LINE GRAPHS AS MODELS OF FAULT-TOLERANT NETWORKS

Martin Knor and Ľudovít Niepel

1. Introduction

Interconnection networks are often modelled by graphs and digraphs. The switching elements or processors are represented by the vertices, and the communication links are represented by edges (if they are bidirectional) or arcs (if they are unidirectional). Definitions of basic notions and references to relevant results in the network modelling can be found in [5] and [6].

The connectivity of a (di)graph is a parameter that often serves as a criterion of the fault tolerance of the corresponding computational network. If some processors or communication links cease to function, it is important that the remaining processors can still intercommunicate. Hence, the underlying (di)graph is expected to have a high connectivity. In particular, it is interesting to know under which circumstances the (di)graph is maximally connected, i.e., its connectivity equals its minimum degree.

Further on, when a message is transmitted from one processor to another one, it is important to send it through as few processors as possible to decrease the communication delay. It means that the distances in the corresponding (di)graph are expected to be small. The maximum distance in a (di)graph is known as the diameter and it is important to determine large (di)graphs with small diameter. There is a famous problem related to this question. The (d,k)-graph (digraph) problem consists in determining the largest number of vertices of a regular (di)graph of diameter d and degree k. It is known that iterated line digraphs yield good lower bound for (d,k)-digraph problem, see [13].

In networks we often do not need small distances between every pair of its vertices. Usually, we expect that some "important" vertices have small distances to all the vertices of the (di)graph, but the distances between "unimportant" vertices may be larger. Hence, we require that the underlying (di)graph has a small radius.

Let F be an operator on (di)graphs. Then iterated F-graphs (digraphs) are defined as

$$F^i(G) = \begin{cases} G & \text{if } i = 0; \\ F(F^{i-1}(G)) & \text{if } i > 0. \end{cases}$$

Considering the sequence

$$G, F(G), F^2(G), \dots, F^i(G), \dots$$

we are interested in how the parameters of $F^i(G)$ depend on the parameters of G and on i. In particular, when a graph has some "good" properties, it is interesting to determine wheather or not these properties are saved by its iterated F-graphs. For instance, a complete digraph K_n is a Moore digraph with diameter 1, and although the iterated line digraphs of K_n are not Moore digraphs, their numbers of vertices are quite close to the Moore bound. Another good "starting" digraphs yield De Bruin and Kautz digraphs.

In this paper we survey the results on connectivity, diameter and radius, when F is the line digraph mapping, line graph mapping and the P_2-path graph mapping.

All necessary definitions and notations as well as the basic results on connectivity, diameter and radius can be found in monographs [8] and [15].

2. Line digraphs

Let D be a digraph, i.e., a directed graph without multiple arcs. By $L(D)$ we denote the **line digraph** of D. The vertices of $L(D)$ are just the arcs of D, and two vertices of $L(D)$, i.e. the arcs of D, say uv and xy, are joined by an arc in $L(D)$ if and only if $v = x$.

A digraph D is said to be **strongly connected** when for every pair of its vertices u and v there exists a $u - v$ path. The **strong connectivity** (or **strong vertex-connectivity**) of D, $\kappa(D)$, is the smallest number of vertices whose deletion results in a digraph that is either not strongly connected or trivial. Analogously, the **strong arc-connectivity** of D, $\lambda(D)$, is the smallest number of arcs whose deletion results in a digraph that is not strongly connected.

The minimum degree of D, $\delta(D)$, is the minimum over all the in-degrees and out-degrees of the vertices of D. For every digraph D we have

$$\kappa(D) \leq \lambda(D) \leq \delta(D),$$

see e.g. [14]. For iterated line digraphs it is enough to consider only the strong vertex-connectivity, as $\lambda(D) = \kappa(L(D))$. By the following theorem, the strong connectivity of loopless i-iterated line digraph attains its theoretical maximum.

