R package to handle Archimax or any user-defined continuous copula construction: *acopula*

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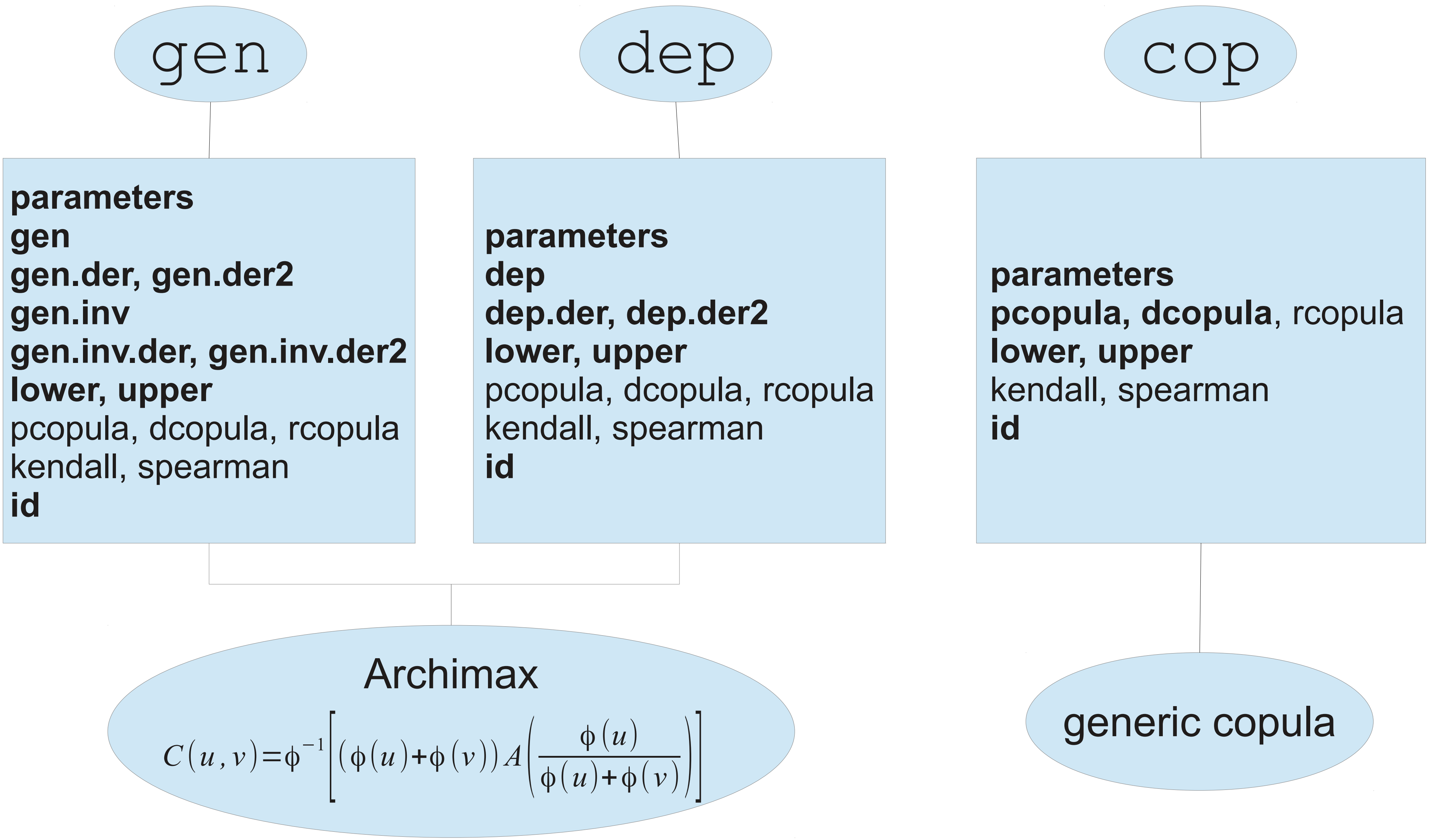
Outline

- alternatives
- Archimax and generic copulae definition lists
- probability functions
- estimation
- testing
- utilities
- conclusion and further plans

