NUMERICAL SOLUTION TO TWO FACTOR MODEL FOR OPTION PRICING WITH STOCHASTIC VOLATILITY

Cyril Ungvarský* — Karol Mikula**

In this paper we discuss numerical solution of the two factor option pricing model represented by a second order parabolic partial differential equation. In spite of the standard Black-Scholes equation, not only the asset price but also volatility follow a stochastic differential equation of diffusive type.

We mainly deal with an application of the finite volume method to the corresponding convection-diffusion equation accompanied with Dirichlet boundary conditions. It seems that such boundary conditions are much more natural and easy to implement in algorithms for computing solution of the two factor option pricing models than ones used by Forsyth, Vetzal and Zvan [2].

Finally, the numerically computed examples for two-factor model are presented and discussed. For a special choice of parameters in the underlying stochastic differential equations it is possible to compare the behaviour of our numerical solution with the exact Black-Scholes solution of the European call option problem. We use such an approach for testing the numerical scheme.

Keywords: option pricing, stochastic volatility, finite volume method

2000 Mathematics Subject Classification: 35Q80, 65M60

1 THE TWO FACTOR MODEL FOR OPTION PRICING

1.1 Governing equations

Let us consider the second order parabolic partial differential equation (PDE)

\[ \frac{\partial V}{\partial t} + (r - D)S \frac{\partial V}{\partial S} + \frac{1}{2} S^2 \frac{\partial^2 V}{\partial S^2} + \rho \sigma S \frac{\partial^2 V}{\partial S \partial v} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial v^2} + (\kappa(\theta - v) - \lambda v) \frac{\partial V}{\partial v} - rV = 0 , \tag{1} \]

where \( dS \) is the change of the stock price in a small time interval \( dt \), \( dv \) is the change of the stock volatility in \( dt \), \( \mu \) is the expected rate of return on \( S \) and \( dW_t \) are standard Wiener processes, \( i = 1, 2 \) with correlation given by \( E(dW_i dW_2) = \rho dt \). If we want to solve PDE (1) uniquely we have to specify initial and boundary conditions. The value of the option at the expiry time \( t = T \) is determined by the option payoff diagram. Then the value of the option \( V(S, v, T) \) at expiry time is given by

\[ V(S, v, T) = \begin{cases} \max(S - E, 0) & \text{for call option,} \\ \max(E - S, 0) & \text{for put option,} \end{cases} \tag{4} \]

which is a well known model for option pricing with stochastic volatility [2,5,7]. In equation (1) the function \( V(S(t), v(t), t) \) represents the value of the option as a function of \( S \), \( v \), \( t \), where \( S(t) \) is the market price of the underlying asset (for example stock), \( v(t) \) is volatility of the underlying asset price and \( t \) is time, \( t \in [0, T] \). The further parameters represent: \( \sigma \) — volatility of the volatility \( v \), \( \lambda \) — the market price of risk, \( \theta \) — the reversion level of volatility \( v \), \( k \) — the speed of reversion parameter for \( v \), \( r \) — the riskfree interest rate and \( \rho \) — the coefficient of correlation. In this two factor model the time evolution of the asset price \( S \) and volatility \( v \) are solutions of the stochastic differential equations [2,6]

\[ dS = \mu Sdt + \sqrt{v} SdW_1, \tag{2} \]

\[ dv = \kappa(\theta - v)dt + \sigma \sqrt{v} dW_2, \tag{3} \]

where \( E \) is the exercise (strike) price. Since we are dealing with 2D problem, we consider Dirichlet boundary conditions for \( S = 0, S \rightarrow \infty, v \rightarrow 0, v \rightarrow \infty \). From the stochastic differential equation (2) it follows that if the price of the stock \( S \) is zero, then the change of the stock price \( dS \) through time interval \( dt \) is zero. By that we get the following condition

\[ V(0, v, t) = 0, \quad S \rightarrow 0. \tag{5} \]

