1. Dané sú dve matice $\mathbb{A}$ a $\mathbb{B}$. Vypočítajte súčiny $\mathbb{A}.\mathbb{B}$ a $\mathbb{B}.\mathbb{A}$ ak existujú.

$$\overset{3\times 3}{\mathbb{A}} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix} \qquad \overset{2\times 3}{\mathbb{B}} = \begin{pmatrix} 2 & -1 & 5 \\ 1 & 3 & -1 \end{pmatrix}$$

$\cancel{A.B} \rightarrow 3\times 3 \neq 2\times 3$

$B.A \rightarrow \underline{2}\times 3 = 3\times \underline{3} \rightarrow 2\times 3$

$$\begin{pmatrix} 2 & -1 & 5 \\ 1 & 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0_{11} & -10_{12} & 13_{13} \\ 7_{21} & 9_{22} & -4_{23} \end{pmatrix}}} \leftarrow B.A$$

$$\bigcirc_{21} = 1.1 + 3.2 + (-1).0 = 7$$

2. Rovnako ako pri mocninách čísel, mocnina matice je súčin matice samej so sebou (ak existuje). Čiže napr. $\mathbb{A}^2 = \mathbb{A}.\mathbb{A}$ alebo $\mathbb{A}^3 = \mathbb{A}.\mathbb{A}.\mathbb{A}$. Vypočítajte:

$$\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}^n, \text{ kde } n \in \mathbb{N}.$$

$n = 1$

$A^1 = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$

$n = 2$

$A^2 = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^2 & 2a \\ 0 & a^2 \end{pmatrix}$

$n = 3$

$A^3 = A^2 . A = \begin{pmatrix} a^2 & 2a \\ 0 & a^2 \end{pmatrix}\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^3 & 3a^2 \\ 0 & a^3 \end{pmatrix}$

$\vdots$

$A^n = \begin{pmatrix} a^n & n a^{n-1} \\ 0 & a^n \end{pmatrix}$ Matematická indukcia

3. Nájdite inverznú maticu ku matici $\mathbb{A}$ pomocou Gauss-Jordanovej metódy.

$$\mathbb{A} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 2 & 1 & 3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ -2R_1 \\ -R_2 \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 2 & 3 & 0 & -1 & 1 \end{array}\right)\begin{matrix} \\ (-1) \\ +2R_2 \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & -1 & -4 & 1 & 1 \end{array}\right)\begin{matrix} +R_3 \\ +2R_3 \\ (-1) \end{matrix} \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 1 & 1 \\ 0 & 1 & 0 & -6 & 1 & 2 \\ 0 & 0 & 1 & 4 & -1 & -1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -3 & 1 & 1 \\ -6 & 1 & 2 \\ 4 & -1 & -1 \end{pmatrix}$$

$A \cdot A^{-1} = E$

$A^{-1} \cdot A = E$

4. Vyriešte maticovú rovnicu:

$$\overset{A}{\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}} - 3\mathbb{X} \cdot \overset{B}{\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}} = \mathbb{X}$$

$A - 3XB = X$

$A = 3XB + X$

$A = X\,3B + X$

$\underbrace{A = X(3B+E)}$

$X = A(3B+E)^{-1}$

$3B = \begin{pmatrix} 6 & 3 \\ 3 & 0 \end{pmatrix}$

$3B + E = \begin{pmatrix} 6 & 3 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 3 & 1 \end{pmatrix}$

$\left(\begin{array}{cc|cc} 7 & 3 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) - 2R_2 \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ 3 & 1 & 0 & 1 \end{array}\right) - 3R_1 \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ 0 & -2 & -3 & 7 \end{array}\right) \left(-\frac{1}{2}\right)$

$\sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & -2 \\ 0 & 1 & \frac{3}{2} & -\frac{7}{2} \end{array}\right) - R_2 \sim \left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & \frac{3}{2} & -\frac{7}{2} \end{array}\right)$

$(3B+E)^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 3 \\ 3 & -7 \end{pmatrix}$

