

1. Nájdiť definičný obor funkcie:  $f(x) = \sqrt{-2 + \log_3(x-1)}$

a)  $\log_3(x-1) > 0$   
 $x-1 > 0 \quad /+1$   
 $x > 1$   
 $(1, \infty)$

b)  $\sqrt{\dots} \geq 0$   
 $-2 + \log_3(x-1) \geq 0 \quad /+2$   
 $\log_3(x-1) \geq 2$   
 $(x-1) \geq 3^2 \quad /+1$   
 $x \geq 9+1$   
 $x \geq 10$   
 $x \in [10, \infty)$

$\log_3 x = 9$   
 $\nearrow$   
 $x = 3^9$

$D(f) = [10, \infty)$

2. Nájdiť definičný obor funkcie:  $f(x) = \operatorname{arctg} \sqrt{\frac{x^2-5x+6}{x^2+x+1}}$

$\operatorname{arctg} 0$ ,  $\sqrt{0}$ ,  $\frac{x^2 \dots}{x^2 \dots}$

a)  $\frac{x^2-5x+6}{x^2+x+1} =$

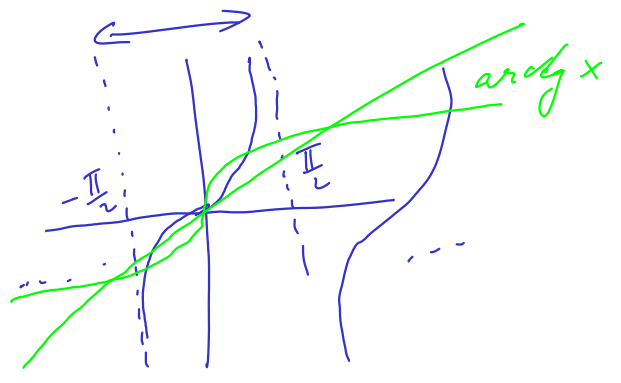
$x^2+x+1=0$

$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$

pre  $x \in \mathbb{R}$ :  $x^2+x+1 \neq 0$

pre  $x=0$ :  $x^2+x+1=1 > 0$   
 $\forall x \in \mathbb{R}$ :  $x^2+x+1 > 0 \leftarrow$

$\boxed{\forall x \in \mathbb{R}} \leftarrow$



b)  $\sqrt{\frac{\dots}{\dots}} \geq 0$

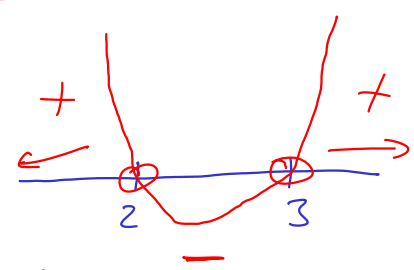
$\frac{x^2-5x+6}{x^2+x+1} \geq 0$

$x^2-5x+6 \geq 0 \leftarrow$

$(x-2)(x-3) \geq 0$

$x_1=2$ ,  $x_2=3 \leftarrow$

ax:  $a > 0$   $\cup$   $a < 0$   $\cap$



$x \in (-\infty, 2] \cup [3, \infty)$

$\boxed{D(f) = (-\infty, 2] \cup [3, \infty)}$

3. Nájďte definičný obor a načrtnite graf funkcie:

$$f(x) = |x^2 - 4|$$

$$x^2 - 4$$

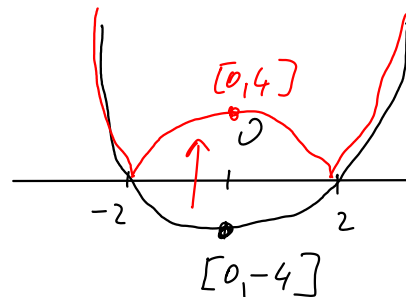


$$\begin{array}{l} x^2 - 4 \\ x \in \mathbb{R} \end{array} \quad \begin{array}{l} | \dots | \\ x \in \mathbb{R} \end{array}$$

$$(x+2)(x-2)$$

$$x_1 = -2 \quad x_2 = 2$$

$$D_f = \mathbb{R} \quad (-\infty, \infty)$$



$$|x| = \begin{cases} x & \text{pre } x \geq 0 \\ -x & \text{pre } x < 0 \end{cases}$$