In the case, when the price of stock rises, i.e. \( S \rightarrow \infty \), we can expect that the owner of the call option will buy the stocks at the expiry date \( T \). Moreover the strike price will be small in comparison with the stock price, so we get the boundary condition \( V(S \rightarrow \infty, v, t) \rightarrow S \). However in the numerical approximation we are looking for the solution on the finite interval. So taking into account the details...
dividends and the time value of the money the boundary condition has the following form

$$V(S, v, t) = Se^{-D(T-t)} - e^{-r(T-t)}$$ for $$S \to \infty$$. (6)

In order to define Dirichlet boundary conditions for $$v \to 0$$ and $$v \to \infty$$ we use the explicit Black-Scholes formula for the value $$c(S, \delta, t)$$ of the European call option given by [4,5,7]

$$c(S, \delta, t) = Se^{-D(T-t)} N(d_1) - e^{-r(T-t)} N(d_2)$$, (7)

where

$$d_1 = \frac{\ln S + (r - D + \frac{\delta}{2})(T - t)}{\delta \sqrt{T-t}}$$,

$$d_2 = d_1 - \delta \sqrt{T-t}$$

and $$\delta$$ is a constant volatility of the stock. Let $$\delta \to 0^+$$. If $$Se^{-D(T-t)} \geq e^{-r(T-t)}$$ then $$d_1, d_2 \to \infty$$ and

$$c(S, \delta, t) = Se^{-D(T-t)} - e^{-r(T-t)}$$. If $$Se^{-D(T-t)} < e^{-r(T-t)}$$ then $$d_1, d_2 \to -\infty$$ and $$c(S, \delta, t) = 0$$. Thus we get the boundary condition

$$V(S, v, t) = \max(Se^{-D(T-t)} - e^{-r(T-t)}, 0), \ v \to 0$$. (8)

In case $$v \to \infty$$ the equation is reduced to [2]

$$\frac{\partial V}{\partial t} + (r - D) S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0$$, (9)

which is nothing else than the Black-Scholes equation and we have

$$V(S, v, t) = c(S, v, t), \ v \to \infty$$. (10)

In the sequel we will consider equation (1) with the expiry condition (4) and boundary conditions (5), (6), (8) and (10).

### 1.2 Transformation of the model

Before numerical discretization, as it is usual in computational finance dealing with option pricing, we transform the model by introducing new variables $$x, y, \tau$$ given by $$x = \ln(S/E), \ y = v$$ and $$\tau = T - t$$. Then $$S = e^x$$, $$t = T - \tau$$ and $$V(S, v, t) = Eu(x, y, \tau)$$. Using such transformation PDE (1) takes the form

$$\frac{\partial u}{\partial \tau} = (r - D - \frac{1}{2} \sigma^2) \frac{\partial u}{\partial x} + \frac{1}{2} \sigma^2 \frac{\partial^2 u}{\partial x^2} + \rho \sigma y \frac{\partial^2 u}{\partial x \partial y}$$

$$+ \frac{1}{2} \sigma^2 \frac{\partial^2 u}{\partial y^2} + (\kappa(y - \theta) - \lambda y) \frac{\partial u}{\partial y} - ru$$ . (11)

In the same way we transform the expiry and boundary conditions, which get the form

$$u(x, y, 0) = \max(e^x - 1, 0)$$, (12)

$$u(x, y, \tau) = 0, \ x \to -\infty$$, (13)

$$u(x, y, \tau) = e^{-Dr} - e^{-rt}, \ x \to \infty$$, (14)

$$u(x, y, \tau) = \max(e^{-D\tau} - e^{-rt}, 0), \ y \to 0$$, (15)

$$u(x, y, \tau) = \frac{c(E^x, y, \tau)}{E}, \ y \to \infty$$ . (16)

### 2 THE FINITE VOLUME METHOD

Before constructing the numerical scheme for PDE (11) we define the mesh $$L$$ of the domain $$\Omega \subset \mathbb{R}^2$$, which consists of a finite (control) volume $$p$$, such that $$\Omega = \bigcup_{p \in L}$$.

