On extensions of the weight-center operator defined over intuitionistic fuzzy sets

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Fuzzy sets

- Ω - universe,
- A - fuzzy set defined on Ω, $A \subseteq \Omega$,
- $\mu_A(x)$ - membership function

$$\mu_A(x) : \Omega \to [0, 1].$$

IF sets

- Ω - universe,
- A - IF set defined on Ω, $A \subseteq \Omega$,
- $\mu_A(x)$ - membership function,
- $\nu_A(x)$ - nonmembership function

$$\mu_A(x), \nu_A(x) : \Omega \to [0, 1]$$

$$\forall x \in \Omega : 0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

We write

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in \Omega\}.$$

Relations and operations defined on IF sets

$$A = B \quad \Leftrightarrow \quad \forall x \in \Omega : \mu_A(x) = \mu_B(x) \;\&\; \nu_A(x) = \nu_B(x)$$

$$A \subseteq B \quad \Leftrightarrow \quad \forall x \in \Omega : \mu_A(x) \leq \mu_B(x) \;\&\; \nu_A(x) \geq \nu_B(x)$$

$$\overline{A} \quad = \quad \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in \Omega\}$$

$$\Box(A) \quad = \quad \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle \mid x \in \Omega\}$$

$$\diamond(A) \quad = \quad \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle \mid x \in \Omega\}$$

Relations and operations defined on IF sets

$$I(A) = \{\langle x, k, l \rangle \mid x \in \Omega\}$$

$$C(A) = \{\langle x, K, L \rangle \mid x \in \Omega\}$$

where

$$k = \inf_{y \in \Omega} \mu_A(y), \quad l = \sup_{y \in \Omega} \nu_A(y)$$

$$K = \sup_{y \in \Omega} \mu_A(y), \quad L = \inf_{y \in \Omega} \nu_A(y)$$

Operator defined on IF sets over a finite universe

Let Ω be the finite set. Then

$$W(A) = \left\{ \left\langle x, \frac{\sum\limits_{y \in \Omega} \mu_A(y)}{card(\Omega)}, \frac{\sum\limits_{y \in \Omega} \nu_A(y)}{card(\Omega)} \right\rangle \mid x \in \Omega \right\}$$

$card(\Omega)$ - the number of the elements of the finite universe Ω.

Remark

$W(A)$ is an IF set.

$$0 \leq \frac{\sum_{y \in \Omega} \mu_A(y)}{card(\Omega)} + \frac{\sum_{y \in \Omega} \nu_A(y)}{card(\Omega)} \leq 1$$

Assumptions for modifications of weight-center operator defined on IF sets A and B over the finite universe Ω

Denote

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle \mid x \in \Omega\}$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle \mid x \in \Omega\}$$

where

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

then

$$H_{0,0}(X) = \{\langle x, 0, \nu_X(x)\rangle \mid x \in \Omega\}$$

$$J_{0,0}(X) = \{\langle x, \mu_X(x), 0\rangle \mid x \in \Omega\}$$

Modification of weight-center operators defined on IF sets A and B over the finite universe Ω

Let Ω be the finite set and $B \neq H_{0,0}(X)$, $B \neq J_{0,0}(X)$. Then

$$W_B^1(A) = \left\{ \left\langle x, \frac{\sum\limits_{y\in\Omega} \mu_A(y)\cdot\mu_B(x)}{card(\Omega)\cdot\sum\limits_{y\in\Omega}\mu_B(y)}, \frac{\sum\limits_{y\in\Omega} \nu_A(y)\cdot\nu_B(x)}{card(\Omega)\cdot\sum\limits_{y\in\Omega}\nu_B(y)} \right\rangle \mid x \in \Omega \right\}$$

Riečan, B., A. Ban, K. Atanassov, Modifications of the weight-center operator, defined over intuitionistc fuzzy sets. Part 1. *Issues in Intuitionistic Fuzzy Sets and Generalized Nets*

