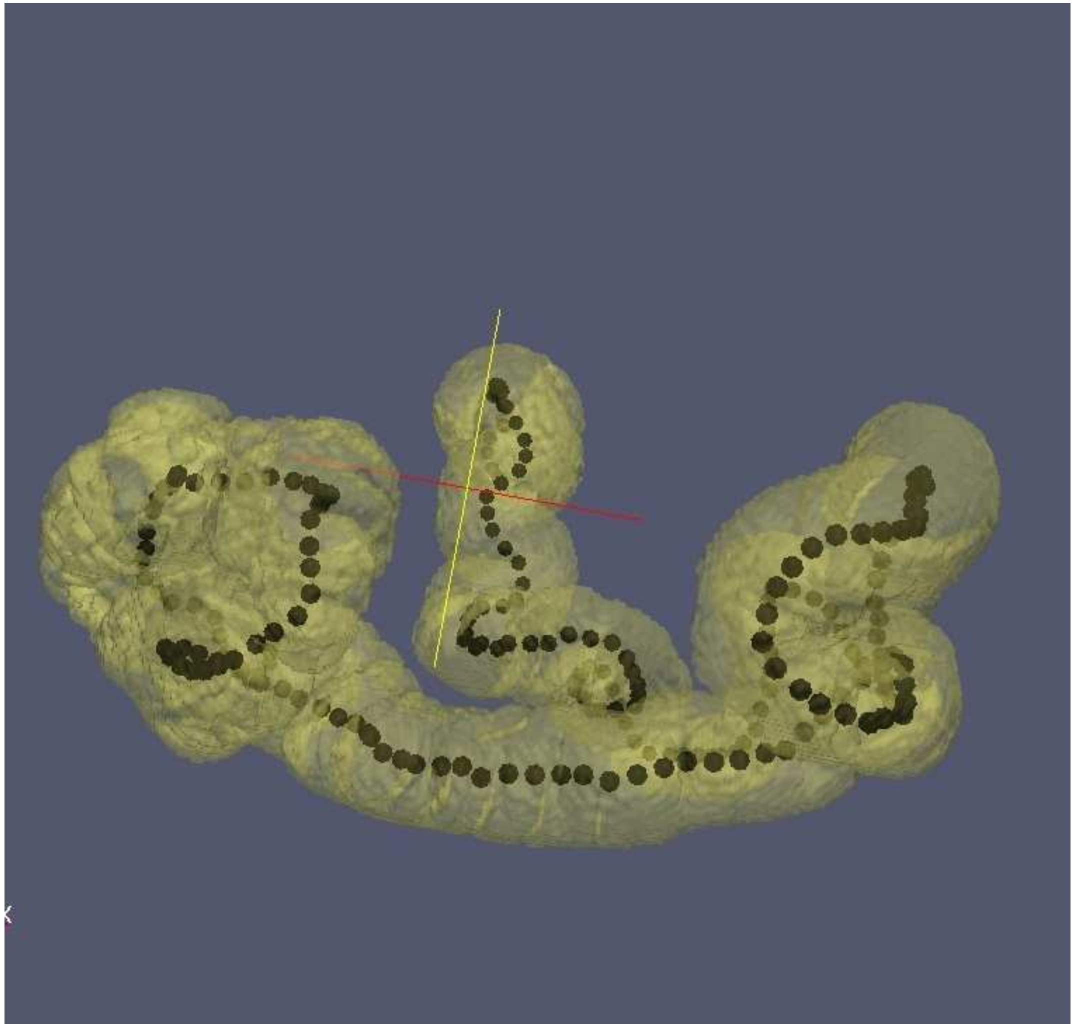


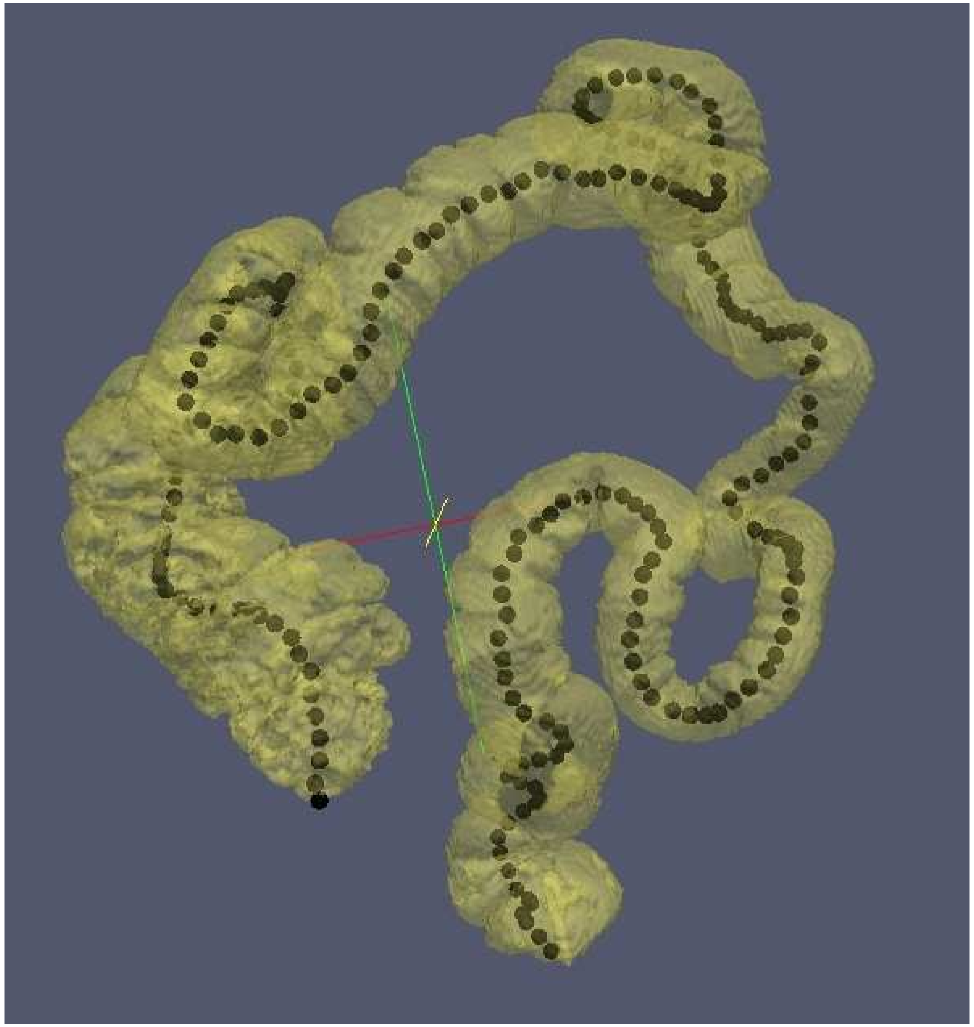




• Comparison of the grid point distances: the basic vector field (red), the projected vector field plus curvature regularization (blue), the final model (violet)







few videos

Thanks for your attention