Since we work with a rectangular domain and we usually construct a simple rectangular grid, we use mesh $$L = \{(p_{ij})_{i=1,...,N_x, j=1,...,N_y}$$ of a domain $$(X_L, X_R) \times (0, Y)$$, satisfying the following assumptions: Let $$N_x, N_y \in N$$, $$h_x > 0, h_y > 0, h_z, h_y \in R$$ such that

$$\sum_{i=1}^{N_x} h_x = X_R - X_L, \ h_y \frac{1}{2} + \sum_{j=2}^{N_y} h_y + h_{N_y+1} = Y,$$

where $$h_1 = h_{N_x+1} = h_y$$. Let

$$x_i = X_L, \ x_{i+1} = x_{i+1} + h_x, \ for \ i = 1, \ldots, N_x,$$

$$y_j = -\frac{h_y}{2}, \ y_{j+1} = y_{j+1} + h_y, \ for \ j = 1, \ldots, N_y + 1,$$

such that

$$x_{N_x+1} = X_R, \ y_{N_y+1} = Y + \frac{h_y}{2}$$

Then

$$P_{ij} = [x_{i-1/2}, x_{i+1/2}] \times [y_{j-1/2}, y_{j+1/2}]$$.

Let $$(x_i)_{i=1,...,N_x}, (y_j)_{j=1,...,N_y}$$ be given by

$$x_i = x_{i-1/2} + \frac{1}{2} h_x, \ for \ i = 1, \ldots, N_x,$$

$$y_j = y_{j-1/2} + \frac{1}{2} h_y, \ for \ j = 1, \ldots, N_y + 1,$$

and $$k = \frac{T}{\tau_n}$$ be a discrete time step, $$\tau_n = nk$$.

<table>
<thead>
<tr>
<th>q7</th>
<th>q8</th>
<th>q9</th>
</tr>
</thead>
<tbody>
<tr>
<td>p_{pq}</td>
<td>p_{pq}</td>
<td>p_{pq}</td>
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<table>
<thead>
<tr>
<th>q4</th>
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<th>p_{pq}</th>
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<tbody>
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<td>n_{pq}</td>
<td>n_{pq}</td>
<td>n_{pq}</td>
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</table>

<table>
<thead>
<tr>
<th>q1</th>
<th>q2</th>
<th>q3</th>
</tr>
</thead>
</table>

Fig. 1. The control volume p, set of its neighbours qj, edges and normals p_{pq} and n_{pq}.  

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2.1 Discretization of PDE

We can write equation (11) in the form

\[ \frac{\partial u}{\partial t} + \vec{A} \cdot \nabla u = \nabla \cdot (B \nabla u) - ru, \tag{17} \]

where

\[ B = \frac{1}{2} \begin{pmatrix} 1 & \rho \sigma \\ \rho \sigma & \sigma^2 \end{pmatrix} \]

\[ \vec{A} = - \begin{pmatrix} r - D - \frac{1}{2}y - \frac{1}{2} \rho \sigma \\ \kappa(\theta - y) - \lambda y - \frac{1}{2} \sigma^2 \end{pmatrix} \tag{18} \]

represent the “diffusion matrix” and “velocity vector”, respectively [1].

In order to define the finite volume numerical scheme we integrate equation (17) over each control volume \( p \in L \)

\[ \int_p \left( \frac{\partial u}{\partial t} + \vec{A} \cdot \nabla u \right) dp = \int_p \left( \nabla \cdot (B \nabla u) - ru \right) dp. \]

Let us denote by \( u^n_p \) the expected approximation of the solution \( u \) at time \( \tau_n \) in the control volume \( p \). This value can be viewed as an approximation of the mean value of \( u \) over volume \( p \) or as an approximation of the value \( u(x_i, y_j, \tau_n) \) [1]. Let \((x_p, y_p)\) be the coordinates of the center of the control volume \( p \). Let \( e_{pq} \) denote the boundary control of the central control volume \( p \) and \( q \). Let \((x_{pq}, y_{pq})\) be the coordinates of the point of intersection of \( e_{pq} \) and the line connecting \((x_p, y_p)\) and \((x_q, y_q)\).