Theorem 1 [10]. *Let D be a digraph without loops. Then there is i_D such that for every $i \geq i_D$*

$$\kappa(L^i(D)) = \delta(L^i(D)).$$

However, as $\delta(L^i(G)) = \delta(G)$, the strong connectivity of iterated line digraphs is bounded from above by a constant depending on D.

Now we turn our attention to distances in iterated line digraphs. Let D be a digraph, and let u be a vertex in D. Then:

out-eccentricity of u is $e^+_D(u) = \max\{d_D(u,v) : v \in V(D)\}$;
in-eccentricity of u is $e^-_D(u) = \max\{d_D(v,u) : v \in V(D)\}$;
eccentricity of u is $e_D(u) = \max\{e^+_D(u), e^-_D(u)\}$.

Using various eccentricities we obtain various radii. The **out-radius** $r^+(D)$ (**in-radius** $r^-(D)$, **radius** $r(D)$) is the minimum value of $e^+_D(u)$ ($e^-_D(u)$, $e_D(u)$) over all vertices u of D. The maximum values of out-eccentricity, in-eccentricity and eccentricity are equal and they are known as the **diameter** of D, $diam(D)$.

Let D' arise from D by reversing the orientation of all arcs. Then $e^-_{D'}(u) = e^+_D(u)$ for every vertex u in D, and hence, $r^+(D) = r^-(D')$. This observation enables us to restrict the considerations to radii r^+ and r, only.

Since $diam(D) < \infty$ if and only if D is strongly connected, the following theorems characterize the behavior of the out-radius, radius and the diameter in iterated line digraphs.

Theorem 2 [18]. *Let D be a nontrivial strongly connected digraph different from a directed cycle. Then there exist positive integers i_D and t_D such that for every $i \geq i_D$ we have*

$$r^+(L^i(D)) = i + t_D.$$

Theorem 3 [18]. *Let D be a nontrivial strongly connected digraph different from a directed cycle. Then there are t_D and t'_D such that for every $i \geq 0$ we have*

$$i + t_D \leq r(L^i(D)) \leq i + t'_D.$$

Theorem 4 [13]. *Let D be a nontrivial strongly connected digraph different from a directed cycle. Then for every $i \geq 0$ we have*

$$diam(L^i(D)) = diam(D) + i.$$

It means that if D is a strongly connected digraph, then the (out-)eccentricity of a central vertex in $L^i(D)$ differs from the eccentricity of any other vertex by at most a constant depending on D but not on i. In fact this is not surprising, as iterated line digraphs are good approximation for the (d, k)-digraph problem.

3. Line graphs

Let G be a graph. The **line graph** of G, $L(G)$, is a graph whose vertices are the edges of G. Two vertices are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G.

If G is a path of length j, then $L^i(D)$ is an empty graph for all $i > j$. If G is a cycle, then each iterated line graph of G is isomorphic to the original cycle; and if G is a claw $K_{1,3}$ then each iterated line graph of G is a triangle. Thus, it is enough to consider connected graphs different from a path, cycle and a claw. Such graphs G are called **prolific**, since each two members of the sequence $G, L(G), L^2(G), \ldots$ are distinct, see [24].

A graph G is said to be **connected** when for every pair of its vertices u and v there exists a $u - v$ path. The **connectivity** (or **vertex-connectivity**) of G, $\kappa(G)$, is the smallest number of vertices whose deletion results in a graph that is either not connected or trivial. Analogously, the **edge-connectivity** of G, $\lambda(G)$, is the smallest number of edges whose deletion results in a graph that is not connected.

It is well-known that for every graph G we have

$$\kappa(G) \leq \lambda(G) \leq \delta(G),$$

where $\delta(G)$ is the minimum degree of G, see e.g. [15]. However, $\lambda(G)$ and $\kappa(L(G))$ are not equal in general. We have only the following inequality

$$\lambda(G) \leq \kappa(L(G)) \leq \delta(L(G)).$$

Let G be a prolific graph. Then there is j_G such that $\delta(L^{j_G}(G)) \geq 3$. Denote $H = L^{j_G}(G)$. Then

$$\delta(L^i(H)) \geq 2^i(\delta(H) - 2) + 2$$

for every $i \geq 0$, see [24]. It means that the minimum degree grows exponentially in iterated line graphs.