Alternatives

- Mathematica 8
- Matlab
- Excel
  - Hoadley Finance Add-in
  - Vose Model Risc
- S-Plus
  - S+ Finmetrics/EVANESCE
- XploRe, ...
- R
  - copula (nacopula)
  - CDVine/VineCopula
  - CopBasic, fCopulae, copulaedas, HAC, fgac, sbgcop, pencopula, penDvine, vines, …
  - acopula

acopula – building blocks



acopula – building blocks

gen

parameters
gen
gen.der, gen.der2
gen.inv
gen.inv.der, gen.inv.der2
lower, upper
pcopula, dcopula, rcopula
kendall, spearman
id

dep

parameters
dep
dep.der, dep.der2
lower, upper
pcopula, dcopula, rcopula
kendall, spearman
id

cop

parameters
pcopula, dcopula, rcopula
lower, upper
kendall, spearman
id

Archimax

$$C(u,v)=\phi^{-1}\left[(\phi(u)+\phi(v))\,A\left(\frac{\phi(u)}{\phi(u)+\phi(v)}\right)\right]$$

generic copula

acopula – building blocks

generator

- Ali-Mikhail-Haq
- Clayton
- Frank
- Gumbel-Hougaard
- Joe
- log

dependence function

- Galambos
- Gumbel-Hougaard
- Husler-Reiss
- Tawn
- 1, max
- (general) convex combination

copula

- Farlie-Gumbel-Morgenstern
- Gumbel-Hougaard
- Normal
- Plackett
- product

```
> pCopula(data=c(0.2,0.3),generator=genGumbel(),gpars=3.5)
> pCopula(data=c(0.2,0.3),copula=copGumbel(),pars=3.5)
> pCopula(data=c(0.2,0.3),gen=genLog(),depfun=depGumbel(),dpars=3.5)
[1] 0.1723903
```

Probability functions

```
> gG <- genGumbel(parameters=3.5)
```

- $P(X<x \,\&\, Y<y) = ?$

```
> pCopula(c(x,y),gG)
```

- $P(X<x \mid Y=y) = ?$

```
> cCopula(c(x,y),con=2,gG)
```

- $P(X<? \,\&\, Y<y) = p$

```
> pCopula(c(p,y),gG,qua=1)
> pCopula(c(NA,y),gG,qua=1,
+ prob=p)
> qCopula(y,qua=1,prob=p,gG)
```

- $P(X<? \mid Y=y) = p$

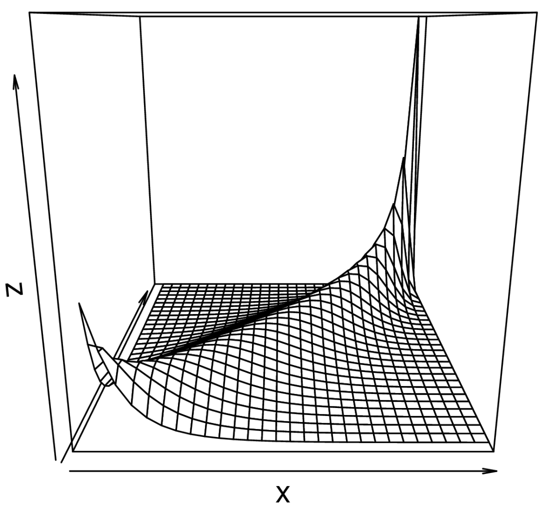
```
> cCopula(c(p,y),con=2,gG,qua=1)
> pCopula(c(NA,y),con=2,gG,qua=1,
+ prob=p)
> qCopula(y,qua=1,prob=p,con=2,
+ gG)
```