Now by backward Euler discretization for time derivative and using relation

\[ \nabla \cdot (\vec{A}u) = \vec{A} \cdot \nabla u + (\nabla \cdot \vec{A})u, \]

for the convective term we can write

\[ \frac{1}{k} (u^n_p - u^n_{p-1}) m(p) + \int_p \nabla \cdot (\vec{A}u^n) dp = \int_p \nabla \cdot (B \nabla u^n) dp + \int_p (\nabla \cdot \vec{A} - r)u^n dp, \tag{19} \]

where \( m(p) = h_x h_y \) is an area of the control volume \( p \). Using divergence theorem

\[ \frac{m(p)}{k} (u^n_p - u^n_{p-1}) + \sum_{q \in N_p} \int_{e_{pq}} \vec{A} \cdot n_{pq} u^n d\gamma \]

\[ = \sum_{q \in N_p} \int_{e_{pq} \cap \tau_p} B \nabla u^n \cdot n_{pq} d\gamma + (\nabla \cdot \vec{A} - r) m(p) u^n_p, \tag{20} \]

where \( n_{pq} \) is a unit normal vector to \( e_{pq} \), outward to \( p \) and \( N_p \) is a set of all neighbour volumes of the control volume \( p \). In our case \( N_p = \{q_1, q_2, \ldots, q_6 \} \) (see Fig. 1). Then we can write equation (20) in the form

\[ \frac{m(p)}{k} (u^n_p - u^n_{p-1}) + \sum_{q \in N_p} Q_{pq} u^n_{p+} \]

\[ = \sum_{q \in N_p} \int_{e_{pq} \cap \tau_p} B \nabla u^n \cdot n_{pq} d\gamma + (\nabla \cdot \vec{A} - r) m(p) u^n_p, \tag{21} \]

where \( Q_{pq} = \int_{e_{pq}} \vec{A} \cdot n_{pq} d\gamma \) and upwind technique is used to define

\[ u^n_{p+} = \begin{cases} u^n_p & \text{for } Q_{pq} \geq 0, \\ u^n_q & \text{for } Q_{pq} < 0. \end{cases} \tag{22} \]

Since in our case \( (\nabla \cdot \vec{A} - r) = \kappa + \lambda - r \) is a given constant, there it remains to approximate the term

\[ \sum_{q \in N_p} \int_{e_{pq} \cap \tau_p} B \nabla u^n \cdot n_{pq} d\gamma. \tag{23} \]

We can approximate

\[ S_2 := \int_{e_{pq}} B \nabla u^n \cdot n_{pq} d\gamma \approx \frac{1}{2} y_{pq} m(e_{pq}) \]

\[ \times \left( \frac{\partial u^n}{\partial x}(x_{pq}, y_{pq}) + \rho \sigma \frac{\partial u^n}{\partial y}(x_{pq}, y_{pq}) \right) \left( \begin{array}{c} 0 \\ 1 \end{array} \right), \]

where \( m(e_{pq}) \) is the length of the edge \( e_{pq} \). Then

\[ S_2 \approx \frac{1}{2} y_{pq} h_x \left( \rho \sigma \frac{\partial u^n}{\partial x}(x_{pq}, y_{pq}) \right. \]

\[ \left. + \sigma^2 \frac{\partial u^n}{\partial y}(x_{pq}, y_{pq}) \right). \tag{24} \]