For prolific graphs we have

Theorem 5 [22]. *Let G be a prolific graph. Then there is i_G such that for every $i \geq i_G$ we have*

$$\kappa(L^{i+2}(G)) \geq 4\delta(L^i(G)) - 6.$$

Recently, it was proved that for every prolific graph G there is k_G such that for every i, $i \geq k_G$, it holds

$$\delta(L^{i+1}(G)) = 2 \cdot \delta(L^i(G)) - 2,$$

see [16]. Hence, we have

Theorem 6. *Let G be a prolific graph. Then there is k_G such that for every $i \geq k_G$ we have*

$$\kappa(L^i(G)) = \delta(L^i(G)).$$

It means that although the minimum degree of iterated line graph grows exponentially in the number of iterations, the connectivity of iterated line graph attains its theoretical maximum (compare this with Theorem 1).

Let G be a graph and let u be a vertex in G. Then

eccentricity of u is $$e_G(u) = \max\{d_G(u, v) : v \in V(G)\}.$$

The **diameter** $diam(G)$ is the maximum value of $e_G(u)$, and the **radius** $r(G)$ is the minimum value of $e_G(u)$, respectively, over all vertices u of G.

The following theorems characterize the behavior of the diameter and the radius in iterated line graphs.

Theorem 7 [24]. *Let G be a prolific graph. Then there are i_G and t_G such that for every $i \geq i_G$ we have*

$$diam(L^i(G)) = i + t_G.$$

Theorem 8 [24]. *Let G be a connected noncomplete graph with the minimum degree at least three. Then for every $i \geq 1$ we have*

$$i + diam(G) - 2 \leq diam(L^i(G)) \leq i + diam(G).$$

Theorem 9 [24]. *Let G be a prolific graph. Then there are t_G and t'_G such that for every $i \geq 0$ we have*

$$\left(i - \sqrt{2\log_2 i}\right) + t_G < r(L^i(G)) < \left(i - \sqrt{2\log_2 i}\right) + t'_G.$$

By Theorems 7 and 9, if G is a prolific graph and k is a number, then there is k_G such that the diameter of $L^i(G)$ differs from the radius of $L^i(G)$ by at least k for every $i \geq k_G$ (compare this with Theorems 3 and 4). Clearly, almost all graphs are prolific. Therefore, the following result may be surprising.

Theorem 10 [17]. *Let $i \geq 0$. Then for almost all graphs G we have*

$$diam(L^i(G)) = r(L^i(G)) = i + 2.$$

We remark that the case $i = 0$ of the previous theorem is a folklore in probabilistic graph theory.

4. Path graphs

Let G be a graph and $k \geq 1$. The **P_k-path graph** $P_k(G)$ of G has as its vertex set the set of all paths of length k in G. Vertices of $P_k(G)$ that correspond to paths A and B of G are joined by an edge in $P_k(G)$ if and only if the edges of $A \cap B$ form a path of length $k-1$ and $A \cup B$ is either a path of length $k+1$ or a cycle of length $k+1$, see [7] and [23].

Path graphs generalize line graphs, as $P_1(G)$ is a line graph of G. Moreover, the P_k-path graph is a special subgraph of k-iterated line graph, see [19], and based on a notion of history, see [24], [18] and [19], we expect that the properties of P_k-path graphs are between those of k-iterated line digraphs and k-iterated line graphs.

If G is a connected graph, then $L(G)$ is connected, too. This is not the case of path graphs.