Probability functions

```
> gG <- genGumbel(parameters=3.5)
```

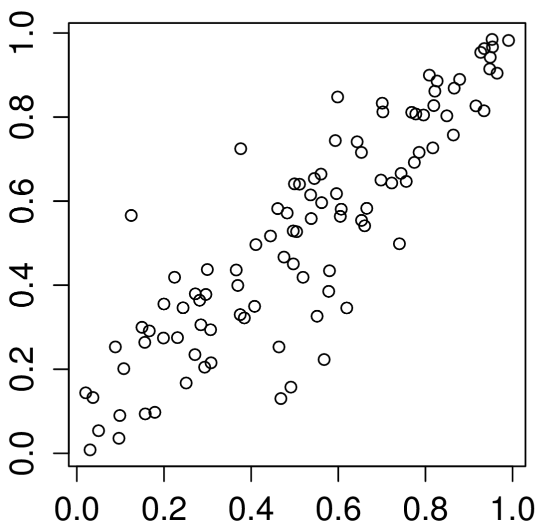
- density

```
> dCopula(c(x,y),gG)
```



- random sampling

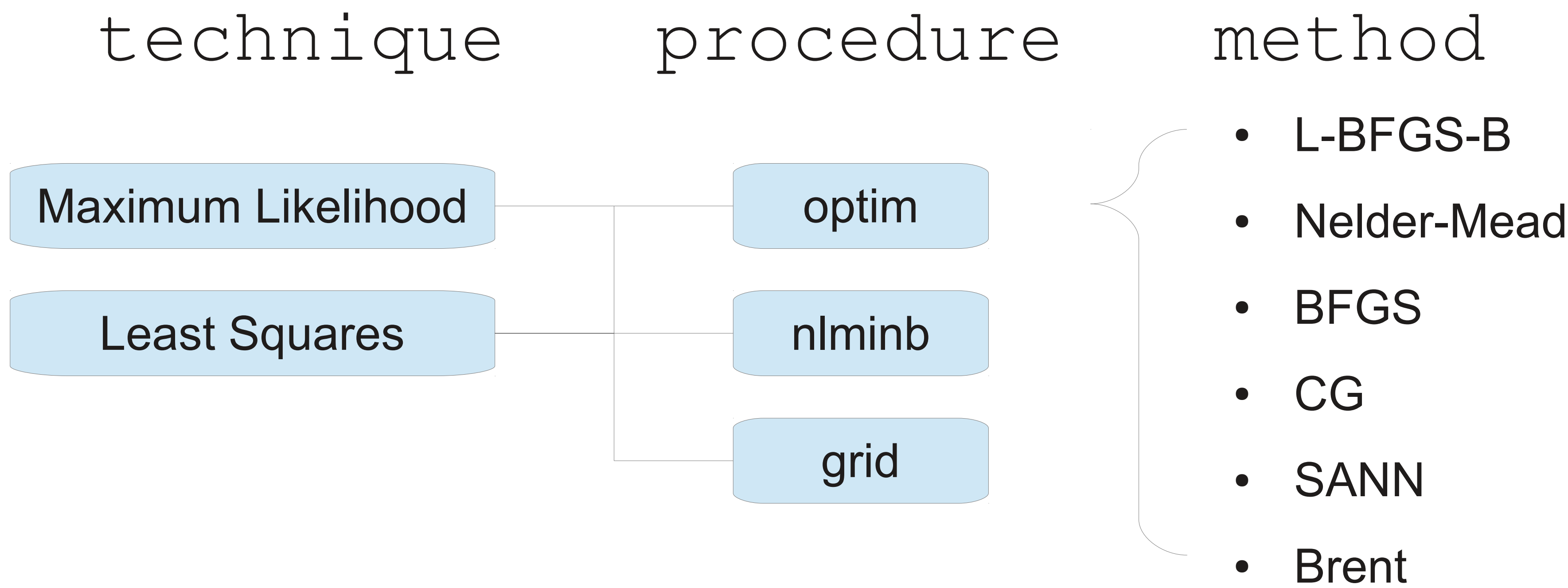
```
> rCopula(100,dim=2,gG)
```



- empirical copula CDF

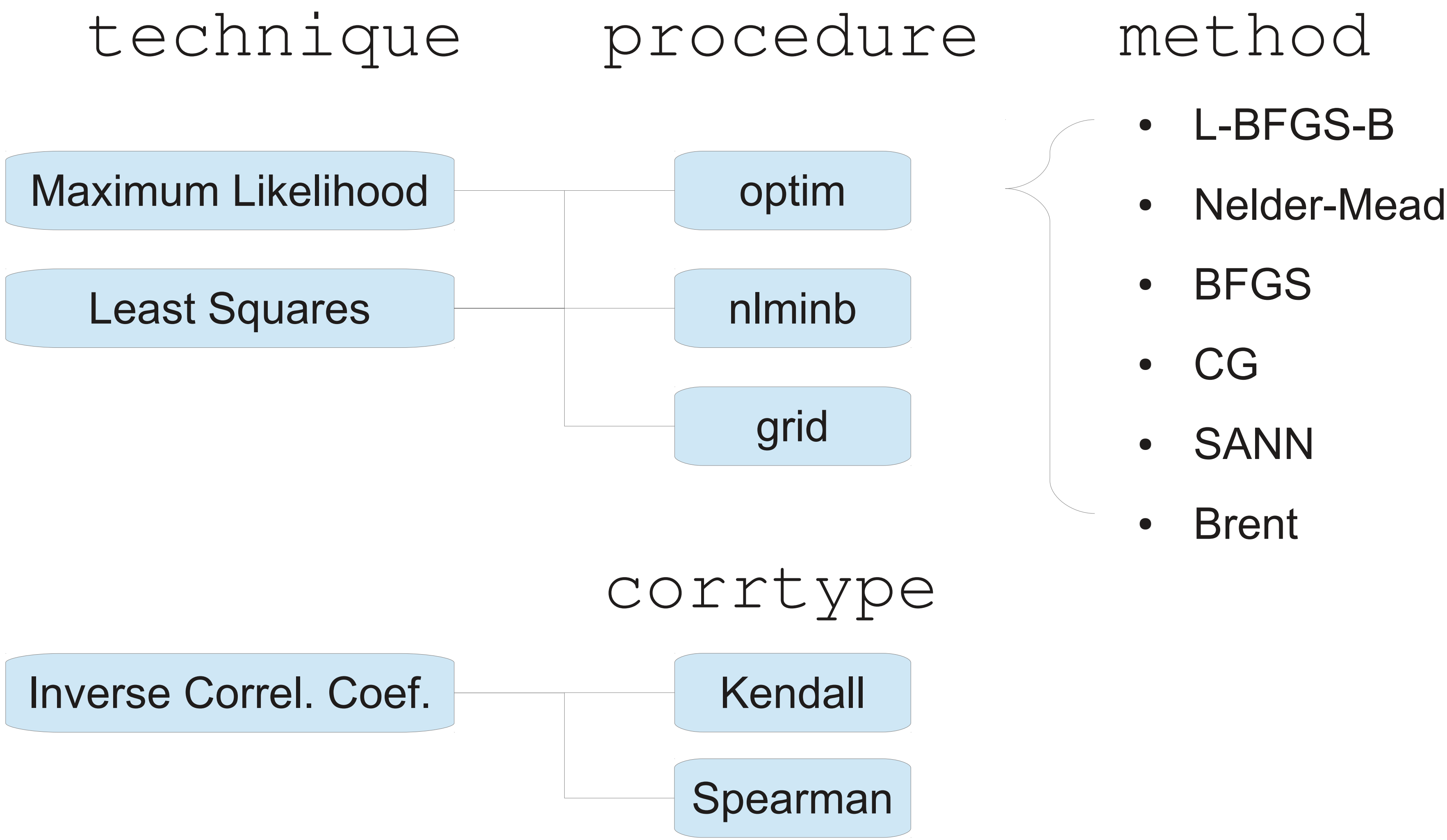
```
> pCopulaEmpirical(c(x,y),base=sample)
```

Estimation




```
> eCopula(sample,gen=genClayton(),dep=depGumbel(),
+ technique="ML",procedure="optim",method="L-BFGS-B")
generator parameters: 0.09357958
depfun parameters: 3.52958
ML function value: 82.63223
convergence code: 0
```

Estimation



Note: Not all copula families has closed-form relation between its parameters and a correlation coefficient, yet it can be approximated by, e.g., 4-parametric Parreto CDF which is easily invertible → max. approx. error at worst ± 0.01 mostly ± 0.001

Estimation

- Kendall's tau

$\tau = 4 \iint_{[0,1]x[0,1]} C(u,v)c(u,v)\, dudv – 1$

- Spearman's rho

$\rho = 12 \iint_{[0,1]x[0,1]} C(u,v)\, dudv – 3$

Estimation

- Kendall's tau

$\tau = 4 \iint_{[0,1]x[0,1]} C(u,v)c(u,v)\, dudv - 1$

  - Gumbel $\tau = (\theta - 1) / \theta$
  - Clayton $\tau = \theta / (\theta + 2)$
  - FGM $\tau = 2\,\theta / 9$

Not many have a closed form, but the relation can be approximated.