For the approximation of the partial derivative with respect to \( y \) we use

\[ \frac{\partial u^n}{\partial y}(x_{pq}, y_{pq}) \approx \frac{u^n_p - u^n_q}{h_y}, \tag{25} \]

For the approximation of the partial derivative with respect to \( x \) we use

\[ \frac{\partial u^n}{\partial x}(x_{pq}, y_{pq}) \approx \frac{1}{2} \left( \frac{u^n_p - u^n_q}{2h_x} + \frac{u^n_q - u^n_{pq}}{2h_x} \right). \tag{26} \]

Using (25) and (26) in (24) we derive the approximation

\[ S_2 \approx \frac{1}{2} y_{pq} h_x \left( \frac{1}{4h_x} \rho \sigma \left( u^n_p - u^n_q + u^n_q - u^n_{pq} \right) \right. \]

\[ \left. + \frac{1}{h_y} \sigma^2 (u^n_p - u^n_{pq}) \right). \tag{27} \]

The remaining terms of the sum (23) can be derived in a similar way.
2.2 Discretization of initial and boundary conditions

Now we introduce a simple indexing of the control volumes in the admissible mesh $L$ of the domain $(X_L, X_R) \times (0, T)$. Let $u^n_{i,j}$, $i = 1, \ldots, N_x$, $j = 1, \ldots, N_y + 1$ be the approximation of the solution corresponding to $u^n_{ij}$ with volume center $(x_i, y_j) \in p$. This volume will be also denoted by $p_{ij}$.

Because the initial condition (12) does not depend on volatility and is not transformed in time, we can write it in the form

$$u^0_{i,j} = \max(e^{x_i} - 1, 0),$$

for $i = 1, \ldots, N_x$, $j = 1, \ldots, N_y + 1$. \hfill (28)

In the same way we can write the approximation of Dirichlet boundary conditions (13) and (14)

$$u^n_{i,j} = 0,$$ \hfill (29)

$$u^n_{N_x,j} = e^{x_{N_x}} - e^{-rT} e^{-rT},$$ \hfill (30)

for $j = 1, \ldots, N_y + 1$ and conditions (15), (16)

$$u^n_{i,1} = \max(e^{x_i} - e^{-rT} - e^{-rT}, 0),$$ \hfill (31)

$$u^n_{i,N_y+1} = c(Ee^{x_i}, y_{N_y+1}, T),$$ \hfill (32)

for $i = 1, \ldots, N_x$ and $c$ is given by (7).

2.3 The numerical scheme

Substitution (27) to (24) and calculating the remaining terms of the sum (23) gives for all neighbouring control volumes of the control volume $p = (i,j)$

$$- \frac{k}{8m(p)} \rho \sigma (y - \frac{1}{2}) u^n_{i-1,j-1} - \frac{k}{2h_y^2} \sigma^2 y - \frac{1}{2} u^n_{i-1,j-1}$$

$$+ \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i+1,j-1} - \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i+1,j-1}$$

$$- \frac{k}{8m(p)} \rho \sigma (y - \frac{1}{2}) u^n_{i,j-1} + \frac{1}{2} + \frac{k}{h_y} \sigma y - \frac{1}{2} u^n_{i,j-1}$$

$$- k(\kappa + \lambda - r) u^n_{i,j} + \frac{k}{m(p)} \sum_{q \in N_{pq}} Q_{pq} u^n_{pq}$$

$$+ \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i+1,j} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i+1,j}$$

$$- \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i,j} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i,j}$$

$$- \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i+1,j} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i+1,j}$$

$$- \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i,j+1} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i,j+1}$$

$$- \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i+1,j+1} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i+1,j+1}$$

$$+ \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i,j} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i,j}$$

$$+ \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i+1,j} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i+1,j}$$

$$+ \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i,j+1} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i,j+1}$$

$$+ \frac{k}{8m(p)} \rho \sigma (y + \frac{1}{2}) u^n_{i+1,j+1} + \frac{k}{2h_y^2} \sigma^2 y + \frac{1}{2} u^n_{i+1,j+1}$$

for $i = 2, \ldots, N_x - 1$, and $j = 2, \ldots, N_y$.

