

Conclusion

Multivariate modelling methods count doubtlessly among the modern approaches in time series processing, allowing us to capture wider area of causalities that are of our interest. Supported by powerful data collecting technologies such as GPS, we are able to model relations of time space entities, namely the varying geometrical positions of points. They reflect processes that interact locally, regionally and globally. In the thesis, we have presented several known methods and proposed some new methods for multivariate time series modelling, as listed below.

In terms of particular application, we concentrated on permanent GPS observations, that serve for regular monitoring of the Earth's crust kinematics (and for other research purposes). The time series of north and east horizontal coordinates clearly show the long-term drift of Eurasian tectonic plate as linear deterministic trend, which is common to every point in the area. Besides this, the time series visibly reflect other effects (e.g., deterministic seasonality), either as a common or as a unique feature. They can be and partly they were distinguished from each other by the methods of multivariate modelling as proposed in this thesis.

The work on the thesis brought several contributions both to theoretical and practical aspects of time series analysis.

First, the thesis has good informative value due to summarizing the recent methods in multivariate context.

Second, we have proposed new extension to regime-switching models when transition variable is determined by aggregation operators. Thanks to variety of such operators, usefulness of this approach is appreciable as it gives more freedom to fit one's empirical feeling without any loss of reliability. The extension causes no limitation to properties of the nonlinear models so the familiar testing and parameter estimation procedures apply as shown in a standalone section.

Third, a more general formula has been derived for Akaike and Schwarz information criteria used to choose the correct order of VAR in individual regimes. The generalisation allows computer aided modelling to encompass

vector variable, arbitrary transition function and regime varying order of autoregression in single procedure.

Fourth, we extend the (modified) Diebold-Mariano test to multivariate case as - to our best knowledge - no such attempt has been published so far.

Fifth, in experimental part of thesis we have shown that investigating common features can significantly improve forecast accuracy of linear models. Improvement in fit is achieved also in using linear convex combination of two copulas, which can be considered as very easy and effective way of constructing multi-parametric copulas. This approach is brand new to copula theory.

At last, the complete source code is available in appendix for the main modelling routines, that is nonlinear model specification, cointegration and common seasonality detection and transformation, nonparametric and semi-parametric fitting of Archimedean copula compared to more universal nonlinear least squares based fit approach. All the routines are coded in computer algebra system Mathematica, properly supplied with description to help practitioners easily implement their specific ideas. The copula fitting program has already been requested from abroad for application in civil engineering and it contains also estimation of standard deviation of copula parameters. Nevertheless, our prime attention was paid to the largest algorithm encapsulated to function `fConditionalRegimeSwitching`. It focuses on conditional estimation of general STAR model parameters and allows one to set variety of input options according to particular problem, e.g., inclusion of exogenous variables, regime specific AR orders, number of regimes, parameters grid, transition and aggregation functions, number of forecast steps and Monte-Carlo cycles. As output - besides parameters estimation and forecasting - it offers the measures for model selection based on both in-sample (sum of squared residuals, information criteria) and out-of-sample fit (forecast errors).

Basically, the whole work is tried to be elaborated keeping in mind the best possible universality thus allowing wide applicability in practice and convenient implementation into computer languages, for instance either as web-based application accessible on Internet or locally executable programme built from *C* source code export.

Publications

- S1 Bacigál, T.: Multivariate Threshold Autoregressive Models in Geodesy, *Journal of Electrical Engineering* 12/S, vol. 55, 91–94 (2004).
- S2 Bacigál, T.: Modeling points position time series with respect to common trend, *Zborník príspevkov konferencie PRASTAN 2004*, Bratislava, 5–10 (2004).
- S3 Bacigál, T., Komorníková, M.: Modelling points position time series in the light of cointegration, *Proceedings of INGeo 2004* (CD-ROM edition), Bratislava (2004).
- S4 Bacigál, T., et al.: Variance components estimate in 2D geodetic network, *Proceedings of INGeo 2004* (CD-ROM edition), Bratislava (2004).
- S5 Bacigál, T.: Testing for Common Deterministic Trends in Geodetic Data, *Journal of Electrical Engineering* 12/S, vol. 56, 115–118 (2005).
- S6 Bacigál, T.: Modelling Relationship Using Archimedean Copula: An Introduction to Experimental Study, *Proceedings of MAGIA 2005*, Bratislava, 60–72 (2005).
- S7 Bacigál, T.: Fitting Archimedean copulas to bivariate geodetic observations, *Proceedings of APLIMAT 2006*, Bratislava, 519–527 (2006).
- S8 Bacigál, T., Komorníková, M.: Fitting Archimedean copulas to bivariate geodetic data, *Proceedings in Computational Statistics COMPSTAT 2006*, Rome, Physica-Verlag Heidelberg, Springer, 649–656 (2006).
- S9 Bacigál, T.: Multivariate LSTAR in geodesy, *Journal of Electrical Engineering* 12/S, vol. 57, 115–118 (2006).
- S10 Bacigál, T.: Multivariate smooth transition AR model with aggregation operators and application to exchange rates, accepted to *Kybernetika* (2007).

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Appendix

Source code for the main procedures described in the thesis. Fully compatible with Mathematica v5.2 and 'TimeSeries' extra package.

1. Modelling bivariate regime-switching nonlinearity in simulated and observed time series.
2. Transformation according to common trend and seasonality. Forecast performance comparison.
3. Definition of functions utilised in above procedures.
4. Fitting of Archimedean copulas.