Estimation

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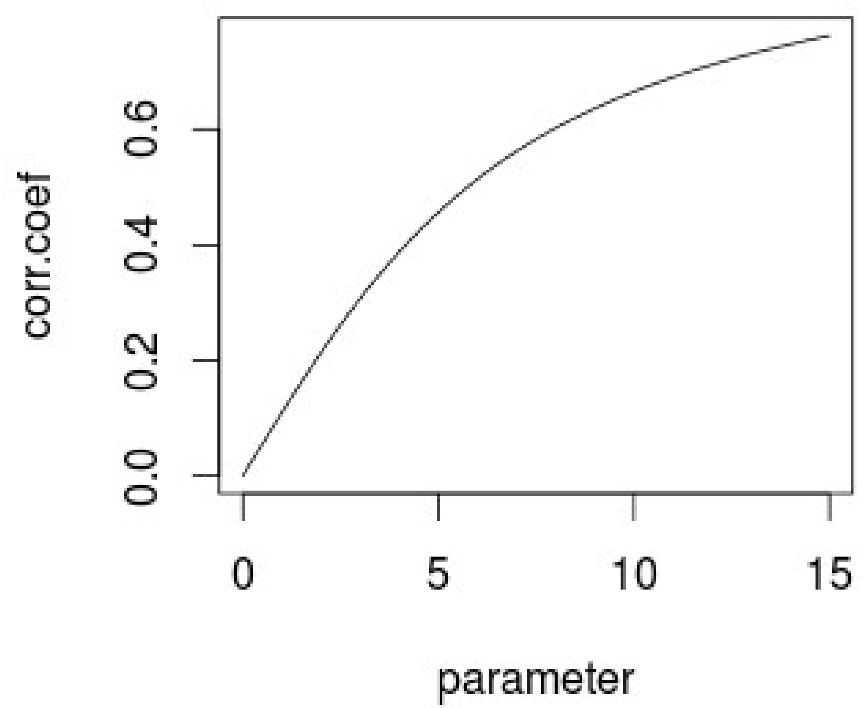
Not many have a closed form, but the relation can be approximated. For example

  - Frank



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Not many have a closed form, but the relation can be approximated. For example

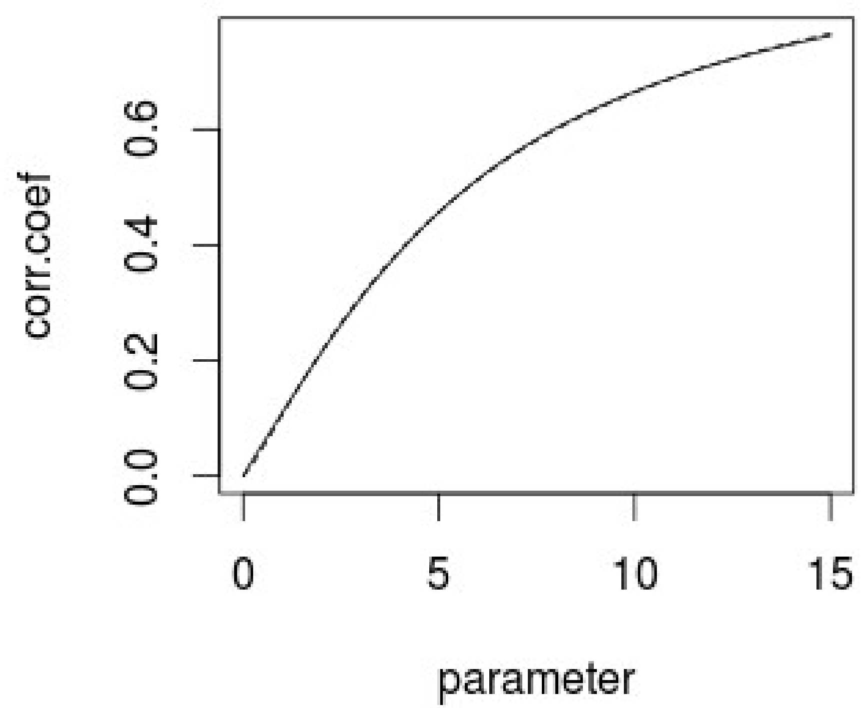
- Frank $\tau = CDF(\theta)$

  $k = 6.1$, $\alpha = 1.1$, $\gamma = 0.8$,

  $\mu = 0$

Pareto type IV distribution

$$CDF(x) = \begin{cases} 1 - \left(1 + \left(\frac{x-\mu}{k}\right)^{\frac{1}{\gamma}}\right)^{-\alpha} & x \geqslant \mu \\ 0 & \text{otherwise} \end{cases}$$



Estimation

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$\tau = 4 \iint_{[0,1]x[0,1]} C(u,v)c(u,v)\, dudv – 1$