Let G be a graph, $k \geq 2$, $0 \leq t \leq k-2$, and let A be a path of length k in G. By $P^*_{k,t}$ we denote an induced subgraph of G which is a tree of diameter $k+t$ with a diametric path $(x_t, x_{t-1}, \ldots, x_1, v_0, v_1, \ldots, v_{k-t}, y_1, y_2, \ldots, y_t)$, such that all endvertices of $P^*_{k,t}$ have distance $\leq t$ either to v_0 or to v_{k-t} and the degrees of $v_1, v_2, \ldots, v_{k-t-1}$ are 2 in $P^*_{k,t}$. Moreover, no vertex of $V(P^*_{k,t}) - \{v_1, v_2, \ldots, v_{k-t-1}\}$ is joined by an edge to a vertex in $V(G) - V(P^*_{k,t})$. We say that the path A is in $P^*_{k,t}$, $A \in P^*_{k,t}$, if $(v_0, v_1, \ldots, v_{k-t})$ is a subpath of A.

$P^*_{6,3}$

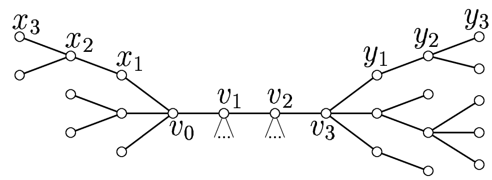

Figure 1

In Figure 1 a $P^*_{6,3}$ is pictured. Note that this graph contains also two $P^*_{6,0}$ and one $P^*_{6,1}$, but it does not contain a $P^*_{6,2}$. We remark that by thin halfedges are outlined possible edges joining vertices of $P^*_{6,3}$ to vertices in $V(G) - V(P^*_{6,3})$. We have

Theorem 11 [20]. *Let G be a connected graph without cycles of length smaller than $k+1$. Then $P_k(G)$ is disconnected if and only if G contains a $P^*_{k,t}$, $0 \leq t \leq k-2$, and a path A of length k such that $A \notin P^*_{k,t}$.*

As a corollary of Theorem 11 we have

Theorem 12 [21]. *Let G be a connected graph. Then $P_2(G)$ is disconnected if and only if G contains two distinct paths A and B of length two, such that the degrees of both endvertices of A are 1 in G.*

The situation is much more complicated for P_k-path graphs if $k > 2$. However, graphs with connected P_3-path graphs are already characterized.

Let G be a graph and let A be a path of length three in G. By P°_3 we denote a subgraph of G induced by vertices of a path of length 3, say (v_0, v_1, v_2, v_3), such that neither v_0 nor v_3 has a neighbour in $V(G) - \{v_1, v_2\}$. We say that the path A is in P°_3, $A \in P^\circ_3$, if $A = (v_0, v_1, v_2, v_3)$.

By P°_4 we denote an induced subgraph of G with a path (x, v_0, v_1, v_2, y), in which every neighbour of v_0 (and analogously every neighbour of v_2), except v_0, v_1

and v_2, has degree 1, or it has degree 2 and in this case it is adjacent to v_1. Moreover, no vertex of $V(P_4^\circ) - \{v_1\}$ is joined by an edge to a vertex of $V(G) - V(P_4^\circ)$ in G. We say that the path A lies in P_4°, $A \in P_4^\circ$, if (v_0, v_1, v_2) is a subpath of A.

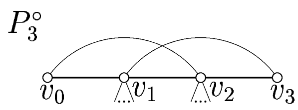

Figure 2

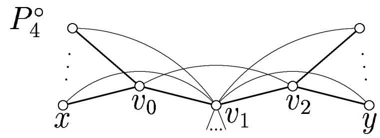

Figure 3

In Figure 2 a P_3° is pictured and a P_4° is in Figure 3. The edges that must be in G are painted thick, while edges, that are not necessarily in G, are painted thin.

Let K_4 be a complete graph on 4 vertices, and let S be a set (possibly empty) of independent vertices. A graph obtained from $K_4 \cup S$ by joining all vertices of S to special vertex of K_4 is denoted by K_4^*, see Figure 4. Let $K_{2,t}$ be a complete bipartite graph, $t \geq 1$, and let (X, Y) be the bipartition of $K_{2,t}$, $X = \{v_1, v_2\}$. Join t sets of independent vertices by edges, each to one vertex of Y; further, glue a set of stars (each with at least 3 vertices) by one endvertex, each either to v_1 or to v_2; glue a set of triangles by one vertex, each either to v_1 or to v_2; and finally, join v_1 to v_2 by an edge. The resulting graph is denoted by $K_{2,t}^*$, see Figure 5.