Assumptions for next modifications of weight-center operators defined on IF sets A and B over the finite universe Ω

$$\|X\| = \frac{\sum\limits_{y\in\Omega} (\mu_X(y) + \nu_X(y))}{card(\Omega)}$$

Modification of weight-center operator defined on IF sets A and B over the finite universe Ω

Let $B \neq H_{0,0}(X)$, $B \neq J_{0,0}(X)$ and $\|A\| \leq \|B\|$. Then

$$W_B^2(A) =$$

$$\left\{ \left\langle x, \frac{\sum\limits_{y\in\Omega} \mu_A(y) \cdot \mu_B(x)}{2\max(\sum\limits_{y\in\Omega} \mu_B(y), \sum\limits_{y\in\Omega} \nu_B(y))}, \frac{\sum\limits_{y\in\Omega} \nu_A(y) \cdot \nu_B(x)}{2\max(\sum\limits_{y\in\Omega} \mu_B(y), \sum\limits_{y\in\Omega} \nu_B(y))} \right\rangle \right\}$$

Riečan, B., A. Ban, K. Atanassov, Modifications of the weight-center operator, defined over intuitionistc fuzzy sets. Part 2. *Notes on Intuitionistic Fuzzy Sets*, Vol. 19, 2013, No. 2, 1-5.

Modification of weight-center operator defined on IF sets A and B over the finite universe Ω

Let $B \neq H_{0,0}(X)$, $B \neq J_{0,0}(X)$ and $\|A\| \leq \|B\|$. Then

$$W_B^3(A) = \left\{ \left\langle x, \frac{\sum\limits_{y\in\Omega} \mu_A(y)\cdot\mu_B(x)}{\sum\limits_{y\in\Omega}(\mu_B(y)+\nu_B(y))}, \frac{\sum\limits_{y\in\Omega} \nu_A(y)\cdot\nu_B(x)}{\sum\limits_{y\in\Omega}(\mu_B(y)+\nu_B(y))} \right\rangle \right\}$$

Riečan, B., A. Ban, K. Atanassov, Modifications of the weight-center operator, defined over intuitionistc fuzzy sets. Part 3. *Notes on Intuitionistic Fuzzy Sets*, Vol. 19, 2013, No. 3, 20-24.

Modification of weight-center operator defined on IF sets A and B over the countable universe Ω

Let Ω be countable and $\sum\limits_{y\in\Omega} \mu_A(y) + \sum\limits_{y\in\Omega} \nu_A(y) < c$, $c < \infty$.

Let $B \neq H_{0,0}(X)$ and $B \neq J_{0,0}(X)$. Then

$$W_B^4(A) = \left\{ \left\langle x, \frac{\sum\limits_{y\in\Omega} \mu_A(y) \cdot \mu_B(x)}{c \cdot \sum\limits_{y\in\Omega} \mu_B(y)}, \frac{\sum\limits_{y\in\Omega} \nu_A(y) \cdot \nu_B(x)}{c \cdot \sum\limits_{y\in\Omega} \nu_B(y)} \right\rangle \mid x \in \Omega \right\}$$

Tomanová, M., Modifications of the weight-center operator, defined over intuitionistc fuzzy sets with a countable universe. *Notes on Intuitionistic Fuzzy Sets*, Vol. 19, 2013, No. 3, 36-42.

Weight-center operator defined on IF sets A and B over the closed interval

Let $\Omega = [\alpha, \beta]$, $\alpha < \beta$. Then

$$W(A) = \left\{ \left\langle x, \frac{\int_\alpha^\beta \mu_A(y)dy}{\beta - \alpha}, \frac{\int_\alpha^\beta \nu_A(y)dy}{\beta - \alpha} \right\rangle \mid x \in \Omega \right\}$$

$W(A)$ is an IF set.