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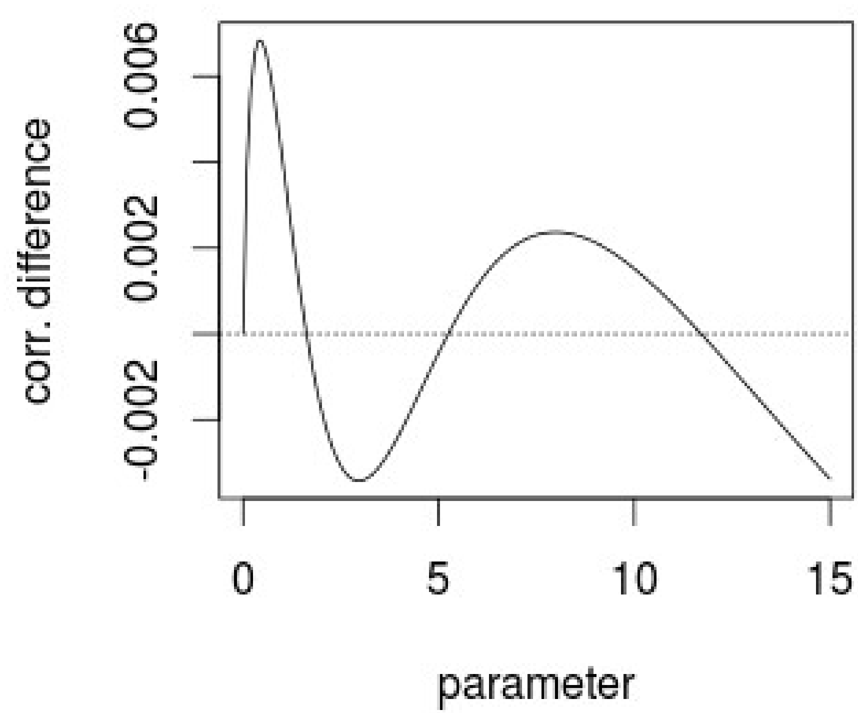

Testing

- goodness-of-fit

```
> gCopula(sample,cop=copNormal(),
+ etechnique="ML",eprocedure="optim",ncores=1,N=100)
|============================================| 100%
Blanket GOF test based on Kendall's transform
statistic       q95    p.value
0.1195500 0.1658125 0.1800000
-----------------------------
data: sample
copula: normal
estimates:
      pars     fvalue
 0.9155766 80.3420886
```

Testing

- goodness-of-fit
- equality of 2 copulas

```
> sampleCl <- rCopula(n=100,dim=2,gen=genClayton(),gpars=5)
> gCopula(list(sample,sampleCl),ncores=1,N=100)
|==================================================| 100%
Test of equality between 2 empirical copulas
 statistic        q95     p.value
0.09791672 0.52893392 0.66000000
-----------------------------
data: sample sampleCl
copula:
estimates:
NULL
```

Testing

- goodness-of-fit
- equality of 2 copulas
- validity of copula properties
  - monotonicity, annihilator and neutral element

```
> isCopula(gen=genGumbel(lower=0),dim=3,glimits=list(0.5,2),
+ dagrid=10,pgrid=4,tolerance=1e-15)
 Does the object appears to be a copula(?):  FALSE
 Showing 2 of 2 issues:
  dim property      value gpar
1   2    monot -0.1534827  0.5
2   3    monot -0.1402209  0.5
```

Utilties

- numerical derivative

```
> fun <- function(x,y,z) x^2*y*exp(z)
> nderive(fun,point=c(0.2,1.3,0),order=c(2,0,1),
+ difference=1e-04,area=0)
[1] 2.600004
```

Utilties

- numerical derivative
- numerical integration

```
> nintegrate(function(x,y) mvtnorm::dmvnorm(c(x,y)),
+ lower=c(-5.,-5.),upper=c(0.5,1),subdivisions=30)
[1] 0.5807843
>#Better alternative:
> cubature::adaptIntegrate(mvtnorm::dmvnorm,
+ lowerLimit=c(-5.,-5.), upperLimit=c(0.5,1))$integral
[1] 0.5817578
> pnorm(0.5)*pnorm(1) #exact solution due to independence
[1] 0.5817583
```

Conclusion

- arbitrary dimension (with `nderive` for dim>2)
- conditional probability and quantile function
- generalization of Archimedean and EV class
- construction method of Pickand's dependence function
- test of equality between two empirical copulas
- numerical check of copula properties
- parallelized goodness-of-fit test based on Kendall's transform
- estimation based on inversion of correlation coefficients available for every 1-parameter copula family

Further plans

- non-parametric estimation for multi-parameter families
- new construction methods
- more GoF test methods
- more copulas definition lists
- connection to other copula packages
- procedures for practical analysis
- bug fixing

Thank you for attention and new ideas.