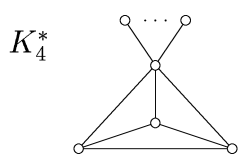

Figure 4

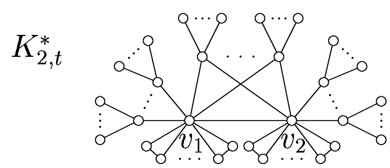

Figure 5

We have

Theorem 13 **[20].** *Let G be a connected graph such that $P_3(G)$ is not empty. Then $P_3(G)$ is disconnected if and only if one of the following holds:*

(1) *G contains a P_t°, $t \in \{3, 4\}$, and a path A of length 3 such that $A \notin P_t^\circ$;*

(2) *G is isomorphic to K_4^*;*

(3) *G is isomorphic to $K_{2,t}^*$, $t \geq 1$.*

The situation is easier if we restrict ourselves to graphs having special properties. We have

Theorem 14 **[11].** *Let G be a connected graph with $\delta(G) > 2(k-1)$. Then*

$$\lambda(P_k(G)) \geq 2(\delta - (k-1)).$$

As a corrolary of this result we receive

Theorem 15 **[11].** *Let G be a connected graph with $\delta(G) > 2(k-1)$. Then*

(1) *$P_k^i(G)$ is connected for every $i \geq 0$;*

(2) *$\lambda(P_k^i(G)) = \delta(P_k^i(G))$ if G is a regular graph.*

Compare the latter result with Theorem 6.

There are some bounds on the diameter of path graphs. However, also here the situation is too complicated for P_k-path graphs if k is “large”.

Theorem 16 **[3].** *Let G be a graph such that $P_k(G)$ is not empty. Then*

$$diam(P_k(G)) \geq diam(G) - k.$$

Theorem 17 **[3].** *Let G be a tree and let H be a component of $P_k(G)$. Then*

$$diam(H) \leq diam(G) + k(k-2).$$

Theorem 18 **[3].** *Let G be a connected graph such that $diam(G) \geq \frac{1}{2}k^2 + 5k - 2$, and $2 \leq k \leq 4$. Then for any component H of $P_k(G)$ we have*

$$diam(H) \leq diam(G) + k^2 - 2$$

We remark that for $k = 2$ the statement of Theorem 18 is valid for connected graphs with arbitrary diameter, see [21].

Up to now, only a few is known about iterated path graphs. We present here results on the diameter of iterated P_2-path graphs. First we introduce some definitions. By G_j, $j \geq 1$, we denote a tree composed of two paths of length two, central vertices of which are joined by a path of length $2j-1$. A **dragon** is a unicyclic graph composed of an even cycle C and a set of vertices, each joined by an edge to some vertex of C. Moreover, each pair of vertices of a dragon that have degree at least three, has an even distance (see Figure 6 for a dragon with cycle of length 8). **Broken dragon** is a tree composed of a diametric path T and a set of vertices, each joined by an edge to some vertex of T. Moreover, each pair of vertices of a broken dragon that have degree at least three, has an even distance (see Figure 8 for a broken dragon with diametric path of length 9). A **dragon’s egg** is a tree composed of a claw $K_{1,3}$ in which each edge is subdivided by one vertex, and a set of vertices, each joined by an edge either to the central vertex or to some endvertex of the subdivided claw (see Figure 7 for a dragon’s egg). If a connected graph G is different from a cycle, dragon, broken dragon, dragon’s egg and the graph G_j, $j \geq 1$, then G is called **2-prolific**.