Let $U^n$ be the vector of the approximate solution. Then the system of linear equations (33) together with the boundary conditions can be written in the matrix form

$$MU^n = F,$$ \hfill (34)

which can be solved, e.g., by Gauss elimination efficiently.

3 DISCUSSION ON NUMERICAL EXPERIMENTS

3.1 The European call option

We present computational results for a two factor European call option model with parameters given in Table 1 (these data were taken from [2,3]).

<table>
<thead>
<tr>
<th>name</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ — exercise price</td>
<td>60</td>
</tr>
<tr>
<td>$r$ — the riskfree interest rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$\tau$ — time to expiration</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma$ — the volatility of volatility</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho$ — the coefficient of correlation</td>
<td>0.5, 0, -0.5</td>
</tr>
<tr>
<td>$\theta$ — the reversion level of the volatility</td>
<td>0.02</td>
</tr>
<tr>
<td>$\lambda$ — the market price of the risk</td>
<td>0.0</td>
</tr>
<tr>
<td>$\kappa$ — the mean reversion</td>
<td>2.0</td>
</tr>
</tbody>
</table>

In Table 2 we present computed values of the two factor European call option changing correlations between the evolution of the price and the volatility of the stock. In the first column there is the stock price $S_i$, in the second column we use three values of volatility $v$, the third column gives the values of the payoff diagram. In the 4th to 6th columns we have the computed values of $V(S, v, t)$ for three different coefficients of correlation and in the two last columns there are classical Black-Scholes values with fixed volatility $\delta = v$.

The differences between the explicit solution of the one factor model and our approximate solution of the two-factor model with stochastic volatility gives a reason for further research, for example, calibrating the two factor model for a specific stock.

If we compare assumptions of the one factor Black-Scholes model and two-factor model with the stochastic volatility, namely stochastic differential equations which represent the evolution of the stock price $S$, we find a correspondence $\delta = \sqrt{\theta}$, where $\delta$ is a constant volatility of the Black-Scholes model and $v$ is stochastic volatility of the two factor model. Then, if $\sigma \to 0$ and $\kappa \to \infty$, the values obtained by the two-factor model should converge to the values of the Black-Scholes model with $\delta = \sqrt{\theta}$. In Table 3 we present the computational results of the two-factor model at the stock price $S = 61.7649$, the volatility of the volatility $\sigma = 0.001$ and the reversion level of the volatility $\theta = 0.2$. As expected, with increasing speed of the reversion $\kappa$ the values of the option converge to the explicit Black-Scholes solution for any level of volatility $v$.
Table 2. Values of the European call option depend on stock price $S$, stock volatility $v$, and the coefficient of the correlation $\rho$.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$v$</th>
<th>payoff</th>
<th>$c(S, v, \tau)$</th>
<th>$c(S, \sqrt{v}, \tau)$</th>
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<tbody>
<tr>
<td>55.64</td>
<td>0.1</td>
<td>0</td>
<td>2.1805</td>
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<tr>
<td>61.77</td>
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<td>1.77</td>
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<td>5.2259</td>
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<td>8.56</td>
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<td>3.4270</td>
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<td>0.3</td>
<td>8.56</td>
<td>12.4993</td>
<td>12.5164</td>
</tr>
</tbody>
</table>

Table 3. Values of the European call option obtained by two-factor model when the stock price $S = 61.7649$, volatility $v$, the volatility of the volatility $\sigma = 0.001$ with increasing speed of the reversion $\kappa$.

<table>
<thead>
<tr>
<th>$S = 61.7649$</th>
<th>Explicit solution of B–S equation: 7.31444</th>
</tr>
</thead>
<tbody>
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<td>$v$</td>
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<td>0.60</td>
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References

[3] Heston, S. L.: A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Cur-

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