Modification of weight-center operator defined on IF sets A and B over the closed interval

Let $\Omega = [\alpha, \beta]$, $\alpha < \beta$.
Let $B \neq H_{0,0}(X)$ and $B \neq J_{0,0}(X)$. Then

$$W_B(A) = \left\{ \left\langle x, \frac{\int_\alpha^\beta \mu_A(y)dy.\mu_B(x)}{(\beta-\alpha).\int_\alpha^\beta \mu_B(y)dy}, \frac{\int_\alpha^\beta \nu_A(y)dy.\nu_B(x)}{(\beta-\alpha)\int_\alpha^\beta \mu_B(y)dy} \right\rangle \mid x \in \Omega \right\}$$

$W_B(A)$ is not an IF set.

Modification of weight-center operator defined on IF sets A and B over the closed interval

Let $\Omega = [\alpha, \beta]$, $\alpha < \beta$. Then

$$W_B^1(A) = \left\{ \left\langle x, \frac{\int_\alpha^\beta \mu_A(y)dy}{\beta - \alpha}.\mu_B(x), \frac{\int_\alpha^\beta \nu_A(y)dy}{\beta - \alpha}.\nu_B(x) \right\rangle \mid x \in \Omega \right\}$$

$W_B^1(A)$ is an IF set.

Assumptions for next modifications of weight-center operators defined on IF sets *A* and *B* over the closed interval

$$\|X\| = \frac{\int_{\alpha}^{\beta}(\mu_X(y) + \nu_X(y))}{(\beta - \alpha)}$$

Modification of weight-center operator defined on IF sets A and B over the closed interval

Let $\Omega = [\alpha, \beta]$, $\alpha < \beta$.
Let $B \neq H_{0,0}(X)$, $B \neq J_{0,0}(X)$ and $\|A\| \leq \|B\|$. Then

$$W_B^2(A) =$$

$$= \left\{ \left\langle x, \frac{\int_\alpha^\beta \mu_A(y)dy . \mu_B(x)}{\int_\alpha^\beta \mu_B(y)dy + \int_\alpha^\beta \nu_B(y)dy}, \frac{\int_\alpha^\beta \nu_A(y)dy . \nu_B(x)}{\int_\alpha^\beta \mu_B(y)dy + \int_\alpha^\beta \nu_B(y)dy} \right\rangle \right\}$$

$W_B^2(A)$ is an IF set.

Modification of weight-center operator defined on IF sets A and B over the closed interval

Let $\Omega = [\alpha, \beta]$, $\alpha < \beta$.
Let $B \neq H_{0,0}(X)$, $B \neq J_{0,0}(X)$ and $\|A\| \leq \|B\|$. Then

$$W_B^3(A) =$$

$$\left\{\left\langle \frac{\int_\alpha^\beta \mu_A(y)dy.\mu_B(x)}{2.\max(\int_\alpha^\beta \mu_B(y)dy, \int_\alpha^\beta \nu_B(y)dy)}, \frac{\int_\alpha^\beta \nu_A(y)dy.\nu_B(x)}{2.\max(\int_\alpha^\beta \mu_B(y)dy, \int_\alpha^\beta \nu_B(y)dy)} \right.\right.$$

$W_B^3(A)$ is an IF set.

Properties of weight-center operators defined on IF sets A and B over the closed interval

Let $W_B^i(A)$, $i = 1, 2, 3$ be the modifications of weight-center operator with their properties.
Then

$$\begin{aligned} \overline{W_B^i(\overline{A})} &= W_B^i(A) \\ I(W_B^i(A) &= W_B^i(I(A)) \\ C(W_B^i(A)) &= W_B^i(C(A)) \end{aligned}$$

Properties of weight-center operators defined on IF sets *A* and *B* over the closed interval

Let $W^i_B(A)$, $i = 1, 2, 3$ be the modifications of weight-center operator with their properties.
Then

$$\begin{aligned} \Box W^i_B(A) &\subseteq W^i_B(\Box A) \\ \diamond(W^i_B(A) &\supseteq W^i_B(\diamond(A)) \end{aligned}$$

Thank you
for your attention!