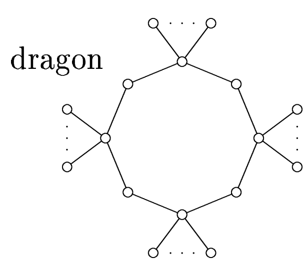

Figure 6

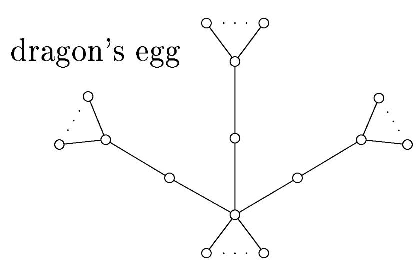

Figure 7

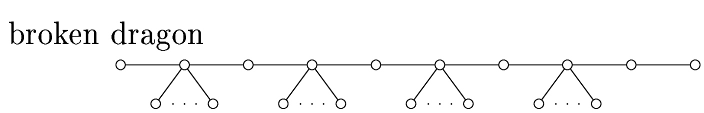

Figure 8

By Theorem 12 at most one component of $P_2(G)$ contains an edge if G is a connected graph. Hence, the following theorem characterizes the behaviour of the diameter in iterated P_2-path graphs.

Theorem 19 [21]. *Let G be a graph with a unique nontrivial component. Denote by H_j a nontrivial component of $P_2^j(G)$ (if it exists), $j \geq 0$.*

(1) *If G is a broken dragon, then there is i_G such that for every $i \geq i_G$ the graph $P_2^i(G)$ is empty.*

(2) *If G is a cycle, or a dragon, dragon's egg, or the graph G_j, $j \geq 1$, then there are i_G and t_G such that for every $i \geq i_G$ we have*

$$diam(H_i) = t_G.$$

(3) *If G is a prolific graph, then there are i_G and t_G such that for every $i \geq i_G$ we have*

$$diam(H_i) = 2i + t_G.$$

For $k > 2$, the situation is extremely complicated in general. However, for graphs with "large" degrees we have

Theorem 20 [11]. *Let G be a connected graph with $\delta(G) > 2(k-1)$. Then for every $i \geq 0$ it holds*

$$diam(P_k^i(G)) \leq diam(G) + 2ki.$$

Observe that by Theorem 15, if G satisfies the hypothesis of Theorem 20, then $P_k^i(G)$ is connected.

5. Concluding remarks

There are several invariants interesting for communication networks, which connect the connectivity with the diameter. The **persistence** of a (di)graph G, denoted $\rho_0(G)$, is the minimum number of vertices of G whose removal either increases the diameter or results in a trivial (di)graph. Similarly, the **line persistence** $\rho_1(G)$, is the minimum number of edges whose deletion increases the diameter, see [9].

The s-**vertex-diameter-vulnerability**, $K(s;G)$, of a (di)graph G is the maximum of the diameters of the (di)graphs obtained by removing s arbitrary vertices of G. Analogously, the s-**edge-diameter-vulnerability**, $\Lambda(s;G)$, is the maximum of the diameters of the (di)graphs obtained by removing s arbitrary edges, see [4].

The **conditional diameter** (or P-**diameter**) of a (di)graph G is the maximum distance among sub(di)graphs of G satisfying a given property P, see [1]. Its consideration could be of some interest if, in some applications, we need to minimize the communication delays between the network clusters modelled by such sub(di)graphs. If P is the property that sub(di)graph consists of a unique vertex, then the conditional diameter coincides with the standard diameter. Analogously as above, the conditional diameter vulnerability is defined, see [2].

The diameter vulnerability and the conditional diameter vulnerability of iterated line digraphs are studied in [25], [12] and [2]. However, there are no bounds for the invariants listed above for iterated line graphs or iterated path graphs at present.

Acknowledgement. The first author acknowledges support for this research by the VEGA grant No. 1/9176/02, and the second author acknowledges support for this research by Kuwait University grant #SM 02/00.

References

[**1**] Balbuena C., Carmona A., Fàbrega J., Fiol M.A., *On the connectivity and the conditional diameter of graphs and digraphs*, Networks **28** (1996), 97-105.

[**2**] Balbuena C., Ferrero D., *Conditional-diameter vulnerability of iterated line digraphs*, *Conditional-diameter vulnerability of iterated line digraphs*, I Jornades de Matematica Discreta y Algorismica, Barcelona, 1998.