$X = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} -1 & 3 \\ 3 & -7 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 & -5 \\ 2 & -4 \end{pmatrix}$

5. Vypočítajte determinant matice:

$$|\mathbb{A}| = \begin{vmatrix} 5 & 1 & 2 & 7 \\ 3 & 0 & 0 & 2 \\ 1 & 3 & 4 & 5 \\ 2 & 0 & 0 & 3 \end{vmatrix} \underset{-3R_1}{=} \begin{vmatrix} 5 & 1 & 2 & 7 \\ 3 & 0 & 0 & 2 \\ -14 & 0 & -2 & -16 \\ 2 & 0 & 0 & 3 \end{vmatrix} =$$

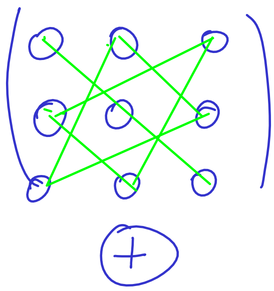
$$= -\begin{vmatrix} 3 & 0 & 2 \\ -14 & -2 & -16 \\ 2 & 0 & 3 \end{vmatrix} (-1) = \begin{vmatrix} 3 & 0 & 2 \\ 14 & 2 & 16 \\ 2 & 0 & 3 \end{vmatrix} = 2 \cdot \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 2(9-4) = 2 \cdot 5 = \underline{\underline{10}}$$

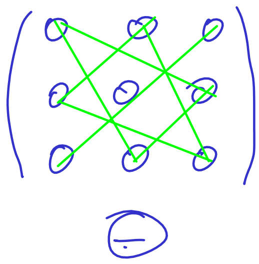
6. Vypočítajte determinant matice:

$$|\mathbb{A}| = \begin{vmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \\ 3 & 0 & 0 & -3 \end{vmatrix} \overset{+S_1}{=} \begin{vmatrix} 1 & -1 & 2 & 0 \\ 2 & 1 & -2 & 0 \\ -1 & 2 & -4 & 0 \\ 3 & 0 & 0 & 0 \end{vmatrix} = \underline{\underline{0}}$$

7. Pomocou determinantov vypočítajte inverznú máticu $\mathbb{A}^{-1}$:

Sarusovo pravidlo!





$$\mathbb{A} = \begin{pmatrix} 3^{+}_{11} & 2^{-}_{12} & -1^{+}_{13} \\ 2^{-}_{21} & -1^{+}_{22} & 4^{-}_{23} \\ -1^{+}_{31} & 3^{-}_{32} & 2^{+}_{33} \end{pmatrix}$$

$|\mathbb{A}| = -20 - 43 = \underline{\underline{-63}} \neq 0$

$\underline{a_{ij}} \to A_{ij} = \underline{(-1)^{i+j}} \cdot | \dots |$

$A_{11} = \begin{vmatrix} -1 & 4 \\ 3 & 2 \end{vmatrix} = \underline{-14}$ $\quad A_{21} = -\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = \underline{-7}$ $\quad A_{31} = \begin{vmatrix} 2 & -1 \\ -1 & 4 \end{vmatrix} = \underline{7}$

$A_{12} = -\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = \underline{-8}$ $\quad A_{22} = \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} = \underline{5}$ $\quad A_{32} = -\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix} = \underline{-14}$

$A_{13} = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = \underline{5}$ $\quad A_{23} = -\begin{vmatrix} 3 & 2 \\ -1 & 3 \end{vmatrix} = \underline{-11}$ $\quad A_{33} = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = \underline{-7}$

$R_1 \qquad R_2 \qquad R_3$

$$\mathbb{A}^{-1} = \underline{-\frac{1}{63}} \begin{pmatrix} -14 & -7 & 7 \\ -8 & -5 & -14 \\ 5 & -11 & -7 \end{pmatrix} = \underline{\frac{1}{63} \begin{pmatrix} 14 & 7 & -7 \\ 8 & 5 & 14 \\ -5 & 11 & 7 \end{pmatrix}}$$