4. Nájdiť definičný obor a načrtnite graf funkcie:

$$\log_{\frac{1}{2}}(x-1)$$

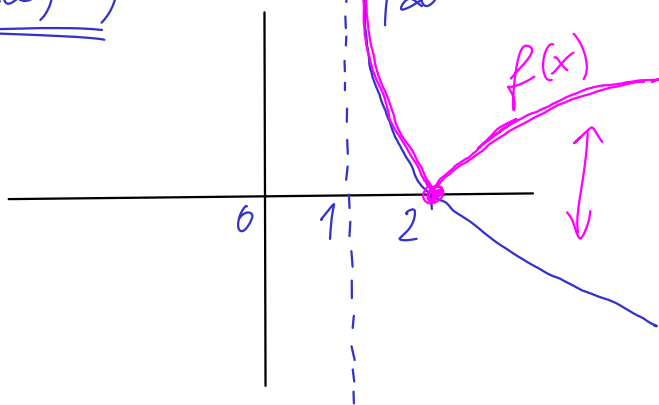
$\begin{matrix} > 0 \\ x-1 > 0 / +1 \\ \underline{x > 1} \end{matrix}$

$$|\log \dots|$$

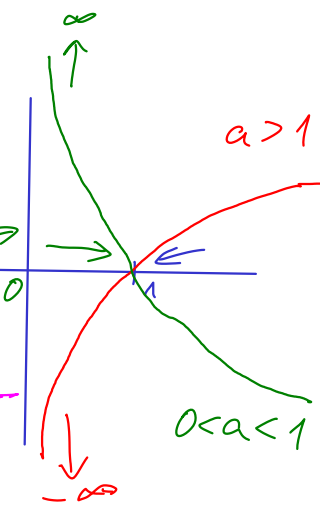
$\underline{D(f) = (1, \infty)}$

$$f(x) = \left| \log_{\frac{1}{2}}(x-1) \right|$$

$\frac{1}{2} < 1$   
 $x-1=0$  pre  $x=1$   
 $x-1=1$  pre  $x=2$



$$\log_a x$$



5. Napíšte zložené funkcie  $f(g(x))$  a  $g(f(x))$  a nájdite ich definičné obory ak  $f(x) = \sin x$  a  $g(x) = 2x - 3$ .

$$f(g(x)) = \sin(\underline{2x-3}) \quad D(f \circ g) = \underline{\underline{\mathbb{R}}} = (-\infty, \infty)$$

$$g(f(x)) = 2 \sin x - 3 \quad D(g \circ f) = \underline{\underline{\mathbb{R}}}$$

6. Nájďte definičný obor zloženej funkcie:

$$f(x) = \ln(2 \cos x - \sqrt{3})$$

a rozložte ju na zložky (elementárne funkcie alebo elementárne funkcie spojené aritmetickými operáciami).

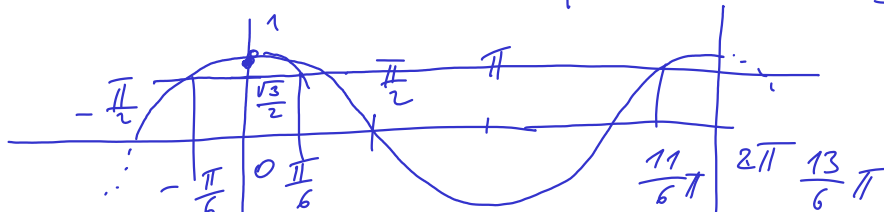
$\ln x$	$2x - \sqrt{3}$	$\cos x$
$a(x)$	$b(x)$	$c(x)$
$x > 0$	$\forall x \in \mathbb{R}$	$\forall x \in \mathbb{R}$
	$-2 - \sqrt{3}$ $2 - \sqrt{3}$	$-1 \leq \cos x \leq 1$

$$f(x) = a(b(c(x)))$$

$$2x - \sqrt{3} > 0 \quad | + \sqrt{3}$$

$$2x > \sqrt{3}$$

$$x > \frac{\sqrt{3}}{2}$$



$$0 < \frac{1}{2} < \frac{\sqrt{2}}{2} < \frac{\sqrt{3}}{2} < 1$$

$\leftarrow \cos x$

$$D(f) = \left(-\frac{\pi}{6} + 2k\pi, \frac{\pi}{6} + 2k\pi\right)$$

7. Určte definičný obor a paritu funkcie:  $f(x) = x^3 - x$

$$\boxed{D(f) = \mathbb{R}}$$

P: párna :  $f(x) = f(-x)$

N: nepárna :  $f(x) = -f(-x) \leftarrow$

P:  $x^2, \cos x, \dots$

N:  $x^3, x, \sin x, \dots$

$x^2 + x^3$  nie je ani P ani N

$$\begin{aligned} f(-x) &= (-x)^3 - (-x) = (-1)^3 x^3 + x = \\ &= -x^3 + x = -(x^3 - x) \end{aligned}$$

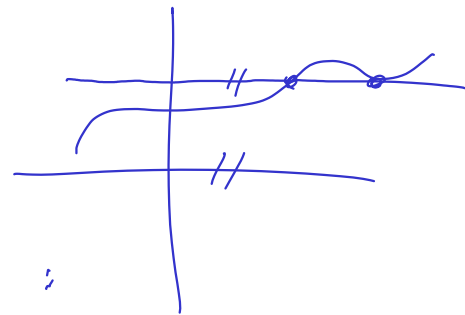
$$f(-x) = -f(x) \Leftrightarrow f(x) = -f(-x)$$

funkcia  $f(x)$  je nepárna.