[**3**] Belan A., Jurica P., *Diameter in path graphs*, Acta Math. Univ. Comenian., LXVIII (1999), 111-126.

[**4**] Bermond J.C., Bond J., Paoli M., Peyrat C., *Graphs and interconnection networks: diameter and vulnerability, in: Surveys in Combinatorics*, London Math. Soc. Lect. Notes Survey, **82** (1983), 1-30.

[**5**] Bermond J.C., Hombono N., Peyrat C., *Large fault-tolerant interconnection networks*, Graphs and Comb., **3** (1989), 107-123.

[**6**] Boesch F.T., Harary F., Kabell J.A., *Graphs as Models of Communication Network. Vulnerability: Connectivity and Persistence*, Networks, **11** (1981), 57-63.

[**7**] Broersma H.J., Hoede C., *Path graphs*, J. Graph Theory, **13** (1989), 427-444.

[**8**] Chartrand G., Lesniak L., *Graphs and Digraphs*, Wadsworth, Monterey, CA, 1986.

[**9**] Exoo G., *On a measure of communication network vulnerability*, Networks, **12** (1982).

[**10**] Fàbrega J., Fiol M.A., *Maximally connected digraphs*, J. Graph Theory, **13** (1989), 657-668.

[**11**] Ferrero D., *Connectivity of path graphs*, Acta Math. Univ. Comenianae, LXXII, **1** (2003), 59-66.

[**12**] Ferrero D., Padró C., *New bounds on the diameter vulnerability of iterated line grapgs*, Discrete Math., **233** (2001), 103-113.

[**13**] Fiol M.A., Yebra J.L.A., Alegre I., *Line digraph iterations and the (d,k) digraph problem*, IEEE Trans. Comput., **C-33** (1984), 400-403.

[**14**] Geller D., Harary F., *Connectivity in digraphs*, Lecture Notes in Mathematics, vol 186, Springer, 1970, pp. 105-114.

[**15**] Harary F., *Graph Theory*, Addison-Wesley, London, 1969.

[16] Hartke S.G., Higgins A.W., *Minimum degree growth of the iterated line graph*, Ars Combinatoria (to appear).

[17] Knor M., *A note on the radius in iterated line graphs*, Acta Math. Univ. Comenianae, LXIV, **2** (1995), 255-258.

[18] Knor M., Niepel Ľ., *Radii and centers in iterated line digraphs*, Discussiones Mathematicae, Graph Theory, **16** (1996), 17-26.

[19] Knor M., Niepel Ľ., *Path, trail and walk graphs*, Acta Math. Univ. Comenian., LXVIII (1999), 253-256.

[20] Knor M., Niepel Ľ., *Connectivity of path graphs*, Discussiones Mathematicae, Graph Theory, **20** (2000), 181-195.

[21] Knor M., Niepel Ľ., *Diameter in iterated path graphs*, Discrete Math., **233** (2001), 151-161.

[22] Knor M., Niepel Ľ., *Connectivity of iterated line graphs*, Discrete Applied Math., **125** (2003), 255-266.

[23] Li H., Lin Y., *On the characterization of path graphs*, J. Graph Theory, **17** (1993), 463-466.

[24] Niepel Ľ., Knor M., Šoltés Ľ., *Distances in iterated line graphs*, Ars Combinatoria, **43** (1996), 193-202.

[25] Padró C., Morillo P., *Diameter vulnerability of iterated line digraphs*, Discrete Math., **149** (1996), 189-204.

Authors: Doc. RNDr. Martin Knor, Dr., Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 813 68 Bratislava, Slovakia,
e-mail: knor@cvt.stuba.sk

Doc. RNDr. Ľudovít Niepel, CSc., Department of Mathematics & Computer Science, Faculty of Science, Kuwait University, P.O. box 5969 Safat 13060, Kuwait,
e-mail: NIEPEL@MATH-1.sci.kuniv.edu.kw