8. Nájďte definičný obor, zistite či je funkcia  $f(x) = \frac{1-x}{1+x}$  injektívna (prostá) a ak je, tak k nej nájdite inverznú funkciu.

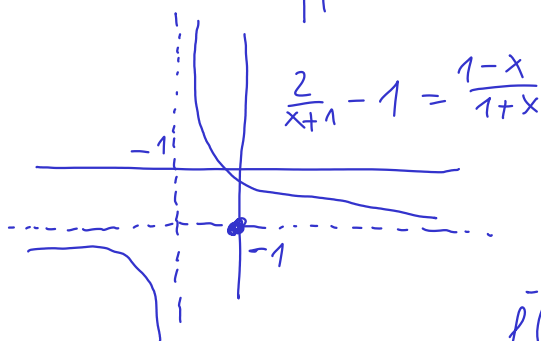
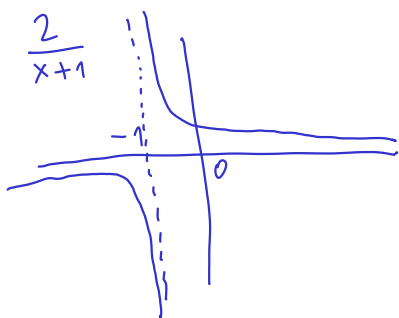
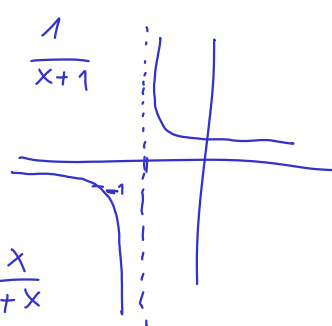
$$\frac{1-x}{1+x} \neq 0 \Rightarrow \begin{cases} 1+x \neq 0 \\ x \neq -1 \end{cases} \Rightarrow \boxed{D(f) = \mathbb{R} - \{-1\}}$$

ak  $x_1, x_2 \in D(f)$  a  $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$   
 $x_1, x_2 \in D(f)$  a  $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



$$\frac{1-x}{1+x} = -\frac{(x-1)}{x+1} = -\frac{\overset{+0}{x-1+1-1}}{x+1} = -\frac{x+1-2}{x+1} = -\left(\frac{x+1}{x+1} - \frac{2}{x+1}\right) =$$

$$= \frac{2}{x+1} - 1$$



$$y = \frac{1-x}{1+x} \quad | \cdot (1+x)$$

$$y(1+x) = 1-x$$

$$y + yx = 1-x \quad | +x - y$$

$$yx + x = 1-y$$

$$x(y+1) = 1-y \quad | : (y+1)$$

$$x = \frac{1-y}{y+1} = \frac{1-y}{1+y}$$

$$\boxed{f^{-1}(x) = \frac{1-x}{1+x}}$$



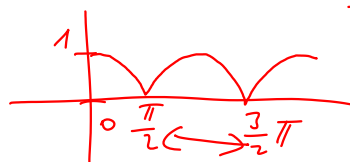
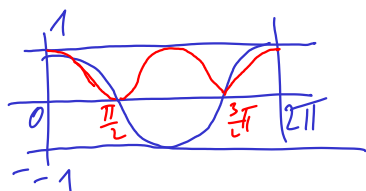
9. Nájďte definičný obor, zistite či je funkcia

$$f(x) = \cos^2 x$$

periodická a ak je, tak určte jej periódu  $p$ .

$$\boxed{D(f) = \mathbb{R}}$$

$\cos x$  je periodická  
mä periódu  $2\pi$



$$\cos^2 x = \cos x \cdot \cos x \\ (\cos x)^2$$

$$\cos^2 x = \cos(x \cdot x)$$

perióda  $\cos^2 x$   
je  $\pi$

$$\cos^2(x + \pi) = \dots = \cos^2 x$$

10. Zistite, či je funkcia  $f(x) = 1 - \text{sgn } x$  periodická. Ak je, určte jej periódu a načrtnite jej graf.

$$\text{sgn } x = \begin{cases} 1 & \text{pre } x > 0 \\ 0 & \text{pre } x = 0 \\ -1 & \text{pre } x < 0 \end{cases}$$

